OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
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12	
TOTAL	

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point $(3, 4.8)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{x+2}}{y(x+1)}$$

Use Euler's step-by-step method with a step length of 0.2 to estimate the value of y when $x = 3.4$

Give your answer to three decimal places.

[5 marks]

Answer _____

5

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2 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

The matrix **B** is defined by $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix}$ where p is a constant and $p > 1$

2 (a) Describe fully the transformation represented by the matrix **A**

[2 marks]

2 (b) State the type of transformation represented by the matrix **B**

[1 mark]

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2 (c) Show that **AB** – **BA** $= k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ where k is a constant.

[4 marks]

2 (d) The matrix **C** represents a reflection in the line $y = (\tan\theta)x$

Find the determinant of **CAB**

[3 marks]

Answer _____

10

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3 The variables X and Y are related by an equation of the form

$$Y = nX + b$$

where n and b are constants.

3 (a) Given that $X = \log_{10} x$ and $Y = \log_{10} y$

show that $y = a\,x^n$ where a is a constant.

[2 marks]

3 (b) An experiment gives the following values of x and y

x	2	4	10	20	40
y	4.6	3.0	1.8	1.2	0.8

Complete the table below.

X	0.30				
Y	0.66				

[2 marks]

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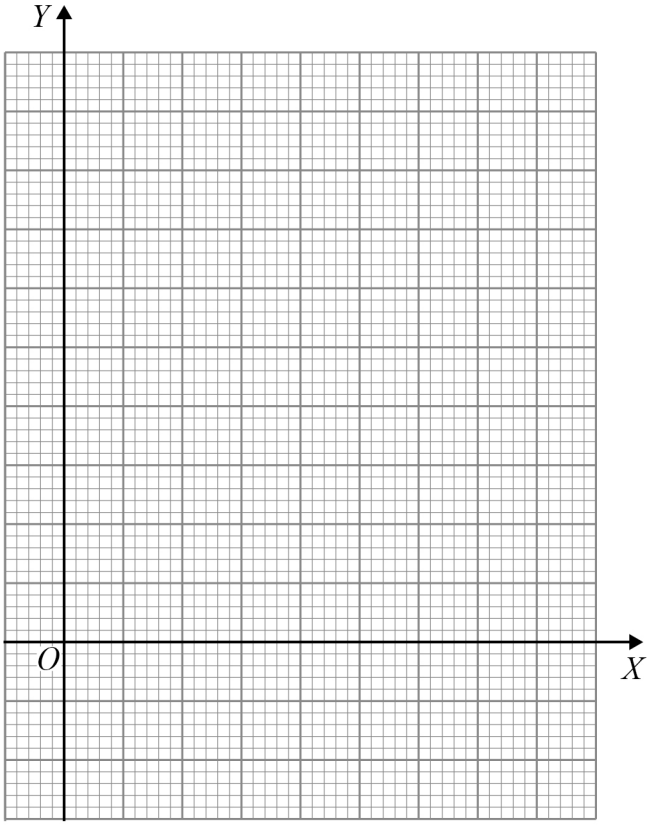
3 (c) (i) On the grid, plot Y against X

Draw a line of best fit for the points you have plotted.

[2 marks]



3 (c) (ii) Use your line of best fit to find estimates for a and n

[4 marks]

$a =$ _____ $n =$ _____

10

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4 The function f is defined by $\mathrm{f}(x) = x^3 - 4x^2 + 6$

4 (a) Show that the equation $\mathrm{f}(x) = 0$ has a root, α, in the interval $3 < \alpha < 4$

