OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA05) Unit M2 Mechanics

Thursday 19 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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ANSWER IN THE SPACES PROVIDED

Answer **all** questions in the spaces provided.

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1 A force **F** newtons acts on a body of mass 0.5 kg.

The position vector **r** metres of the body at time t seconds is given by

$$\mathbf{r} = \begin{bmatrix} 3t^2 \\ \cos(4t) \\ e^{t^2} \end{bmatrix}$$

Find **F** in terms of t

[5 marks]

Answer _____

5

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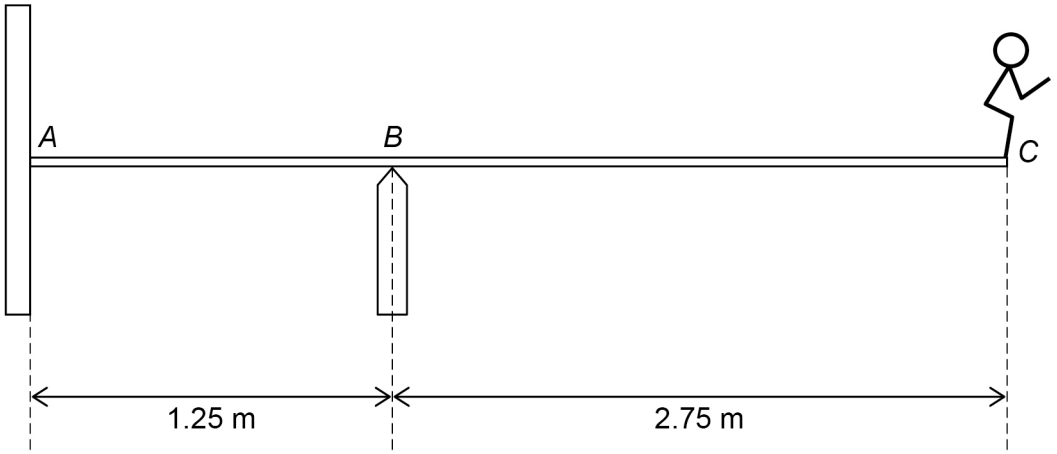
2 A person stands at the end of a uniform horizontal diving board AC, as shown below.



The diving board is attached to a vertical wall at point A and rests on a support at point B

The mass of the person is 60 kg.

The mass of the uniform diving board is 25 kg.

The distance AB is 1.25 metres.

The distance BC is 2.75 metres.

The diving board is in equilibrium.

2 (a) Find the magnitude of the normal reaction force acting on the diving board at B

[3 marks]

Answer _____

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2 (b) Find the force acting on the diving board at A, giving its magnitude and direction.

[3 marks]

Magnitude _____

Direction _____

6

Turn over for the next question

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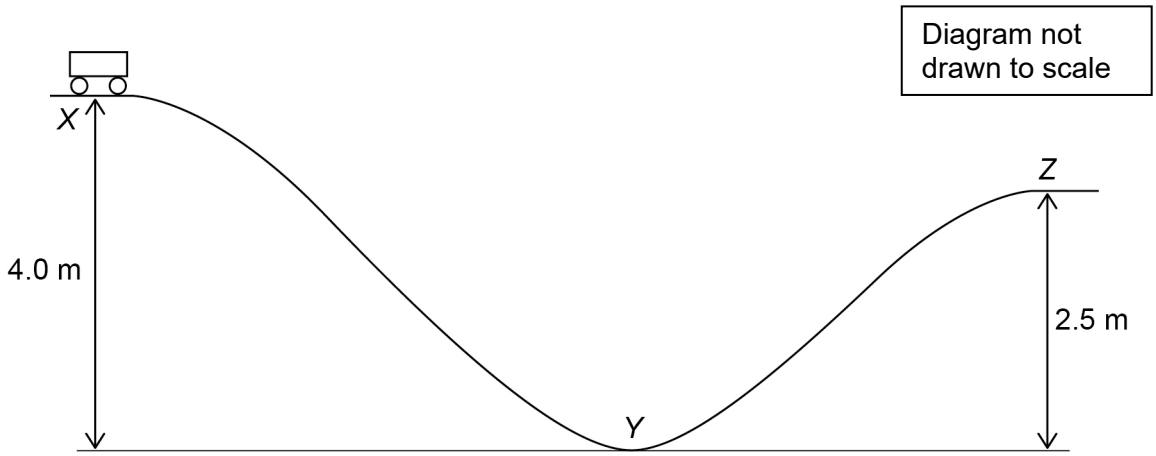
3 A motorless cart starts from rest at the point X and moves along a curved track through the point Y before coming to rest at the point Z, as shown below.



