OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working: otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
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6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

Answer **all** questions in the spaces provided.

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1 (a) Use the mid-ordinate rule with 4 strips to find an estimate for

$$\int_0^{0.8} \sin(e^x)\,dx$$

Give your answer to three decimal places.

[4 marks]

Answer _____

Do not write outside the box

1 (b) A curve has equation $y = \sin(e^x)$

The curve intersects the line $y = 3x - 2$ at a single point where $x = \alpha$

1 (b) (i) Show that α lies between 0.8 and 0.9

[2 marks]

1 (b) (ii) The equation $\sin(e^x) = 3x - 2$ can be rearranged into the form $x = \dfrac{\sin(e^x) + 2}{3}$

Use the iterative formula

$$x_{n+1} = \frac{\sin(e^{x_n}) + 2}{3}$$

with $x_1 = 0.8$ to find the values of x_2 and x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

8

Turn over ►

Do not write outside the box

2 (a) Given that $y = \dfrac{1-3x}{2x+5}$ show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(2x+5)^2}$ where k is a constant.

[2 marks]

2 (b) Given that $y = \ln\left(\dfrac{1-3x}{2x+5}\right)$ find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

[2 marks]

Answer _____

4

Do not write outside the box

3 (a) Express $16\sin\theta + 30\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give your value of α to three significant figures.

[3 marks]

Answer _____

3 (b) Using your answer to **part (a)**

3 (b) (i) write down the minimum value of $16\sin\theta + 30\cos\theta$

[1 mark]

Answer _____

3 (b) (ii) find a value of θ at which this minimum value occurs.

[1 mark]

Answer _____

5

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4 The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = 18x^3 + bx^2 + cx - 4$$

where b and c are constants.

When $\mathrm{f}(x)$ is divided by $(2x+1)$ the remainder is 0

When $\mathrm{f}(x)$ is divided by $(3x-1)$ the remainder is -5

4 (a) (i) Find the value of b and the value of c

[4 marks]

$b =$ _____ $c =$ _____

Do not write outside the box

4 (a) (ii) Show that

$$\mathrm{f}(x) = (2x+1)(px+q)(px-q)$$

where p and q are constants.

[2 marks]

4 (b) Hence show that

$$\frac{\mathrm{f}(x)}{(3x+2)(x^2-2)} = k + \frac{\mathrm{g}(x)}{x^2-2}$$

where k is a constant and g is a linear function.

[2 marks]

8

Do not write outside the box

5 (a) By using a suitable trigonometrical identity, solve the equation

$$3\sec^2(2x-1) = 2 - 4\tan(2x-1)$$

giving all values of x to two decimal places in the interval $0 < x < \pi$

[5 marks]

Answer _____

Do not write outside the box

5 (b) Given that $\cos 2x(1-\cos 4x) \neq 0$, show that

$$\frac{\sin 4x(1-\cos 2x)}{\cos 2x(1-\cos 4x)} = \tan x$$

[4 marks]

9

Turn over for the next question

Do not write outside the box

6 (a) Find the binomial expansion of $(1-x)^{-\frac{1}{3}}$ up to and including the term in x^3

[3 marks]

Answer _____

6 (b) (i) Find the binomial expansion of $\frac{1}{\sqrt[3]{1-2x}}$ up to and including the term in x^3

[2 marks]

Answer _____

Do not write outside the box

6 (b) (ii) State the values of x for which the binomial expansion of $\dfrac{1}{\sqrt[3]{1-2x}}$ is valid.

[2 marks]

Answer _____

6 (c) Use your binomial expansion in **part (b)(i)** with $x = 0.1$ to find an estimate for $\sqrt[3]{10}$ giving your answer to three decimal places.

[3 marks]

Answer _____

10

Do not write outside the box

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \cos x$ on to the graph of $y = \frac{1 + \cos x}{3}$

[4 marks]

Do not write outside the box

7 (b) The function f is defined by

$$f(x) = \frac{1 + \cos x}{3} \quad \text{for } 0 \leq x \leq \pi$$

Find an expression for $f^{-1}(x)$

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

8 (a) (i) Show that

$$\frac{6}{x^3+x} = \frac{a}{x} + \frac{bx}{x^2+1}$$

where a and b are constants.

[2 marks]

8 (a) (ii) Using **part (a)(i)**, show that

$$\int_1^2 \frac{6}{x^3+x}\,\mathrm{d}x = \ln k$$

where k is a rational number.

[4 marks]

Do not write outside the box

8 (b) (i) By writing $\sec y$ as $(\cos y)^{-1}$ use the chain rule to show that $\dfrac{\mathrm{d}(\sec y)}{\mathrm{d}y} = \sec y \tan y$

[2 marks]

8 (b) (ii) Use the substitution $u = \sin x$ to find the exact value of

$$\int_0^{0.5} \frac{u}{\sqrt{(1-u^2)^3}}\,\mathrm{d}u$$

You may use $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$ and $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

[5 marks]

Answer _____

13

Do not write outside the box

9 (a) Express $\dfrac{1}{(30-x)(10-x)}$ in partial fractions.