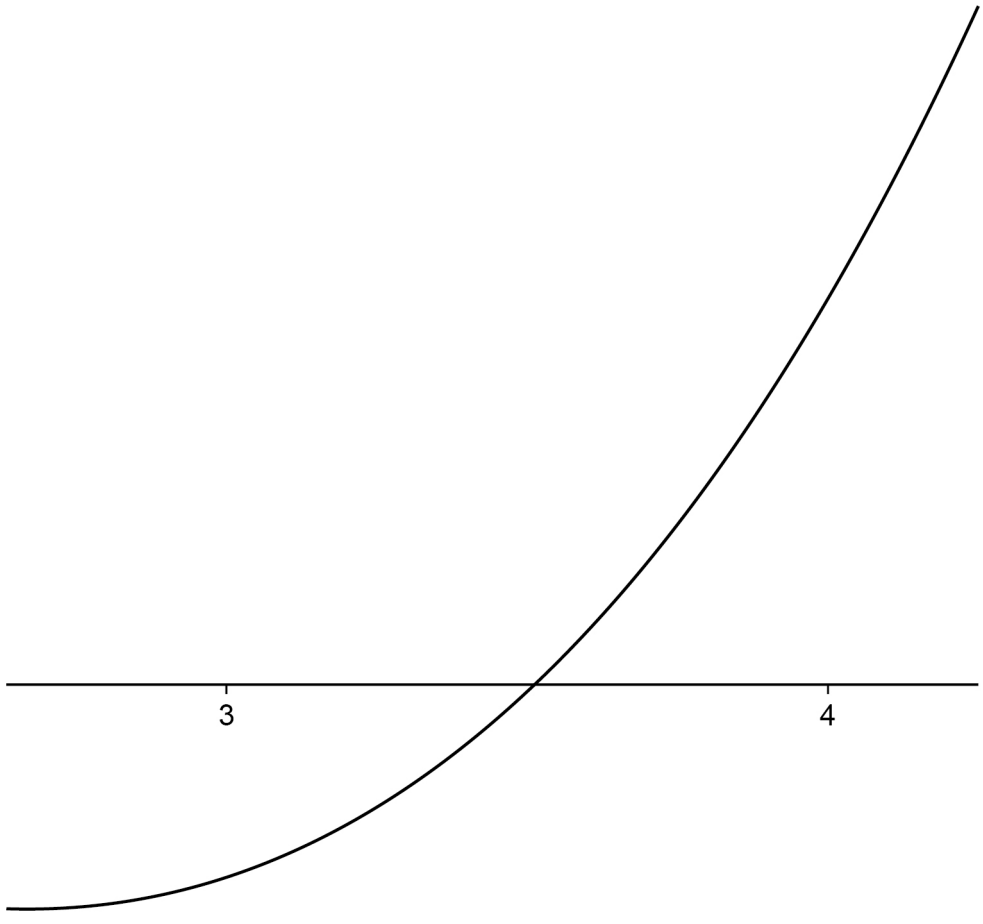
[2 marks]

4 (b) The diagram below shows part of the graph of $y = \mathrm{f}(x)$

Draw suitable lines on the diagram to show how linear interpolation can be used to find the first two approximations, x_1 and x_2, to α

Mark the positions of x_1 and x_2 on the x-axis.

[4 marks]



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4 (c) Use linear interpolation **three** times to find an estimate for α

Give your answer to two decimal places.

[4 marks]

Answer _____

10

Turn over for the next question

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5 The matrix $\mathbf{Q}$ is defined by $\mathbf{Q} = \begin{bmatrix} -6 & 4 \\ -2 & 1 \end{bmatrix}$

The matrix $\mathbf{R}$ is defined by $\mathbf{R} = \begin{bmatrix} -2 & 3 \\ -16 & 37 \end{bmatrix}$

The matrix $\mathbf{P}$ is such that $4\mathbf{P} + \mathbf{PQ} = \mathbf{R}$

Find $\mathbf{P}$

[5 marks]

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Answer _____

5

Turn over for the next section

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 Customers at a tennis club may rent a racket or use their own racket.

The probability that a randomly selected customer rents a racket is 0.18

The random variable R represents the number of customers on a particular day up to and including the first customer who rents a racket.

