

INTERNATIONAL A-LEVEL MATHEMATICS

MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

January 2023

Version: 1.0 Final



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Key to mark scheme abbreviations

м	Mark is for method
m	Mark is dependent on one or more M marks and is for method
Α	Mark is dependent on M or m marks and is for accuracy
В	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
\checkmark or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
– <i>x</i> EE	Deduct <i>x</i> marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\left[f(x+1)-f(x-2)=\right] 3^{x+1}-3^{x-2}$	B1	
	$=3^{x}(3-3^{-2})$	M1	oe , e.g. $3^{x-2}(3^3 - 1)$ Factorises
	$=\frac{26}{9}f(x)$	A1	Correct simplified value of k
		3	

Q	Answer	Marks	Comments
1(b)(i)	$x = \frac{3 - y}{5 + 2y}$ $5x + 2xy = 3 - y$	M1	Interchanges <i>x</i> and <i>y</i>
	5x + 2xy = 3 - y		
	2xy + y = 3 - 5x	M1	Attempt to rearrange
	$2xy + y = 3 - 5x$ $\left[y = g^{-1}(x) = \right] \frac{3 - 5x}{1 + 2x}$	A1	ACF , e.g. $3 - \frac{11x}{1+2x}$
		3	

1(b)(ii) $g^{-1}(x) \in \Box$, $g^{-1}(x) \neq -2.5$ B1Oe Condone omission of $g^{-1}(x) \in \Box$ Allow $y \neq -2.5$ and no other values1	Q	Answer	Marks	Comments
1	1(b)(ii)	$g^{-1}(x) \in \Box$, $g^{-1}(x) \neq -2.5$	B1	Condone omission of $g^{-1}(x) \in \Box$
			1	

Question 1 Total 7	
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Q	Answer	Marks	Comments
2(a)	$\begin{bmatrix} 8\cos\theta + 15\sin\theta = \end{bmatrix}$ $R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$	M1	
	<i>R</i> = 17	B1	
	$\alpha = 62^{\circ}$	A1	AWRT 62°
	$\begin{bmatrix} 8\cos\theta + 15\sin\theta = \end{bmatrix} 17\cos(\theta - 62^\circ)$		
		3	

Q	Answer	Marks	Comments
2(b)(i)	0	B1	
		1	

Q	Answer	Marks	Comments
2(b)(ii)	152°	B1	AWRT 152° Any correct value eg 332°, 512°
		1	

Q	Answer	Marks	Comments
2(c)	$\begin{bmatrix} \text{Let } X = 2y + 10^{\circ} \\ 8 \cos \operatorname{ec} X + 15 \sec X = 8.5 \tan X + 8.5 \cot X \end{bmatrix}$		
	$\frac{8}{\sin X} + \frac{15}{\cos X} = 8.5 \left(\frac{\sin X}{\cos X} + \frac{\cos X}{\sin X}\right)$	B1	PI
	$8\cos X + 15\sin X = 8.5(\sin^2 X + \cos^2 X)$	M1	Eliminate fractions
	$17\cos(X-62)=8.5$	A1ft	ft their part (a)
	$17\cos(2y+10-62)=8.5$		
	$\left[\cos\left(X-62\right)=0.5\right]$		
	$X - 62 = \pm 60$		
	$2y + 10 = -238^{\circ}, 2^{\circ}, 122^{\circ}, 362^{\circ}$		
	<i>y</i> = -124°, -4°, 56°, 176°	B1 B1	At least one correct answer All four correct and no others
		5	

Question 2 Total 10	
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Q	Answer	Marks	Comments
3(a)(i)	$16(-1.5)^{3} + b(-1.5)^{2} + c(-1.5) = -45$ $16(1.25)^{3} + b(1.25)^{2} + c(1.25) = 10$	M1	One correct substitution or M1 for clear use of long division
	$\frac{9}{4}b - \frac{3}{2}c = 9$ $\frac{25}{16}b + \frac{5}{4}c = -21.25$	A1	Correct equations 3b-2c = 12 oe , e.g. 5b+4c = -68
	11 <i>b</i> = -44	m1	Attempt to solve for b or c PI by correct final answers
	b = -4 c = -12	A1	Both answers
		4	

