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MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
3(a)	Change in GPE of cart $= 16 \times 9.8 \times (4.0 - 2.5)$ $= 235.2 \text{ [J]}$ Work done against resistance force $235.2 = F \times 48$ $F = 4.9 \text{ [N]}$	M1 A1 M1 A1	Correct change in GPE, PI ft their change in GPE CAO SC2 for a correct answer from a method that assumes equivalence with constant acceleration
		4	

Q	Answer	Marks	Comments
3(b)	Work done against resistance force $= 4.9 \times 30$ $= 147 \text{ [J]}$ Kinetic energy of cart at Y $= 16 \times 9.8 \times 4.0 - 147$ $= 480.2 \text{ [J]}$ Speed of cart at Y $\left[v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \times 480.2}{16}} \right]$ $[v =] 7.7 \text{ [m s}^{-1}\text{]}$	B1ft M1 A1	ft their answer to part (a) ft their work done against friction oe , e.g. 7.74758... [m s ⁻¹] Exact answer is $\frac{49\sqrt{10}}{20} \text{ [m s}^{-1}\text{]}$
		3	

	Question 3 Total	7	
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Q	Answer	Marks	Comments
4(a)	30 [cm]	B1	Allow any mention of symmetry or that the centres of mass of the square and rectangle lie on the line connecting the midpoints of OC and AB
	The line joining the midpoint of OC and the midpoint of AB is a line of symmetry	E1	
		2	

Q	Answer	Marks	Comments						
4(b)	<table border="1"> <thead> <tr> <th>Area / cm²</th> <th>COM from OC / cm</th> </tr> </thead> <tbody> <tr> <td>4200 [= 60 × 70]</td> <td>35</td> </tr> <tr> <td>256 [= 16 × 16]</td> <td>56</td> </tr> </tbody> </table>	Area / cm ²	COM from OC / cm	4200 [= 60 × 70]	35	256 [= 16 × 16]	56	B1	Use or sight of all four values
	Area / cm ²	COM from OC / cm							
4200 [= 60 × 70]	35								
256 [= 16 × 16]	56								
$4200\sigma \times 35 - 256\sigma \times 56 = 3944\sigma\bar{Y}$ $[\bar{Y} =] 33.6$ [cm]	M1 A1	At least two terms correct Condone omission of σ CAO to 3 sf							
		3							

Q	Answer	Marks	Comments
4(c)	Angle OBA $\tan^{-1}\left(\frac{35}{30}\right) = 49.39\dots^\circ$	B1	or Angle OBC $\tan^{-1}\left(\frac{30}{35}\right) = 40.60\dots^\circ$
	Angle MBA $\tan^{-1}\left(\frac{70 - 33.636\dots}{30}\right) = 50.47\dots^\circ$ $50.47\dots^\circ - 49.39\dots^\circ = 1.1^\circ$	B1 M1 A1	or Angle MBC $\tan^{-1}\left(\frac{30}{70 - 33.636\dots}\right) = 39.52\dots^\circ$ or $40.60\dots^\circ - 39.52\dots^\circ$ CAO
		4	

	Question 4 Total	9	
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Q	Answer	Marks	Comments
5(a)(i)	$[ma = mg \sin(25^\circ) = 24.849...N]$ $a = [9.8 \sin(25^\circ) =] 4.1 [m s^{-2}]$	B1	Answer to 3 sf is 4.14 [m s ⁻²]
		1	

Q	Answer	Marks	Comments
5(a)(ii)	$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2 \times 4.141... \times 10$ Speed of particle at Y $v^2 = 82.833...$ $v = 9.101... [m s^{-1}]$	M1 A1	$\frac{1}{2}mv^2 = mg\Delta h \Rightarrow v^2 = 2g\Delta h$ or $v^2 = 2 \times 9.8 \times (10 \times \sin(25^\circ))$ AG Must be convincingly shown Value of speed shown to more than 2 sf or correct unsimplified equation for v leading to the given result
		2	

Q	Answer	Marks	Comments
5(a)(iii)	Speed at Y would be greater Component of weight down the slope would increase [therefore acceleration increases]	B1 E1	Allow explanations based upon conversion of GPE to KE
		2	

