OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Tuesday 10 January 2023 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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8	
9	
10	
11	
12	
TOTAL	

Answer **all** questions in the spaces provided.

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1 (a) The function f is defined by $\mathrm{f}(x) = 3^x$ for all real values of x

Show that

$$\mathrm{f}(x+1) - \mathrm{f}(x-2) = k\mathrm{f}(x)$$

where k is a constant.

[3 marks]

Do not write outside the box

1 (b) The function g is defined by

$$g(x) = \frac{3-x}{5+2x} \quad \text{for all real values of } x, \quad x \neq -2.5$$

The inverse of g is g^{-1}

1 (b) (i) Find $g^{-1}(x)$

[3 marks]

Answer _____

1 (b) (ii) State the range of g^{-1}

[1 mark]

Answer _____

7

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2 (a) Express $8\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α to the nearest degree.

[3 marks]

Answer _____

2 (b) (i) Hence write down the minimum value of

$$\left| 8\cos\theta + 15\sin\theta \right|$$

[1 mark]

Answer _____

2 (b) (ii) Hence find a value of θ at which the minimum value in **part (b)(i)** occurs, giving your answer to the nearest degree.

[1 mark]

Answer _____

Do not write outside the box

2 (c) Use your answer to **part (a)** to solve the equation

$$8\text{cosec}\left(2y+10^\circ\right)+15\sec\left(2y+10^\circ\right)=8.5\tan\left(2y+10^\circ\right)+8.5\cot\left(2y+10^\circ\right)$$

giving all solutions to the nearest degree in the interval $-180^\circ < y < 180^\circ$

[5 marks]

Answer _____

10

Turn over ►

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3 The polynomial $f(x)$ is defined by

$$f(x) = 16x^3 + bx^2 + cx$$

where b and c are constants.

When $f(x)$ is divided by $(2x+3)$ the remainder is -45

When $f(x)$ is divided by $(4x-5)$ the remainder is 10

3 (a) (i) Find the value of b and the value of c

[4 marks]

$b =$ _____ $c =$ _____

Do not write outside the box

3 (a) (ii) Express $\mathrm{f}(x)$ as a product of three linear factors.

[2 marks]

Answer _____

3 (b) Hence express $\dfrac{\mathrm{f}(x)}{16x^2 - 9}$ in the form

$$px + q + \frac{r}{mx + n}$$

where p, q, r, m and n are constants.

[3 marks]

Answer _____

9

Turn over ►

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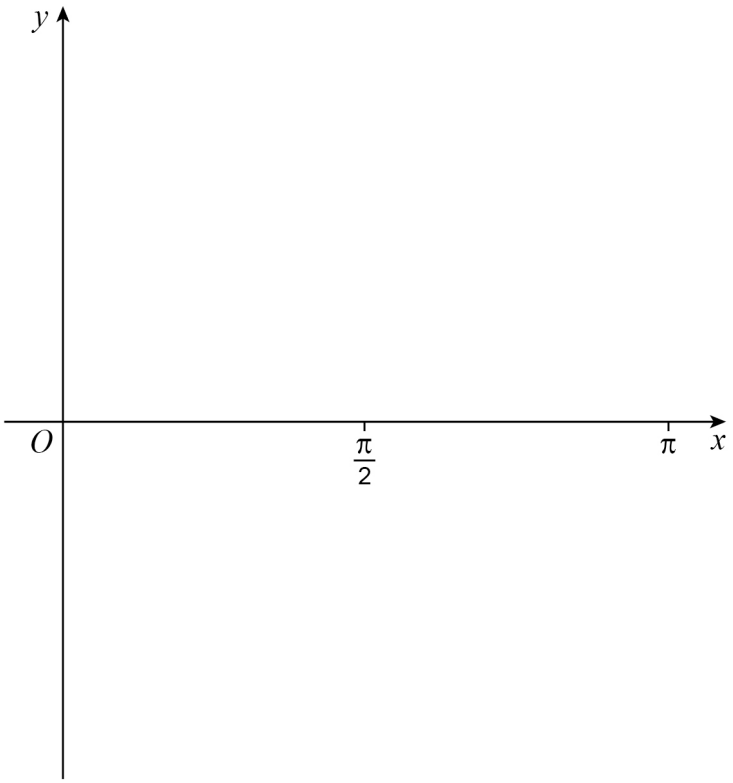
4 (a) Sketch on the axes below the graph of the curve that has equation

$$y = \sec x \quad \text{for} \quad 0 \leq x \leq \pi$$

[2 marks]