Q	Answer	Marks	Comments
3(a)(ii)	$\left[f(x)=\right] 4x(4x+3)(x-1)$	M1 A1	M1 : $[f(x) =]kx(px+q)(rx+s)$ A1 : Any correct form, ISW
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{f(x)}{16x^2 - 9} = \frac{4x(4x + 3)(x - 1)}{(4x + 3)(4x - 3)} = \frac{4x(x - 1)}{4x - 3}$	M1	or M1 for correct use of long division
	$\frac{4x^2 - 4x}{4x - 3} = x - \frac{x}{4x - 3}$	M1	PI by correct final answer
	$\left[\frac{x}{4x-3} = \frac{(4x-3)+3}{16x-12} = \frac{1}{4} + \frac{3}{16x-12}\right]$		
	$=x-\frac{1}{4}-\frac{3}{16x-12}$	A1	Condone $x - \frac{1}{4} - \frac{3}{4(4x-3)}$
		3	

Question 3 Tot	I 9	
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Q	Answer	Marks	Comments
4(a)		M1	Two sections with approx. correct curvature
		A1	End points correct (approx.) and asymptote correct (approx.)
	$\begin{array}{c} O \\ -1 \end{array}$		
	· · ·	2	

Q	Answer	Marks	Comments
4(b)(i)	$f(x) = \sec x - 10x + 5$		
	f(0.6) = 0.21 f(0.7) = -0.69	M1	or reverse Both values rounded or truncated to at least 1sf
	Change of sign, $0.6 < \alpha < 0.7$	A1	Must have both statement and interval in words or symbols or comparing 2 sides: at 0.6, $\sec 0.6 > 6-5$; at 0.7, $\sec 0.7 < 7-5$ (M1) Conclusion as before (A1)
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$[x_2 =] 0.621$	B1	
	$[x_3 =] 0.623$	B1	
		2	

Q		Answer	Marks	Comments	
4(B1	All five correct <i>x</i> values (and no extrast	
4(c)	x	У		used) PI by four correct <i>y</i> values to 3 dp	
	0.61	1.22003589		They four confectly values to 5 up	
	0.63	1.2375816	M1	At least four correct <i>y</i> values in exact	
	0.65	1.2561492		form or as decimals which are rounde	
	0.67	1.2758004		or truncated correct to 2 dp or better May be seen in a table or a formula	
	0.69	1.2966031		PI by AWRT 1.2572	
	0.02×(1.2	2003589+1.2375816+1.2561492 +1.2758004+1.2966031)	m1	Correct sub into formula with $h = 0.02$ oe and at least four correct <i>y</i> values either listed, with + signs, or totalled	
	= 0.125723	3	A1	CAO Must see this value exactly and no errors made	

Question 4 Total	10
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Q	Answer	Marks	Comments
5(a)	$\left[\left(1-px\right)^{-\frac{1}{2}}=\right]$		
	$1 + \left(-\frac{1}{2}\right)(-px) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-px)^{2}$	М1	At least 2 terms in <i>x</i> correct
	$+ \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6}\left(-px\right)^{3}$		
	$=1 + \frac{1}{2}px + \frac{3}{8}p^2x^2 + \frac{5}{16}p^3x^3$	A1	AG Must be convincingly shown
		2	

