

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

**INTERNATIONAL AS
FURTHER MATHEMATICS
FM02**

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2023

Version: 1.0 Final



2 3 1 X F M 0 2 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2023 Oxford International AQA Examinations and its licensors. All rights reserved.

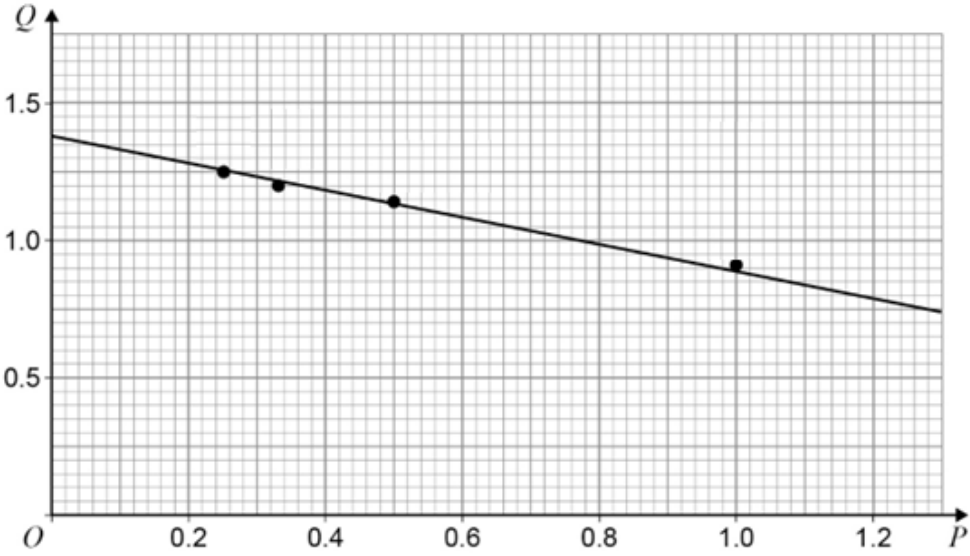
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(x,y) = 0.1 \times \left(3 \times 1 + \frac{2 \times 1^3}{-1} \right)$ $= 0.1$ $y_2 = -1 + 0.1$ $= -0.9$ $y_3 = -0.9 + 0.1 \times \left(3 \times 1.1 + \frac{2 \times 1.1^3}{-0.9} \right)$ $= -0.9 + \frac{77}{2250}$ $= -0.9 + 0.03422$ $= -0.8657778$ $= -0.8658$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>correct use of formula</p> <p>PI by values to at least 5 decimal places or equivalent eg</p> $y_3 = \frac{-974}{1125}$ <p>CAO</p>
		4	

	Question 1 Total	4	
--	-------------------------	----------	--

Q	Answer	Marks	Comments										
2(a)	<table border="1"> <tr> <td>P</td> <td>1</td> <td>0.5</td> <td>0.33</td> <td>0.25</td> </tr> <tr> <td>Q</td> <td>0.91</td> <td>1.14</td> <td>1.20</td> <td>1.25</td> </tr> </table>	P	1	0.5	0.33	0.25	Q	0.91	1.14	1.20	1.25	B1	CAO Condone 1.2 for 1.20
	P	1	0.5	0.33	0.25								
Q	0.91	1.14	1.20	1.25									
	1												

Q	Answer	Marks	Comments
2(b)	Their points plotted correctly	B1ft	All their points plotted ± 0.5 squares
	Line of best fit drawn	B1ft	Their line of best fit drawn
			2

Q	Answer	Marks	Comments
2(c)(i)	$b =$ their intercept $b = 1.4$	B1ft	Intercept must be from a suitable line of best fit for their points
	$a =$ their gradient $a = -0.45$	B1ft	Gradient must be from a suitable line of best fit for their points
		2	

Q	Answer	Marks	Comments
2(c)(ii)	$\frac{1}{y} = \frac{-0.45}{x} + 1.4$	B1ft	alternative forms accepted eg $y = \frac{x}{1.4x - 0.45}$ ft their values of a and b
		1	

Q	Answer	Marks	Comments
2(d)	$y = \frac{1.6}{1.4 \times 1.6 - 0.45}$ $= 0.89$	B1ft	ft their values of a and b
		1	

	Question 2 Total	7	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
3(a)	$\begin{bmatrix} 4+4k & 24k & -3+12k \\ 0 & 33 & 12 \\ 4 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 20k & 24k & 0 \\ 0 & 33 & 12 \\ 4 & 6 & 9 \end{bmatrix}$	M1	equating their obtained matrix with the $\mathbf{A}+4\mathbf{B}$ and using at least one expression to evaluate k
		A1	CSO
		2	