Q	Answer	Marks	Comments
5(b)	Friction acting on particle between Y and Z $= \mu mg \cos(25^\circ)$ $= 0.5 \times 6 \times 9.8 \times \cos(25^\circ)$ $= 26.645... \text{ N}$ Resultant force acting on particle between Y and Z $24.849... - 26.645... = -1.795... \text{ [N]}$ Acceleration of particle between Y and Z $a = \frac{-1.795...}{6} = -0.299... \text{ m s}^{-2}$ Distance YZ $v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$ $s = \frac{0^2 - 9.1^2}{2 \times -0.299...} = 138.362... \text{ [m]}$ Distance XZ $[138.362... + 10 =] 150 \text{ [m]} \text{ (2 sf)}$	M1 A1 m1 A1ft M1 A1	Attempt to find friction acting on particle. Correct value for friction to at least 2 sf, PI ft their friction, PI ft their resultant force Uses their acceleration and the initial velocity of 9.1 m s^{-1} to find their distance YZ AWRT 150 [m] Answer to 3 sf is 148 m
		6	

Q	Answer	Marks	Comments
5(c)	Resultant force on rough part of slope must not be negative $mg \sin \alpha - \mu mg \cos \alpha \geq 0$ $\tan \alpha \geq \mu$ $\alpha \geq \tan^{-1}(0.5)$ $\alpha \geq 26.565...$ $\alpha = 26.6 \text{ (1 dp)}$	M1 A1	PI CAO
		2	

	Question 5 Total	13	
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Q	Answer	Marks	Comments
6(a)	$[\mathbf{v} =] \left(-\frac{1}{2}e^{-2t} + c_1 \right) \mathbf{i} + \left(-\frac{1}{1+t} + c_2 \right) \mathbf{j}$	M1 A1	M1 : At least one term correct A1 : Both components correct Condone omission of constants of integration for M1 A1
	$[\mathbf{v} =] \frac{1}{2}(1 - e^{-2t}) \mathbf{i} + \left(1 - \frac{1}{1+t} \right) \mathbf{j}$	m1 A1	m1 : At least one component correct A1 : Both components correct
		4	

Q	Answer	Marks	Comments
6(b)	$[\mathbf{r} =] \left(\frac{1}{4}e^{-2t} + \frac{1}{2}t + c_3 \right) \mathbf{i} + (t - \ln(1+t) + c_4) \mathbf{j}$	M1 A1	M1 : At least correct exponential term in i component or correct logarithm term in j component A1 : Both components fully correct Condone omission of constants of integration for M1 A1
	$[\mathbf{r} =] \left(\frac{1}{4}e^{-2t} + \frac{1}{2}t + \frac{11}{4} \right) \mathbf{i} + (t - \ln(1+t) + 2) \mathbf{j}$	B1	Correct position vector
	[When $t = 5$] $[\mathbf{r} =] \sqrt{\left(\frac{1}{4}e^{-10} + \frac{5}{2} + \frac{11}{4} \right)^2 + (5 - \ln(6) + 2)^2}$	M1	Evaluates their distance at $t = 5$
	$[\mathbf{r} =] 7.4 \text{ [m]}$	A1	Answer to 3 sf is 7.40
		5	

	Question 6 Total	9	
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Q	Answer	Marks	Comments
7	Time to cover the 12 m to P $T = \frac{12}{15 \cos \alpha}$ or $T = \frac{x}{v \cos \alpha}$ Vertical displacement $s = 15 \sin \alpha \times T - 0.5 \times 9.8 \times T^2$ or $s = v \sin \alpha \times T - 0.5 \times g \times T^2$ $s = 15 \sin \alpha \times \frac{12}{15 \cos \alpha}$ $- 0.5 \times 9.8 \times \left(\frac{12}{15 \cos \alpha} \right)^2$ $5 = 15 \sin \alpha \times \frac{12}{15 \cos \alpha}$ $- 0.5 \times 9.8 \times \left(\frac{12}{15 \cos \alpha} \right)^2$ $3.136 \tan^2 \alpha - 12 \tan \alpha + 8.136 = 0$ $\alpha = [41.370\dots^\circ,] 71.249\dots^\circ$ $v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$ $s = \frac{0^2 - (15 \sin(71.249\dots^\circ))^2}{2 \times -9.8}$ $[s =] 10.3 [m]$	B1 M1 A1 m1 m1 m1 A1 M1 A1	M1: Use of $s = ut + \frac{1}{2}at^2$ with $u = 15 \sin \alpha$ and $a = \pm 9.8$ PI A1: Correct result for s in terms of T PI Eliminates T PI Use of $s = 5$ PI Dependent on M1 Forms quadratic equation in $\tan \alpha$ PI Note $3.136 = \frac{392}{125}$ and $8.136 = \frac{1017}{125}$ PI Solves equation to find the required value of α Uses their (larger) angle [$< 90^\circ$] with $v^2 = u^2 + 2as$ CAO
		9	
	Question 7 Total	9	