The mass of the cart is 16 kg.

The point X is 4.0 metres vertically above the level of Y

The point Z is 2.5 metres vertically above the level of Y

The distance travelled by the cart along the track between X and Y is 30 metres.

The distance travelled by the cart along the track between Y and Z is 18 metres.

Assume that throughout the motion a constant resistance force of magnitude F newtons acts on the cart.

3 (a) Find the value of F

[4 marks]

Answer _____

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3 (b) Find the speed of the cart at *Y*

[3 marks]

Answer _____

7

Turn over for the next question

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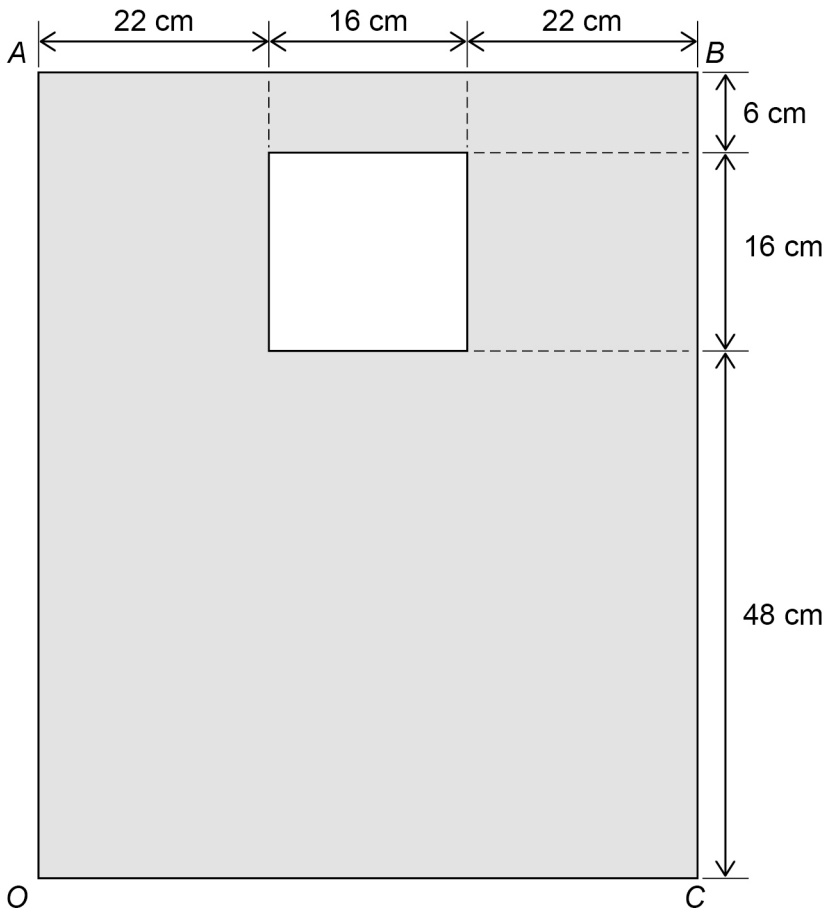
4 The diagram below shows the dimensions of a rectangular uniform lamina $OABC$ in which a square hole has been cut.