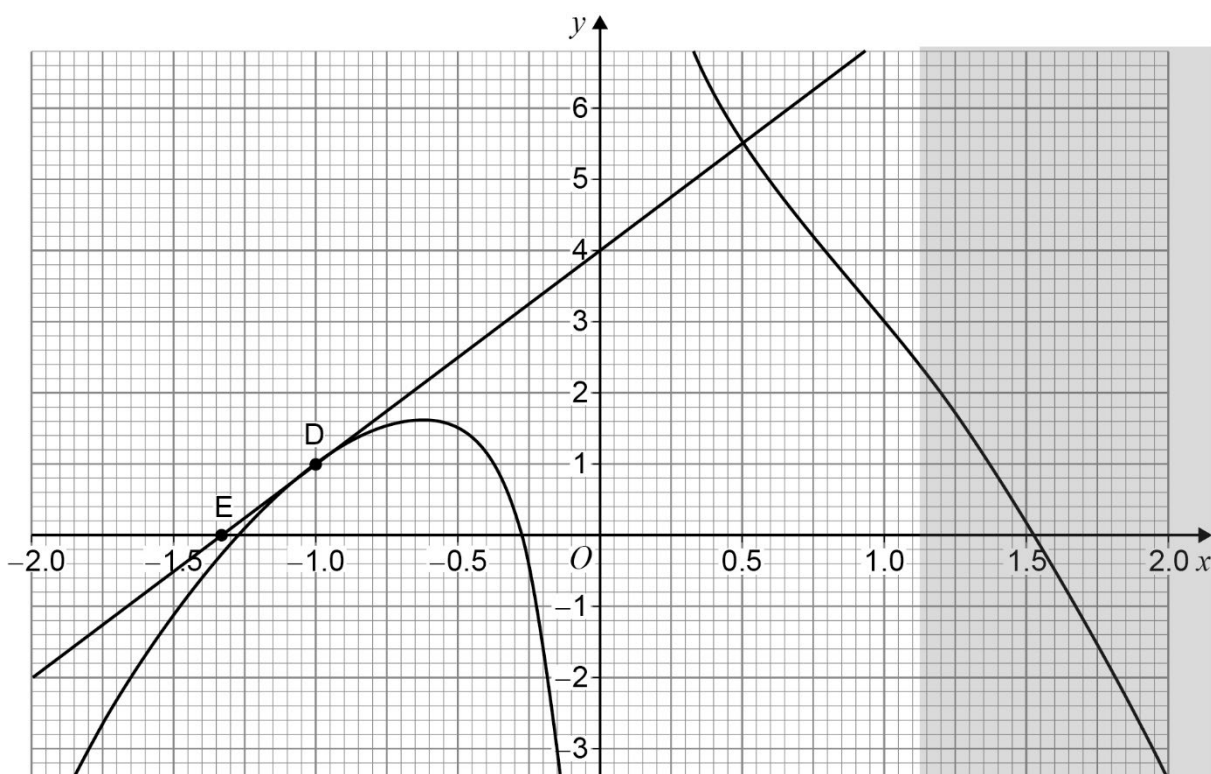
Q	Answer	Marks	Comments
3(b)(i)	$\begin{bmatrix} 4 & 0 & -3 \\ -2 & 1 & -2 \\ 4 & -2 & 5 \end{bmatrix} \begin{bmatrix} 0.25 & 1.5 & 0.75 \\ 0.5 & 8 & 3.5 \\ 0 & 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1	showing correct multiplication allowing for ft with their value of k
		A1ft	ft their value of k
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$\mathbf{C} \begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 0 & 8 & 4 \end{bmatrix} = \mathbf{AB}$ $\mathbf{C}(4\mathbf{B}) = \mathbf{AB}$ $\mathbf{C} = \frac{1}{4}\mathbf{A}$ $= \begin{bmatrix} 1 & 0 & -0.75 \\ -0.5 & 0.25 & -0.5 \\ 1 & -0.5 & 1.25 \end{bmatrix}$	M1	equating $\begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 4 & 6 & 9 \end{bmatrix} = 4\mathbf{B}$
		A1	correct Matrix for \mathbf{C}
		2	

	Question 3 Total	6	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
4(a)	$f(1) = 3$ $f(2) = -3.5$ As there has been a change of sign between $x = 1$ and $x = 2$, and as the curve is continuous [on this interval], then there is a root $1 < \gamma < 2$	M1	Correct evaluation of a suitable interval
		A1	Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
4(b)	Tangent drawn at $x = -1$ The next approximation for a root is closer to α rather than β [see diagram below]	B1	Gives justification that using $x_1 = -1$ will converge to the root α where $-2 < \alpha < -1$
		E1	