4 (b) A curve has equation $y = \sec x$ where $0 \leq x \leq \frac{\pi}{4}$

This curve intersects the line $y = 10x - 5$ at a single point where $x = \alpha$

4 (b) (i) Show that α lies between 0.6 and 0.7

[2 marks]

Do not write outside the box

4 (b) (ii) The equation $\sec x = 10x - 5$ can be rearranged into the form

$$x = 0.1\sec x + 0.5$$

Use the iterative formula

$$x_{n+1} = 0.1\sec x_n + 0.5$$

with $x_1 = 0.6$ to find the values of x_2 and x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

4 (c) Use the mid-ordinate rule with five strips to find an estimate for

$$\int_{0.6}^{0.7} \sec x \, dx$$

Give your answer to **six** decimal places.

[4 marks]

Answer _____

10

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5 (a) Show that the binomial expansion of $(1-px)^{-\frac{1}{2}}$ up to and including the term in x^3 where p is a constant is

$$1+\frac{1}{2}px+\frac{3}{8}p^2x^2+\frac{5}{16}p^3x^3$$

[2 marks]

5 (b) Find the binomial expansion of $\sqrt{(4+px)}$ up to and including the term in x^3

Give all numerical coefficients as simplified fractions.

[3 marks]

Answer _____

Do not write outside the box

5 (c) For particular values of p the binomial expansion up to and including the term in x^3 gives

$$\frac{3}{4}px + \sqrt{(4+px)} - 2(1-px)^{-\frac{1}{2}} = -x^2 + qx^3$$

where q is rational.

5 (c) (i) Show that $p = \pm\frac{8}{7}$

[4 marks]

5 (c) (ii) Hence find the exact values of q

[2 marks]

