OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 The diameters of the metal discs produced by a machine have a normal distribution with standard deviation 1 millimetre.

The machine breaks down. After it is repaired, a random sample of 101 metal discs produced by the machine is taken.

The sample standard deviation is found to be 1.2 millimetres.

The owner of the machine is concerned that the population standard deviation may have increased.

Investigate whether the population standard deviation has increased, using the 1% level of significance.

[7 marks]

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2 Wind speed can be measured in knots where 1 knot $= 1$ nautical mile per hour.

A town has a weather station which measures the wind speed. The maximum daily wind speed at the station has a normal distribution with standard deviation 1.6 knots.

Emma constructs a 94% confidence interval of width 0.4 knots for the mean maximum daily wind speed using data from the weather station.

Emma uses a sample of size of n days.

2 (a) Find n

[3 marks]

Answer _____

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2 (b) The total of the maximum daily wind speeds in Emma's sample is 2300 knots.

Find Emma's confidence interval, giving your values to two decimal places.

[3 marks]

Answer _____

6

Turn over for the next question

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3 At a particular company, a random sample of the employees was selected.

Each employee was asked whether they were 'satisfied' or 'not satisfied' working for the company.

The results are shown in the following table.

	Satisfied	**Not satisfied**	**Total**
Male	21	19	40
Female	27	13	40
Total	48	32	80

Test if there is an association between gender and survey response, using the 10% level of significance.

[9 marks]

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9

Turn over for the next question

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4 The number of cars passing Jane's house per minute has a Poisson distribution with mean λ

Jane conducts a hypothesis test at the 6% level of significance with the hypotheses

$$H_0: \lambda = 5.5$$

$$H_1: \lambda \neq 5.5$$

Jane counts the number of cars passing her house in a randomly chosen minute.

4 (a) The actual value of λ is 5

4 (a) (i) Find the probability that Jane makes a Type II error, giving your answer to three decimal places.

[6 marks]

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Answer _____

4 (a) (ii) Find the power of Jane's hypothesis test, giving your answer to three decimal places.

[2 marks]

Answer _____

4 (b) State the critical region of Jane's hypothesis test.

[1 mark]

Answer _____

9

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5 Huang is investigating the weekly household income in a country.

In this country, the weekly household income has a normal distribution.

Huang takes a random sample of 5 households.

Their weekly household income, in dollars, was recorded in a particular week in 2008 and then recorded again in 2018 in the corresponding week.

The results are shown in the following table.

Household	2008 ($)	2018 ($)
A	600	610
B	597	600
C	602	602
D	605	603
E	598	598

Huang claims that the mean weekly household income in 2018 has not changed compared with the mean weekly household income in 2008

Test Huang's claim, using the 10% level of significance.

[11 marks]

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11

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6 Justine is investigating the heights of women who play basketball.

She takes a random sample of 400 women who play basketball and measures their heights, X centimetres.

She also takes a random sample of 300 women who do not play basketball and measures their heights, Y centimetres.

Her results are summarised as

$$\sum x = 65\,760 \quad \text{and} \quad \sum x^2 = 10\,814\,020$$

$$\sum y = 49\,197 \quad \text{and} \quad \sum y^2 = 8\,070\,022$$

The heights of women who play basketball and the heights of women who do not play basketball are independent.

Justine claims that the mean height of women who play basketball is greater than the mean height of women who do not play basketball.

Test Justine's claim, using the 5% level of significance.

[12 marks]

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12

Turn over ►

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7 The random variables $B_i \ \{i = 1, 2, \ldots, k\}$ are independent and each have a binomial distribution with parameters n and p

7 (a) Show that the random variable $R = \frac{B_1}{n}$ is an unbiased estimator of p

[2 marks]

7 (b) Show that the random variable $T = \frac{1}{kn}\sum_{i=1}^{k} B_i$ is a consistent estimator.

[4 marks]

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7 (c) The random variable T is an unbiased estimator of p

Evaluate the efficiency of estimator T relative to estimator R

Interpret your answer.

[5 marks]

11

Turn over for the next question

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8 Each of the random variables X_i $\{i = 1, 2, \ldots, n\}$ has an exponential distribution with parameter λ

8 (a) Show that the moment generating function of X_i is given by

$$M_{X_i}(t) = \left(1 - \frac{t}{\lambda}\right)^{-1} \quad \text{where} \quad t < \lambda$$

[4 marks]

8 (b) The random variable X satisfies

$$X = \sum_{i=1}^{n} X_i \quad \text{where } X_i \text{ and } X_j \text{ are independent for } i \neq j$$

Using moment generating functions, find $\mathrm{Var}(X)$

[7 marks]

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Answer _____

Question 8 continues on the next page

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8 (c) The random variable Y is such that $Y = 2\lambda X$ where λ is a constant

8 (c) (i) Find $\mathrm{E}(Y)$

[2 marks]

Answer _____

8 (c) (ii) Find $\mathrm{Var}(Y)$

[2 marks]

Answer _____

15

END OF QUESTIONS

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