Q	Answer	Marks	Comments
5(b)	$(4+px)^{\frac{1}{2}} = 2\left(1+\frac{px}{4}\right)^{\frac{1}{2}}$	B1	
	$= 2 \left(1 + \left(\frac{1}{2}\right) \left(\frac{px}{4}\right) + \frac{\left(\frac{1}{2}\right) \left(-\frac{1}{2}\right)}{2} \left(\frac{px}{4}\right)^2 + \frac{\left(\frac{1}{2}\right) \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right)}{6} \left(\frac{px}{4}\right)^3 \right) \right)$ $= 2 + \frac{1}{4} px - \frac{1}{64} p^2 x^2 + \frac{1}{512} p^3 x^3$	M1 A1	At least 2 terms in <i>x</i> correct
		3	

Q	Answer	Marks	Comments
5(c)(i)	$\begin{bmatrix} LHS = \end{bmatrix} \frac{3}{4} px + \left(2 + \frac{1}{4} px - \frac{1}{64} p^2 x^2 + \frac{1}{512} p^3 x^3\right) \\ -2\left(1 + \frac{1}{2} px + \frac{3}{8} p^2 x^2 + \frac{5}{16} p^3 x^3\right)$	M1	Use of their part (b)
	$= (2-2) + \left(\frac{3}{4} + \frac{1}{4} - 1\right) px - \left(\frac{1}{64} + \frac{3}{4}\right) p^2 x^2 + \left(\frac{1}{512} - \frac{5}{8}\right) p^3 x^3$	A1ft	Correctly collecting their x^2 terms
	$-\frac{49}{64}p^2x^2 \left[-\frac{319}{512}p^3x^3\right] = -x^2 \left[+qx^3\right]$	m1	Equating their <i>x</i> ² terms and attempting to solve
	$\left[-\frac{49}{64}p^2 = -1 \Rightarrow \right] p^2 = \frac{64}{49}$		
3	$p = \pm \frac{8}{7}$	A1	AG Must be convincingly shown
		4	

Q	Answer	Marks	Comments
5(c)(ii)	$-\frac{319}{512} \left(\pm\frac{8}{7}\right)^3 x^3 = qx^3$	M1	Equating their x^3 terms and attempting to solve PI by at least one correct value for q (which may be a truncated decimal)
	$q = \pm \frac{319}{343}$	A1	CAO
		2	

Question 5 Total 11		
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Q	Answer	Marks	Comments
6(a)		B1 B1 B1	Graph in first and second quadrant only Correct curvature Correct intercepts (In16, 0) and (0, 3) shown or stated Allow (2.8, 0) or better
		3	

Q	Answer	Marks	Comments
6(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0.5 \mathrm{e}^{0.5x}$	M1	$k e^{0.5x}$
	When $x = \ln 25$ $\frac{dy}{dy} = 0.5 e^{0.5 \times \ln 25}$		
	$\frac{dy}{dx} = 0.5 e^{0.5 \times \ln 25}$ $\frac{dy}{dx} = 0.5 e^{\ln 5}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2.5$	A1	ое
	When $x = \ln 25$		
	$y = \left e^{0.5 \ln 25} - 4 \right = 1$	B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x_{\text{normal}}} = -\frac{2}{5}$	M1	M1 for the negative reciprocal of their 2.5
	$y-1=-\frac{2}{5}(x-\ln 25)$		
	$2x + 5y = 5 + 2\ln 25$	A1	ое
		5	

Q	Answer	Marks	Comments
6(c)	$x = 0, \ y = \frac{(5 + \ln 625)}{5}$ $y = 0, \ x = \frac{(5 + \ln 625)}{2}$	M1	Either correct
	$A = \frac{1}{2} \times \frac{(5 + \ln 625)}{5} \times \frac{(5 + \ln 625)}{2}$	M1	
	$A = \frac{1}{20} \left(5 + \ln 625 \right)^2$	A1	oe, e.g. $A = \frac{1}{5} \left(\frac{5}{2} + \ln 25 \right)^2$ or $A = \frac{4}{5} \left(\frac{5}{4} + \ln 5 \right)^2$
		3	
	Question 6 Total	11	