Answer _____

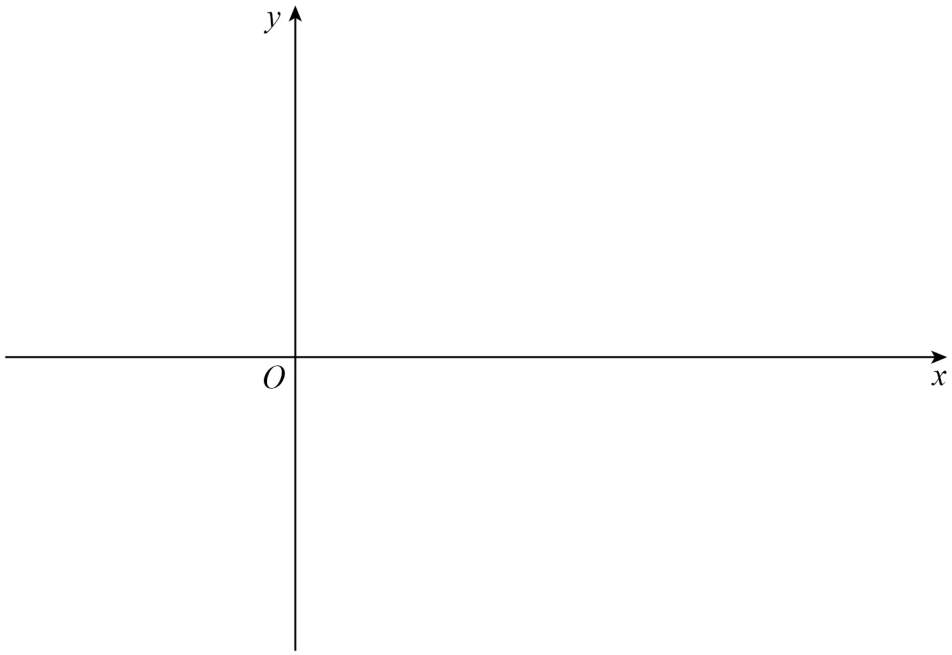
11

Do not write outside the box

6 A curve C has equation $y = \left| e^{0.5x} - 4 \right|$

6 (a) Sketch on the axes below the graph of C indicating the coordinates of any points where the curve meets or cuts the axes.

[3 marks]



6 (b) Find an equation of the normal to C at the point P where $x = \ln 25$

Give your answer in the form $ax + by = k$ where a, b and k are constants.

[5 marks]

Do not write outside the box

Answer _____

6 (c) The normal to C at the point P intersects the coordinate axes at A and B

Find the exact value of the finite area OAB

Give your answer in the form $p(q+\ln r)^2$ where p, q and r are constants.

[3 marks]

Answer _____

11

Do not write outside the box

7 The coordinates of the points A and B are $(1, 5, -3)$ and $(-2, 3, 4)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -1 \\ c \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$ where c is a constant.

7 (a) (i) Find the vector $\overrightarrow{AB}$

[1 mark]

Answer _____

7 (a) (ii) Find $\left|\overrightarrow{AB}\right|$

[2 marks]

Answer _____

7 (a) (iii) Calculate the acute angle between $\overrightarrow{AB}$ and the line l, giving your answer to the nearest 0.1°

[3 marks]

Answer _____

Do not write outside the box

7 (a) (iv) The line AB intersects the line l

Find the value of c

[3 marks]

Answer _____

7 (b) (i) Find the shortest distance from l to the origin.

[4 marks]

Answer _____

7 (b) (ii) Explain which of the line l or the line AB, is nearest to the origin.

[2 marks]

15

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8 A curve has equation $x + y = (x - 2y)^2$

Find the equation of the tangent to the curve at (2, 2)

Give your answer in the form $y = mx + c$ where m and c are rational numbers.

[6 marks]

Answer _____

6

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9 (a) Describe a single geometrical transformation that maps the graph of $y = \ln x$ onto the graph of $y = \ln(2x)$

[2 marks]

9 (b) The region bounded by the curve $y = \ln(2x)$, the line $x = 4$ and the x-axis from $x = 0.5$ to $x = 4$ is rotated through 2π radians about the x-axis to form a solid.

Find the value of the volume of the solid generated, giving your answer in an exact form.

[9 marks]

Answer _____

11

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10 It is given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = b(3a - 2y)(a - y)$$

where a and b are positive constants.

Solve the differential equation such that $y = 0$ when $x = 0$

Give your answer in the form $y = \mathrm{f}(x)$

[10 marks]

Do not write outside the box

Answer _____

10

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11 (a) Find $\int 4\cos^2\theta \, \mathrm{d}\theta$

[2 marks]

Answer _____

11 (b) Use the substitution $t = \sin x$ to find

$$\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{3+\cos^2 x} \, \mathrm{d}x$$

giving your answer in the form $\ln\left(\frac{a}{b}\right)$ where a and b are integers.

[7 marks]

Do not write outside the box

Answer _____

9

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12 A curve C is defined by the parametric equations

$$x = 2\cos\theta \quad \text{and} \quad y = 3\sin\theta \quad \text{for} \quad 0 \leq \theta \leq 2\pi$$

12 (a) Find a Cartesian equation of C

[2 marks]

Answer _____

12 (b) Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{6}$

Give your answer in the form $y + ax + b = 0$ where a and b are constants.

$$\left[\text{You are given } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \text{ and } \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\right]$$

[4 marks]

Answer _____

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12 (c) Show that C intersects the curve with equation $xy = k^2$ where k is a positive real constant in four distinct points only if $k^2 < 3$

[5 marks]

11

END OF QUESTIONS

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