4 (a) State, with a reason, the distance of the centre of mass of the lamina from OA

[2 marks]

4 (b) Find the distance of the centre of mass of the lamina from OC giving your answer to three significant figures.

[3 marks]

Do not write outside the box

Answer _____

4 (c) The lamina hangs in equilibrium, freely suspended from B

Find the angle between OB and the vertical, giving your answer to the nearest 0.1°

[4 marks]

Answer _____

9

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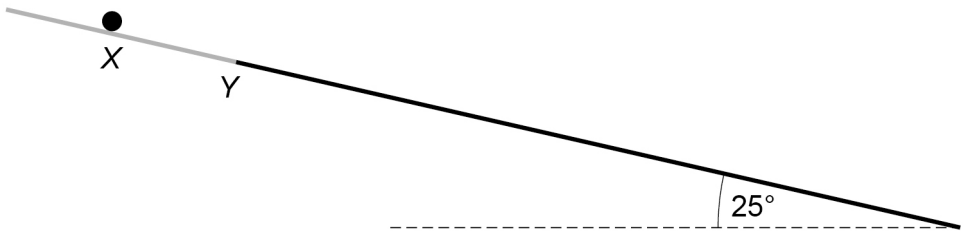
5 A particle of mass 6 kg is released from rest at the point X on a long slope. The particle begins to slide down the slope.

The slope is inclined at 25° to the horizontal as shown below.



The slope is smooth between X and the point Y where the distance $XY = 10$ metres.

The slope is rough from Y downwards. The coefficient of friction between the particle and this part of the slope is 0.5

5 (a) (i) Find the magnitude of the particle's acceleration between X and Y

[1 mark]

Answer _____

5 (a) (ii) Show that the speed of the particle at Y is 9.1 m s^{-1} correct to two significant figures.

[2 marks]

5 (a) (iii) State, with a reason, how the speed given in **part (a)(ii)** would change if the slope was inclined at an acute angle greater than 25° to the horizontal.

[2 marks]

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5 (b) The particle comes to rest at a point Z on the slope.

Find the distance XZ

[6 marks]

Answer _____

5 (c) The slope is now adjusted so that it is inclined at α degrees to the horizontal.

The particle is again released from rest at X and begins to slide down the slope.

This time the particle **does not** come to rest on the slope.

Find the smallest possible value of α, giving your answer to one decimal place.

[2 marks]

Answer _____

13

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6 The acceleration $\mathbf{a}$ m s^{-2} of a particle at time t seconds is given by

$$\mathbf{a} = \mathrm{e}^{-2t}\mathbf{i} + \frac{1}{(1+t)^2}\mathbf{j} \qquad \text{for } t \geq 0$$

where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.

6 (a) The particle is initially at rest.

Find the velocity of the particle in terms of t

[4 marks]

Answer _____

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6 (b) The particle's initial position is $3\mathbf{i} + 2\mathbf{j}$ relative to a fixed origin O

Find the distance of the particle from O when $t = 5$

[5 marks]

Answer _____

9

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7 A particle is projected from a point O on horizontal ground with a speed of 15 m s^{-1} at an angle α above the horizontal, where $0° < \alpha < 90°$

The particle passes through the point P which has a horizontal displacement of 12 metres from O and a vertical displacement of 5 metres from O

Find the maximum possible height that the particle can reach, giving your answer to three significant figures.