Q	Answer	Marks	Comments
7(a)(i)	$\overrightarrow{AB} = \begin{bmatrix} -3\\ -2\\ 7 \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
7(a)(ii)	$\left \overrightarrow{AB}\right = \sqrt{\left(-3\right)^2 + \left(-2\right)^2 + 7^2}$	M1	
	= $\sqrt{62}$	A1	
		2	

Q	Answer	Marks	Comments
7(a)(iii)	$\begin{bmatrix} 1\\-2\\-3\end{bmatrix} \cdot \begin{bmatrix} -3\\-2\\7\end{bmatrix} \begin{bmatrix} =-20\end{bmatrix}$	M1	PI by –20 seen or used
	$\cos\theta = \frac{\begin{bmatrix} 1\\ -2\\ -3 \end{bmatrix} \cdot \begin{bmatrix} -3\\ -2\\ 7 \end{bmatrix}}{\sqrt{62} \times \sqrt{1^2 + 2^2 + 3^2}}$	М1	
	<i>θ</i> = 132.75°		
	$[\Rightarrow acute angle =] 47.2^{\circ}$	A1	
		3	

Q	Answer	Marks	Comments
7(a)(iv)	The line <i>AB</i> has vector equation $\mathbf{r} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$		
	$4 + \lambda = 1 - 3\mu$	M1	
	$4 + \lambda = 1 - 3\mu$ -1-2 $\lambda = 5 - 2\mu$ $\mu = 0, \ \lambda = -3$ -3 = $c + (-3)(-3)$ c = -12	A1	
	<i>c</i> = -12	A1	
		3	

Q	Answer	Marks	Comments
7(b)(i)	$\begin{bmatrix} 4+p \\ -1-2p \\ -12-3p \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} = 0$ 4+p+2+4p+36+9p = 0	M1	
	p3	A1	
	$ OP = \sqrt{1^2 + 5^2 + (-3)^2}$ = $\sqrt{35}$	m1	
	$=\sqrt{35}$	A1	
		4	

Q	Answer	Marks	Comments
7(b)(ii)	$ OB = \sqrt{29}$	B1	2 oe Note, shortest distance from line AB to origin is $\frac{13\sqrt{93}}{31} = 4.044$
	The shortest distance from l to the origin is $\sqrt{35}$, so the line <i>AB</i> must be nearer	E1ft	ft their $\sqrt{35}$ and their equivalent of $\sqrt{29}$ with a consistent conclusion
		2	

Question 7 Tota	15	
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Q	Answer	Marks	Comments
8	$1 + \frac{\mathrm{d}y}{\mathrm{d}x} = 2\left(x - 2y\right)\left(1 - 2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	M1 A1	M1: LHS or RHS correct A1: Both correct $\begin{bmatrix} \frac{dy}{dx} (1+4(x-2y)) = 2(x-2y) - 1 \\ \frac{dy}{dx} = \frac{2(x-2y) - 1}{(1+4(x-2y))} \text{oe} \end{bmatrix}$
	At (2, 2) $1 + \frac{dy}{dx} = -4 + 8 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{5}{7}$	M1 A1	Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{dy}{dx} = \frac{5}{7}$ $y = \frac{5}{7}x + c$ $2 = \frac{5}{7} \times 2 + c$ $c = \frac{4}{7}$ $y = \frac{5}{7}x + \frac{4}{7}$	m1	Attempt to find <i>c</i>
	$y = \frac{5}{7}x + \frac{4}{7}$	A1	
		6	
	Question 8 Total	6	

Q	Answer	Marks	Comments
9(a)	Stretch +either I or IIParallel to x-axisISF 0.5II	M1 A1	or Translation $\begin{bmatrix} 0\\k \end{bmatrix}$ M1 $k = \ln 2$ A1
		2	

