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Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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I declare this is my own work.

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 16 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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9	
TOTAL	



J A N 2 3 F M 0 4 0 1

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FM04

1 A random sample of size 10 is taken from a normal distribution with population standard deviation σ

$$H_1: \sigma \neq \sigma_0$$

The test conclusion is that there is no reason to reject the null hypothesis.

[4 marks]

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1 (b) Find the largest possible value for σ_0

Give your answer correct to three significant figures.

[2 marks]

Answer _____

6

Turn over for the next question

Turn over ►



- 2** A bag contains 5 cent and 50 cent coins only.
It is given that 70% of the coins in the bag are 5 cent coins.

A coin is selected at random and its value recorded.

The coin is returned to the bag and this process is repeated a further two times.

- 2 (a)** Complete the sampling distribution table for V the total value (in cents) of the **three** coins selected.

[3 marks]

Total value v (cents)	15			150
$P(V = v)$	0.343		0.189	

- 2 (b)** By using your sampling distribution table in **part (a)** or otherwise

- 2 (b) (i)** Complete the sampling distribution table of the modal value M of a coin in the sample.

[2 marks]

Modal value m (cents)	5	50
$P(M = m)$		



2 (b) (ii) Determine $\text{Var}(M)$ giving your answer correct to three significant figures.

[3 marks]

Answer _____

8

Turn over for the next question

Turn over ►



[illegible]

- 3 (c)** The company target is for a mean conference call time of 30 minutes.

State with a reason whether or not Raj's sample provides evidence that the company target has been met.

[2 marks]

- 3 (d)** Another business analyst thinks Raj's statistical test is invalid as the sample size is too small.

Identify **two** features of the distribution that justify Raj's calculation.

[2 marks]

8

Turn over for the next question

Turn over ►



$$M_Z(t) = e^{\frac{1}{2}t^2}$$

[5 marks]

[illegible]

- 4 (b) The random variable X is given by

$$X = a + bZ$$

where a and b are real numbers.

Determine the moment generating function $M_X(t)$ giving your answer in the form $e^{f(t)}$

[2 marks]

Answer _____

- 4 (c) State $E(X)$ and $\text{Var}(X)$ in terms of a and b

[1 mark]

$E(X) =$ _____ $\text{Var}(X) =$ _____

- 4 (d) Use your results in **parts (b) and (c)** to find the moment generating function $M_Y(t)$ for

$$Y \sim N(\mu, \sigma^2)$$

giving your answer in terms of μ , σ and t

[2 marks]

Answer _____



- 5** Two independent random variables X and Y have Poisson distributions with parameters λ and 2λ respectively.

Samples of n observations are taken from each population and sample means $\bar{X}$ and $\bar{Y}$ are determined.

Two estimators S and T for λ are proposed where

$$S = \frac{\bar{X} + \bar{Y}}{3} \quad \text{and} \quad T = \bar{Y} - \bar{X}$$

- 5 (a)** Show that S and T are unbiased estimators of λ

[3 marks]

- 5 (b)** Show that the relative efficiency of S to T is independent of the sample size n

[5 marks]



[2 marks]

[illegible]

10

Turn over ►



$$f(t) = \begin{cases} -\frac{\pi}{200} \sin\left(\frac{\pi t}{100}\right) & 100 \leq t \leq 200 \\ 0 & \text{otherwise} \end{cases}$$
$$F(t) = \begin{cases} 0 & t < 100 \\ \frac{1}{2} \cos\left(\frac{\pi t}{100}\right) + \frac{1}{2} & 100 \leq t \leq 200 \\ 1 & t > 200 \end{cases}$$
[illegible]

Interval (milliseconds)	100–120	120–140	140–160	160–180	180–200
Number of sprints	145	390	561	470	154



- [3 marks]**

Interval (milliseconds)	100–120	120–140	140–160	160–180	180–200
Expected number of sprints	164	430			

- [7 marks]**

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- 7** A researcher is undertaking a study comparing the lengths of two species of penguins: Galapagos penguins and Fairy penguins.

The researcher collects data from two independent random samples, one for each species of penguin.

The table below shows the summary of her results.

Penguin species	Sample size	Sample mean length (cm)
Galapagos	60	53.4
Fairy	80	45.0

The distributions of lengths for each species of penguin are assumed to come from populations that are normally distributed.

The standard deviation for Galapagos penguins is 6 cm

The standard deviation for Fairy penguins is 4 cm

The researcher believes that the population mean length of Galapagos penguins is 10 cm longer than the population mean length of Fairy penguins.

- 7 (a) (i)** The researcher begins to test her belief by using the hypotheses

$$H_0: \mu_G - \mu_F = 10$$

$$H_1: \mu_G - \mu_F \neq 10$$

where μ_G is the population mean length for the Galapagos penguins
and μ_F is the population mean length for the Fairy penguins.

State whether the researcher is performing a one-tailed test or a two-tailed test.

[1 mark]



[6 marks]

[illegible]

[1 mark]

A test is used to investigate the population mean of a normal distribution with standard deviation 10

The hypotheses are

$$H_1: \mu > 100$$

It is given that $P(\text{Type I error}) > P(\text{Type II error})$

[5 marks]

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- 9** A new version of a computer operating system is designed to reduce the start-up times of the system. The operating system is tested on 10 computers.

The table below shows the start-up times in seconds of the old version and the new version of the operating system, with the exception of one missing value a

Operating system	Computer start-up time (seconds)									
	1	2	3	4	5	6	7	8	9	10
Old version	52.3	29.2	113.5	92.6	37.5	95.7	47.8	130.1	62.3	72.8
New version	50.1	37.3	a	86.0	35.0	88.1	47.9	134.1	58.1	74.0

A test is conducted for evidence that the start-up times with the new version have reduced.

- 9 (a)** Show that the test statistic to be used has value

$$\frac{\sqrt{10} (0.1a - 12.32)}{\sqrt{0.1a^2 - 22.48a + 1286}}$$

where numbers shown in the denominator are given correct to **four** significant figures.

[6 marks]



Test at the 10% level of significance whether there is evidence that the start-up times with the new version of the operating system have reduced.

[illegible]

12



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2 4



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