[9 marks]

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Answer _____

9

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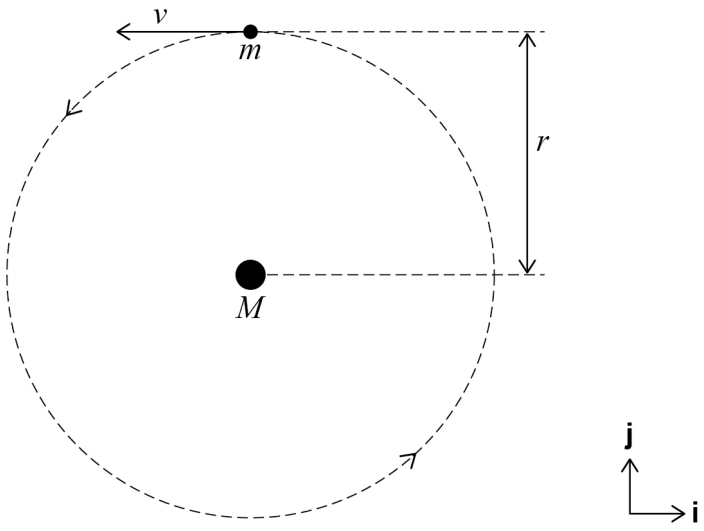
8 A satellite of mass m kg moves in a circular orbit of radius r metres with constant speed v m s^{-1} around a planet of mass M kg

The initial position of the satellite is shown in the diagram.



The unit vectors **i** and **j** are directed as shown in the diagram.

The only force which acts on the satellite is always directed towards the centre of the planet. This force has magnitude F newtons such that

$$F = \frac{GMm}{r^2}$$

where G m^3 kg^{-1} s^{-2} is a constant.

The time for the satellite to make one revolution around its orbit is T seconds.

8 (a) Show that

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

[4 marks]

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8 (b) It is given that $r = 4.2 \times 10^7$ and $M = 6.0 \times 10^{24}$

8 (b) (i) Find the angular speed of the satellite.

Take the value of G to be 6.7×10^{-11}

[3 marks]

Answer _____

8 (b) (ii) The initial position vector of the satellite relative to the planet is $4.2 \times 10^7\,\mathbf{j}$ metres.

Find the velocity of the satellite t seconds after leaving its initial position.

Give your answer in the form $a\mathbf{i} + b\mathbf{j}$

[3 marks]

Answer _____

10

Turn over ►

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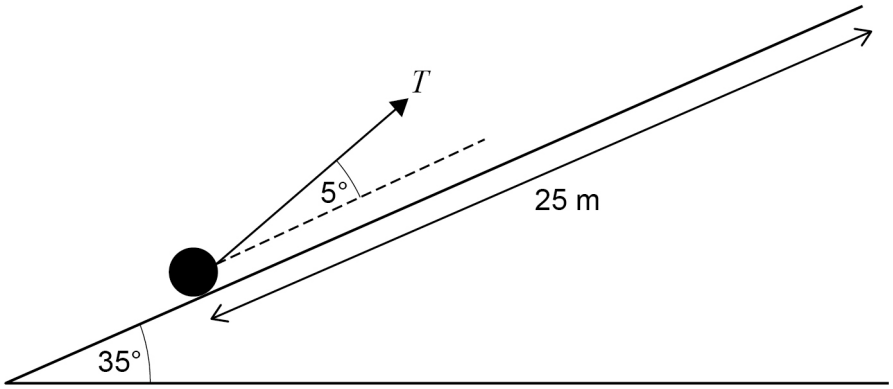
9 A particle of mass 15 kg is pulled a distance of 25 metres up a rough inclined plane with constant speed 4 m s^{-1} by a force of magnitude T newtons which acts at 5° above the inclined plane.

The inclined plane is at 35° to the horizontal and the particle moves along a line of greatest slope, as shown below.



The coefficient of friction between the particle and inclined plane is 0.25

9 (a) State, with **two** reasons, whether the particle is in equilibrium.

[3 marks]

Reason 1

Reason 2

9 (b) Find the value of T giving your answer to three significant figures.

[6 marks]

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Answer

9 (c) (i) Show that the work done by the force of magnitude T newtons in pulling the particle 25 metres up the inclined plane is 2800 J, correct to three significant figures.

[1 mark]

9 (c) (ii) Find the rate at which the force T newtons does work on the particle.

[2 marks]

Answer

12

END OF QUESTIONS

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