Q	Answer	Marks	Comments
9(b)	$V = \pi \int_{0.5}^{4} \left(\ln \left(2x \right) \right)^2 \mathrm{d}x$	B1	Complete correct statement
	$u = \left(\ln\left(2x\right)\right)^2, \frac{\mathrm{d}v}{\mathrm{d}x} = 1$	M1	Attempt at parts
	$\frac{\mathrm{d}u}{\mathrm{d}x} = 2\ln(2x) \times \frac{1}{x}, v = x$	A1	All 4 terms correct
	$\int \ln(2x)^2 dx = x \left(\ln(2x)\right)^2 - \int x \times \frac{2\ln(2x)}{x} dx$	m1	Correct substitution into parts formula
	$\int \ln(2x) dx$ $u = \ln(2x), \frac{dv}{dx} = 1$ $\frac{du}{dx} = \frac{1}{x}, v = x$	М1	Attempt at parts
	$\int \ln(2x) dx = x \ln(2x) - \int x \times \frac{1}{x} dx$	m1	Correct substitution into parts formula
	$= x \ln 2x - x$	A1	
	$\left[\int \ln(2x)^2 dx = x(\ln(2x))^2 - 2x\ln(2x) + 2x\right]$		
	$V = \pi \int_{0.5}^{4} \ln \left(2x\right)^2 \mathrm{d}x$		
	$=\pi \Big(4 \Big(\ln 8\Big)^2 - 8\ln 8 + 8 - 1\Big)$	M1	Subst limits into their expression (must be in form $ax(\ln(2x)^2) + bx\ln(2x) + cx$)
	$=\pi\Big(4\big(\ln 8\big)^2-8\ln 8+7\Big)$	A1	ACF eg $\pi (36(\ln 2)^2 - 24\ln 2 + 7)$
		9	

Question 9 Tota	I 11	
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Q	Answers	Marks	Comments
	$\int \frac{\mathrm{d}y}{(3a-2y)(a-y)} = \int b \mathrm{d}x$	M1	Separate variables
	$\frac{1}{(3a-2y)(a-y)} = \frac{A}{3a-2y} + \frac{B}{a-y}$	M1	Use of partial fractions
	$1 = A(a - y) + B(3a - 2y)$ $A = -\frac{2}{a}, B = \frac{1}{a}$	A1	
	$-\frac{2}{a} \times \left(\frac{1}{-2}\right) \ln (3a - 2y) + \frac{1}{a} \times (-1) \ln (a - y) = bx + c$ x = 0, y = 0	M1 A1	M1: Attempt to integrate A1: Fully correct integration
	$\frac{1}{a}\ln 3a - \frac{1}{a}\ln a = c$	m1	Attempt to find <i>c</i>
	$c = \frac{1}{a} \ln 3$	A1	
	$\begin{bmatrix} \ln\left(\frac{3a-2y}{a-y}\right) = abx + \ln 3\\ \ln\left(\frac{2y-3a}{3(y-a)}\right) = abx \end{bmatrix}$		
	$\left[\ln\left(\frac{2y-3u}{3(y-a)}\right) = abx\right]$		
	$\frac{2y-3a}{3(y-a)} = e^{abx}$	M1	Eliminates logarithms
	$2y - 3a = 3y e^{abx} - 3a e^{abx}$		
	$y(2-3e^{abx}) = 3a(1-e^{abx})$	M1	Attempt to find <i>y</i>
	$y = \frac{3a\left(1 - e^{abx}\right)}{2 - 3e^{abx}}$	A1	ое
		10	

Question 10

Q	Answer	Marks	Comments
11(a)	$\begin{bmatrix} \cos 2\theta = 2\cos^2 \theta - 1 \end{bmatrix}$ $\int 4\cos^2 \theta d\theta = \int (2\cos 2\theta + 2) d\theta$ $= \sin 2\theta + 2\theta [+c]$	M1A1	M1 for $a \sin 2\theta + b\theta$ A1 correct with no errors seen
		2	