		2	
--	--	---	--

Q	Answer	Marks	Comments
4(c)	$f(x) = \frac{1}{x} - 2x^2 + 4$ $f'(x) = \frac{-1}{x^2} - 4x$ $f'(-1) = \frac{-1}{(-1.3)^2} - 4(-1.3)$ $= 4.60828402$ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $x_2 = -1.3 - \frac{\frac{1}{-1.3} - 2(-1.3)^2 + 4}{\frac{-1}{(-1.3)^2} - 4(-1.3)}$ $= -1.3 - \frac{\frac{1}{-1.3} - 2(-1.3)^2 + 4}{4.60828402}$ $= -1.3 + 0.0323832$ $= -1.267616846$ $= -1.2676$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>correct derivative</p> <p>PI AWRT 4.608</p> <p>correctly substituting into formula or obtaining correct value for $\frac{f(x)}{f'(x)}$</p> <p>correct answer to 4 dp</p>
		4	
	Question 4 Total	8	

Q	Answer	Marks	Comments
5(a)(i)	$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ p \end{bmatrix}$ $\Rightarrow 2\cos 2\theta = -1 \text{ and } 2\sin 2\theta = p$ $\Rightarrow \cos 2\theta = -\frac{1}{2}$ $\cos^2 2\theta + \sin^2 2\theta = 1$ $\Rightarrow \left(\frac{-1}{2}\right)^2 + \left(\frac{p}{2}\right)^2 = 1$ $\Rightarrow \left(\frac{p}{2}\right)^2 = \frac{3}{4}$ $\Rightarrow p = \pm\sqrt{3} \text{ but } p < 0$ $\therefore p = -\sqrt{3}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Obtains correct expressions containing p, $\cos 2\theta$ and $\sin 2\theta$ PI</p> <p>Eliminates θ to gain an equation to find p or finds $2\theta = 240^\circ$ oe</p> <p>CSO</p>
		3	

Q	Answer	Marks	Comments
5(a)(ii)	$\mathbf{M} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	B1ft	ft their value of p or θ
		1	

Q	Answer	Marks	Comments
5(a)(iii)	$\cos 2\theta = -\frac{1}{2}$ and $\sin 2\theta = -\frac{\sqrt{3}}{2}$ $\tan 2\theta = \sqrt{3}$ $2\theta = 240^\circ$ $\theta = 120^\circ$ $y = \tan 120^\circ x$ or $y = -\sqrt{3}x$ Reflection in the line $y = -\sqrt{3}x$	<p>M1</p> <p>A1ft</p> <p>A1</p>	<p>PI attempts to find the angle for the equation of the line of reflection</p> <p>ft their values from (a)(ii) describing fully with equation of the line of reflection</p> <p>CSO</p>
		3	

Q	Answer	Marks	Comments
5(b)(i)	$\begin{bmatrix} c & d \\ d & -c \end{bmatrix} \begin{bmatrix} -1 \\ -\sqrt{3} \end{bmatrix} = \begin{bmatrix} -1 \\ -\sqrt{3} \end{bmatrix}$ $\Rightarrow -c - \sqrt{3}d = -1$ and $\Rightarrow -d + \sqrt{3}c = -\sqrt{3}$ $d = \frac{\sqrt{3}}{2} \text{ and } c = -\frac{1}{2}$	<p>M1</p> <p>A1ft</p> <p>A1</p>	<p>PI obtains correct matrix equation could be in terms of p ft their value of p</p> <p>PI obtains both correct simultaneous equations could be in terms of p ft their value of p</p>
		3	

Q	Answer	Marks	Comments
5(b)(ii)	$\mathbf{NM} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ $= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	M1 A1	using correct order of multiplication NM using their <i>c</i> and <i>d</i> and their M CSO
		2	

Q	Answer	Marks	Comments
5(b)(iii)	$\Rightarrow \cos \theta = -\frac{1}{2} \text{ and } \sin \theta = -\frac{\sqrt{3}}{2}$ $\theta = 240^\circ$ <p>Single transformation is a rotation about the origin of 120° clockwise</p>	M1 B1 A1	Correct method to find θ Identifies transformation as rotation Full description oe such as 240° [anticlockwise]
		3	