Q	Answer	Marks	Comments
8(a)	$\omega = \frac{2\pi}{T}$ or $v = \frac{2\pi r}{T}$	B1	May be seen in elimination of ω or v
	$\frac{GMm}{r^2} = m\omega^2 r$ or $\frac{GMm}{r^2} = \frac{mv^2}{r}$	M1	Relates given force to resultant force
	$\frac{GM}{r^3} = \left(\frac{2\pi}{T}\right)^2$ or $\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$	m1	Eliminates ω or v
	$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$ or $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$		
	$T^2 = \frac{4\pi^2 r^3}{GM}$	A1	AG Must be convincingly shown Must see correct expansion of bracket
		4	

Q	Answer	Marks	Comments
8(b)(i)	$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 \times (4.2 \times 10^7)^3}{6.7 \times 10^{-11} \times 6.0 \times 10^{24}}}$		
	$T = 85298.3... [s]$	B1	PI by correct answer Value is 85000 [s] to 2 sf
	$\omega = \frac{2\pi}{T} = \frac{2\pi}{85298.3...}$	M1	ft their T
	$[\omega =] 7.4 \times 10^{-5} [\text{rad s}^{-1}]$	A1ft	ft their T
		3	

Q	Answer	Marks	Comments
8(b)(ii)	$v = r\omega = 4.2 \times 10^7 \times 7.366... \times 10^{-5}$		
	$v = 3093.7... [\text{m s}^{-1}]$	B1ft	AWRT 3100 ft their ω from part (b)(i)
	$[\mathbf{v} =] (-3100 \cos(7.4 \times 10^{-5} t) \mathbf{i} - 3100 \sin(7.4 \times 10^{-5} t) \mathbf{j}) [\text{m s}^{-1}]$	M1 A1	M1 : ft their ω with use of sine and cosine A1 : Both components fully correct
		3	

	Question 8 Total	10	
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Q	Answer	Marks	Comments
9(a)	Particle is in equilibrium	B1	Condone 'Yes' for B1
	as the particle is not accelerating [so there is no resultant force acting on the particle]	E1	
	and there is no resultant moment on a particle	E1	
		3	

Q	Answer	Marks	Comments
9(b)	Forces parallel to slope $T \cos(5^\circ) = mg \sin(35^\circ) + \mu N$	M1 A1	M1 : At least two terms in an equation correct A1 : Both equations fully correct
	Forces perpendicular to slope $T \sin(5^\circ) + N = mg \cos(35^\circ)$		
	$T \cos(5^\circ) = mg \sin(35^\circ)$ $+ \mu(mg \cos(35^\circ) - T \sin(5^\circ))$	M1 A1	M1 : Eliminating normal reaction using their two equations A1 : Correct elimination of normal reaction
	$T = \frac{mg(\sin(35^\circ) + \mu \cos(35^\circ))}{\cos(5^\circ) + \mu \sin(5^\circ)}$	A1	oe , e.g. $T = 112.398\dots$
	$T = \frac{15 \times 9.8 \times (\sin(35^\circ) + 0.25 \cos(35^\circ))}{\cos(5^\circ) + 0.25 \sin(5^\circ)}$ $T = 112$	A1	CAO to 3 sf
		6	

Q	Answer	Marks	Comments
9(c)(i)	$W = 112.398... \cos(5^\circ) \times 25$ $W = 2800 \text{ [J]}$	B1	AG Must be convincingly shown 2810 J or 112×25 scores B0
		1	

Q	Answer	Marks	Comments
9(c)(ii)	Time taken to move 25 m up the slope $\left[= \frac{25}{4} \right] = 6.25 \text{ s}$ $\left[\text{Power} = \frac{2800}{6.25} = \right] 450 \text{ [J s}^{-1}\text{]}$	M1 A1ft	or use of $P = Fv$ with their tension from part (b) AWRT to 450 $[\text{J s}^{-1}]$
		2	

	Question 9 Total	12	
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