Q	Answer	Marks	Comments
11(b)	$t = \sin x, \mathrm{d}t = \cos x \mathrm{d}x$	B1	ое
	$\begin{bmatrix} t \end{bmatrix}_{0}^{\frac{1}{2}} = \begin{bmatrix} \sin x \end{bmatrix}_{0}^{\frac{\pi}{6}} \\ \int \frac{\sin 2x}{3 + \cos^{2} x} dx = \int \frac{2t}{4 - t^{2}} dt$	B1	Change of limits
	$\int \frac{\sin 2x}{3 + \cos^2 x} dx = \int \frac{2t}{4 - t^2} dt$	M1	
	$\int \frac{2t}{4-t^2} dt = -\ln(4-t^2)$	m1 A1	m1: $k \ln(4-t^2)$ A1: Correct, or $-\ln(2-t) - \ln(2+t)$ oe
	$= -\ln\left(4 - \frac{1}{4}\right) - \left(-\ln(4)\right)$	M1	Substituting into $k \ln(4-t^2)$ oe
	$=\ln\left(\frac{16}{15}\right)$	A1	
		7	

Question 11 Tota

Q	Answer	Marks	Comments
12(a)	$\cos\theta = \frac{x}{2}, \sin\theta = \frac{y}{3}$	M1	
	$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$	A1	oe
		2	

Q	Answer	Marks	Comments
12(b)	$\theta = \frac{\pi}{6}, x = \sqrt{3}, y = \frac{3}{2}$ $\frac{dx}{d\theta} = -2\sin\theta \frac{dy}{d\theta} = 3\cos\theta$	B1	$\left[\frac{2x}{4} + \frac{2y}{9}\frac{dy}{dx} = 0\right]$
	$\frac{dy}{dx} = -\frac{3\cos\theta}{2\sin\theta} \left[= -1.5\cot\theta \right]$	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9x}{4y}$
	$\theta = \frac{\pi}{6}, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3\sqrt{3}}{2}$	A1	PI
	$y - 1.5 = -\frac{3\sqrt{3}}{2} \left(x - \sqrt{3} \right)$		
	$y + \frac{3\sqrt{3}}{2}x - 6 = 0$	A1	
		4	