	Question 5 Total	15	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
6(a)	$G_X(t) = 0.008 + 0.096t + 0.384t^2 + 0.512t^3$	M1	expands $G_X(t)$ or uses chain rule to differentiate to $k(0.2 + 0.8t)^2$
	$G'_X(t) = 0.096 + 0.768t + 1.536t^2$	A1	Obtains correct $G'_X(t)$ oe , chain rule gives $2.4(0.2 + 0.8t)^2$
	$G'_X(1) = 0.096 + 0.768(1) + 1.536(1)^2$	M1	attempts to find $G'_X(1)$
	$E(X) = G'_X(1) = 2.4$	A1	
		4	

Q	Answer	Marks	Comments
6(b)	$P(X \geq 2) = 0.384 + 0.512 = 0.896$	B1ft	ft their expanded $G_X(t)$, their 0.384 + their 0.512 oe
		1	

	Question 6 Total	5	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
7(a)	$\frac{1-p}{p^2} = 3.75$ $3.75p^2 + p - 1 = 0$ $p = 0.4$	M1	forms correct equation
		M1	rearranges to three term quadratic = 0 oe PI by correct final answer
		A1	if $-2/3$ seen, it must be rejected
		3	

Q	Answer	Marks	Comments
7(b)	$E(X) = 2.5$	B1ft	ft their $\frac{1}{p}$
		1	

Q	Answer	Marks	Comments
7(c)	$P(X \leq 5) = 1 - (1 - 0.4)^5$ $= 0.92224$	M1	attempts to calculate $(1 - \text{their } p)^5$ oe PI
		A1	oe
		2	

	Question 7 Total	6	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
8(a)	$E(X) = 4$	B1	oe, PI
	$E(Y) = 5.5$	B1	oe, PI
	$E(4X - 3Y) = 4E(X) - 3E(Y)$	M1	applies formula
	$E(4X - 3Y) = -0.5$	A1	
		4	

Q	Answer	Marks	Comments
8(b)	$\text{Var}(X) = 4$	B1	oe
	$\text{Var}(Y) = 8.25$	B1	oe
	$\text{Cov}(X, Y) = \sqrt{4 \times 8.25} \rho$	M1	express covariance in terms of ρ PI
	$10 = 4 + 8.25 + 2\sqrt{4 \times 8.25} \rho$	M1	form correct equation to find ρ oe
	$\rho = -0.196$	A1	AWRT
		5	

	Question 8 Total	9	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
9	$MLT^{-2} = [k](LT^{-1})^n$	M1 A1	M1: dimensions equation with at least one side correct Condone consistent use of units
	$MLT^{-2} = [k]L^nT^{-n}$ $[k] = ML^{1-n}T^{n-2}$	A1	A1: correct dimensions equation Condone consistent use of units correct dimensions for k
		3	

	Question 9 Total	3	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
10(a)	$I = 0.3 \times 4 - 0.3 \times (-6)$	M1	uses impulse equation condone sign errors
	$I = 3 \text{ N s}$	A1	obtains correct magnitude must include units
		2	

Q	Answer	Marks	Comments
10(b)	$\frac{1}{2} \times 75T = 3$	M1	finds impulse in terms of T from graph and sets equal to their impulse
	$T = \frac{6}{75} = 0.08$	A1	correct T
		2	

	Question 10 Total	4	
--	--------------------------	----------	--

Q	Answer	Marks	Comments
11	$-20 + 2t = t + 80$ $t = 100$ $450 - 0.5 \times 100 = 150 + 100U$ $U = 2.5$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>forms equation to find the time when they meet oe condone sign errors</p> <p>correct time</p> <p>forms equation to find U oe</p> <p>correct U</p>
		4	
	Question 11 Total	4	

Q	Answer	Marks	Comments
12(a)	$7 \times 0.8 = 5.6$	B1	finds speed after collision with the wall
	$0.3 \times 2 + 0.5 \times (-5.6) = 0.3v_P + 0.5v_Q$ $-22 = 3v_P + 5v_Q$	M1	equation for conservation of momentum condone sign errors
	$v_P - v_Q = -0.5(2 - (-5.6))$	A1	correct equation
	$v_P - v_Q = -3.8$	M1	applies equation for restitution condone sign errors
	$v_P = -5.125$	A1	correct velocity for P or Q note: $v_Q = -1.325$
	$I = 0.3 \times (-5.125) - 0.3 \times (2)$ $= -2.1375 \text{ N s}$	M1	applies impulse equation with their velocities condone sign errors
$ I = 2.14 \text{ to } 3 \text{ sf}$	A1	correct magnitude	
		7	

Q	Answer	Marks	Comments
12(b)	$2.1375 = 0.075F$	M1	uses $I = Ft$ with their impulse
	$F = \frac{2.1375}{0.075} = 29 \text{ N (to 2 sf)}$	A1	AWRT 29 N
		2	

	Question 12 Total	9	
--	--------------------------	----------	--