[2 marks]

Answer _____

9 (b) A chemical experiment produces a substance.

The mass of the substance is x grams after time t minutes.

The mass of the substance increases at a rate directly proportional to $(30-x)(10-x)$

When $t=0$, $x=0$

When $t=2$, $x=6$

Find an expression for x in terms of t

[9 marks]

Do not write outside the box

Answer _____

11

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10 A curve is defined by the parametric equations $x = \cos^3 t$ and $y = \left(2 + \cos^2 t\right)\sin t$

10 (a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\cot t$

[4 marks]

10 (b) Find an equation of the normal to the curve at the point P where $t = p$

[2 marks]

Answer _____

Do not write outside the box

10 (c) The normal to the curve at P intersects the x-axis at the point A and intersects the y-axis at the point B

Find the length of AB

[6 marks]

Answer _____

12

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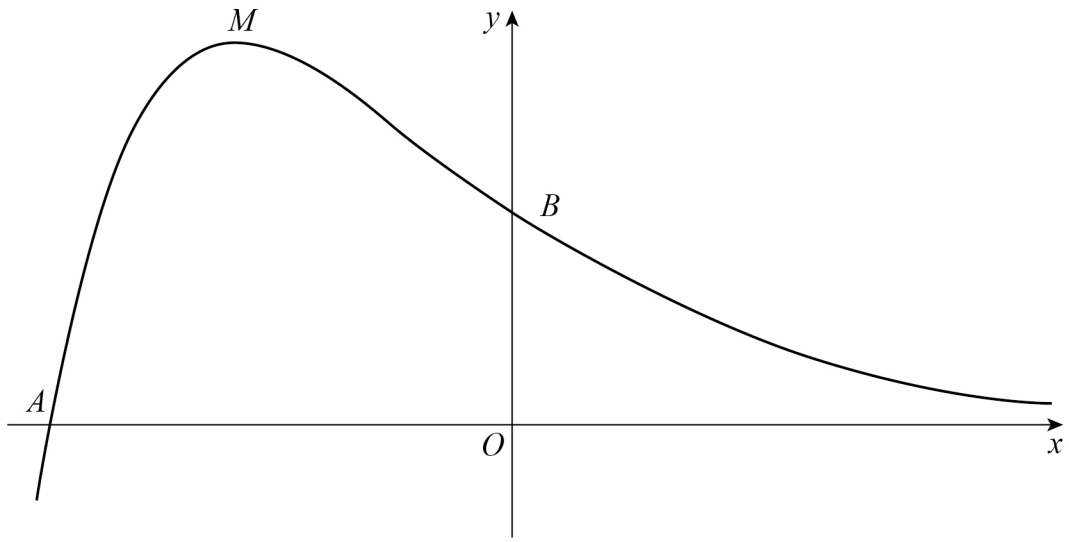
11 The diagram shows a sketch of the curve with equation $y = (5 + 2x)e^{-x}$



The curve crosses the axes at *A* and *B* and has a stationary point at *M*

11 (a) Find the coordinates of *A* and the coordinates of *B*

[1 mark]

Answer _____

11 (b) The curve $y = (5 + 2x)e^{-x}$ has a stationary point at *M*

11 (b) (i) Find $\frac{dy}{dx}$

[2 marks]

Answer _____

Do not write outside the box

11 (b) (ii) Find the exact value of the coordinates of *M*

[2 marks]

Answer _____

11 (b) (iii) Show that *M* is a local maximum.

[2 marks]

11 (c) Find the exact value of the area of the region enclosed by the curve $y = (5 + 2x)e^{-x}$ and the line *AB*

[6 marks]

Turn over ►

Do not write outside the box

Answer _____

13

Do not write outside the box

12 A curve has equation $y = x\ln(x+y)$

Show that

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} + 1$$

[6 marks]

Answer _____

6

Turn over ►

Do not write outside the box

13 (a) The point *A* has coordinates $(2, -3, 7)$. The point *B* has coordinates $(16, -1, -1)$.

Find the distance *AB*

[2 marks]

Answer _____

13 (b) The line l has equation $\mathbf{r} = \begin{bmatrix} 9 \\ -2 \\ q \end{bmatrix} + \mu \begin{bmatrix} 5 \\ -4 \\ 5 \end{bmatrix}$

13 (b) (i) The line l intersects the line *AB*

Find the value of q, where q is an integer.

[3 marks]

Answer _____

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13 (b) (ii) Find the acute angle between the line l and the line AB, giving your answer in degrees to three significant figures.

[4 marks]

Answer _____

13 (c) The point D has coordinates $(-1, 2, 3)$.

The perpendicular from D to the line l meets l at the point C

Show that ABC is a right-angled triangle.

[6 marks]

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Answer _____

15

END OF QUESTIONS

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