12(c) $xy = k^{2} \implies y = \frac{k^{2}}{x}$ $\frac{x^{2}}{4} + \left(\frac{k^{2}}{3x}\right)^{2} = 1$ $9x^{4} + 4k^{4} = 36x^{2}$ $9x^{4} - 36x^{2} + 4k^{4} = 0$ $(-36)^{2} - 4 \times 9 \times 4k^{4} > 0$ $(-36)^{2} - 4 \times 9 \times 4k^{4} > 0$ $x^{2} = 2 \pm \frac{2}{3}\sqrt{9 - k^{4}}$ Given that k is positive, for x^{2} to have two distinct positive real values then $x^{2} = 2 \pm \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ or $x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $\mathbf{A1}$ $\mathbf{M1}$ $xy = k^{2} \implies x = \frac{k^{2}}{y}$ $\frac{k^{4}}{4y^{2}} + \frac{y^{2}}{9} = 1$ $gk^{4} + 4y^{4} = 36y^{2}$ 41 $y^{4} - 36y^{2} + 9k^{4} = 0$ $\mathbf{A1}$ $y^{2} = \frac{9}{2} \pm \frac{3}{2}\sqrt{9 - k^{4}} > 0$ $\mathbf{M1}$ $y^{2} = \frac{9}{2} \pm \frac{3}{2}\sqrt{9 - k^{4}}$ $\mathbf{Given that } k \text{ is positive, for } y^{2} \text{ to have two distinct positive real values then}$ $y^{2} = \frac{9}{2} \pm \frac{3}{2}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ or $y^{2} = \frac{9}{2} - \frac{3}{2}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ $\mathbf{A1}$ $\mathbf{A1}$ $\mathbf{M2}$ $\mathbf{M2}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M4}$ $\mathbf{M4}$ $\mathbf{M4}$ \mathbf	Q	Answer16	Marks	Comments
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	12(c)	$xy = k^2 \implies y = \frac{k^2}{r}$	M1	$xy = k^2 \implies x = \frac{k^2}{y}$
$yx^{2} + 4k^{2} = 36x^{2}$ $yx^{4} - 36x^{2} + 4k^{4} = 0$ $(-36)^{2} - 4 \times 9 \times 4k^{4} > 0$ $x^{2} = 2 \pm \frac{2}{3}\sqrt{9 - k^{4}}$ Given that k is positive, for x ² to have two distinct positive real values then $x^{2} = 2 \pm \frac{2}{3}\sqrt{9 - k^{4}} > 0 \Rightarrow k^{2} < 3$ or $x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \Rightarrow k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $A1$ $4y^{4} - 36y^{2} + 9k^{4} = 0$ $(-36)^{2} - 4 \times 9 \times 4k^{4} > 0$ $B1$ $(-36)^{2} - 4 \times 9 \times 4k^{4} > 0$ $y^{2} = \frac{9}{2} \pm \frac{3}{2}\sqrt{9 - k^{4}}$ Given that k is positive, for y ² to have two distinct positive real values then $y^{2} = \frac{9}{2} \pm \frac{3}{2}\sqrt{9 - k^{4}} > 0 \Rightarrow k^{2} < 3$ or $y^{2} = \frac{9}{2} - \frac{3}{2}\sqrt{9 - k^{4}} > 0 \Rightarrow k^{2} < 3$ $\therefore k^{2} < 3$ $(k^{2} < 3)$ $(hen there will be 4 distinct points of intersection. (A1)$		$\frac{x^2}{4} + \left(\frac{k^2}{3x}\right)^2 = 1$., .
$x^{2} = 2 \pm \frac{2}{3}\sqrt{9-k^{4}}$ Given that k is positive, for x^{2} to have two distinct positive real values then $x^{2} = 2 \pm \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ or $x^{2} = 2 - \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $x^{1} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $\therefore k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$ $x^{2} = x^{2} + \frac{2}{3}\sqrt{9-k^{4}} > 0 \Rightarrow k^{2} < 3$			A1	
Given that k is positive, for x^2 to have two distinct positive real values then $x^2 = 2 + \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ or $x^2 = 2 - \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore k^2 < 3$ then there will be 4 distinct points of intersection. $x^2 = 2 - \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore k^2 < 3$ A1 $x^2 = 2 - \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore k^2 < 3$ A1 $x^2 = 2 - \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore k^2 < 3$ A1 $x^2 = 2 - \frac{2}{3}\sqrt{9 - k^4} > 0 \Rightarrow k^2 < 3$ $\therefore k^2 < 3$		$(-36)^2 - 4 \times 9 \times 4k^4 > 0$	B1	$(-36)^2 - 4 \times 9 \times 4k^4 > 0$
distinct positive real values then $x^{2} = 2 + \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ or $x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. distinct points of $x^{2} = \frac{9}{2} + \frac{3}{2}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. A1 distinct points of intersection. A1		5	M1	
or $x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ $x^{2} < 3$ $x^{2} < 3$ $x^{2} < 3$ $x^{2} < 3$				distinct positive real values then
$x^{2} = 2 - \frac{2}{3}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $y^{2} = \frac{9}{2} - \frac{3}{2}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$ $\therefore k^{2} < 3$ then there will be 4 distinct points of intersection. $A1$ then there will be 4 distinct points of intersection.		5		2 2
then there will be 4 distinct points of intersection.				$y^{2} = \frac{9}{2} - \frac{3}{2}\sqrt{9 - k^{4}} > 0 \implies k^{2} < 3$
intersection.		$\therefore k^2 < 3$		$\therefore k^2 < 3$
ое			A1	
				oe
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Question 12 Total 11		Question 12 Total	11	
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