6 (a) Find the exact probability that the third customer is the first to rent a racket.

[1 mark]

Answer _____

6 (b) Find the probability that the first six customers do not rent a racket, giving your answer to three significant figures.

[2 marks]

Answer _____

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6 (c) Find $\mathrm{E}(R)$ giving your answer to three significant figures.

[1 mark]

Answer _____

6 (d) Find $\mathrm{Var}(R)$ giving your answer to three significant figures.

[1 mark]

Answer _____

6 (e) The random variable S represents the number of customers on a particular day up to and including the first customer who rents a pair of shorts.

The variance of S is 3

R and S are independent.

Find $\mathrm{Var}(R-S)$ giving your answer to three significant figures.

[1 mark]

Answer _____

6

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7 The probability generating function of a Bernoulli random variable X_i is given by

$$G_{X_i}(t) = 0.22 + 0.78t$$

X_1 and X_2 are independent.

Let Y be the random variable such that $Y = X_1 + X_2$

7 (a) Find $G_Y''(t)$

[4 marks]

Answer _____

Do not write outside the box

7 (b) Find $\mathrm{Var}(Y)$

[3 marks]

Answer _____

7

Turn over for the next question

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8 In a bag there are 35 balls.

Each ball has a unique whole number on it from 1 to 35

8 (a) A ball is randomly selected from the bag.

Find the probability that the number on the ball is greater than 28

[1 mark]

Answer _____

8 (b) In a game, a ball is randomly selected from the bag and its number is recorded.

The ball is **not** replaced in the bag.

A second ball is then randomly selected from the bag and its number is recorded.

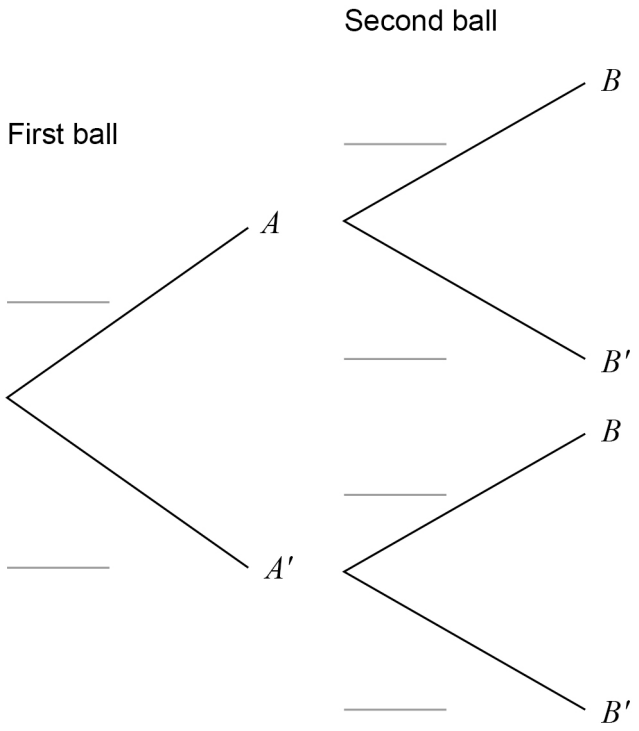
Event A is 'the **first** ball selected from the bag has a number greater than 28'

Event B is 'the **second** ball selected from the bag has a number greater than 28'

The game is won if the first ball has a number greater than 28 and the second ball has a number less than or equal to 28

8 (b) (i) Complete the tree diagram by giving the probability associated with each branch.

[3 marks]



8 (b) (ii) Jamie plays the game.

Find the probability that Jamie wins the game given that the second ball has a number less than or equal to 28

[3 marks]

Answer _____

7

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 A ball of mass 0.3 kg is moving at a speed of 7 m s^{-1} when it collides with a vertical wall.

The velocity of the ball is perpendicular to the wall when the ball collides with the wall.

The ball rebounds with speed 5 m s^{-1}

Calculate the magnitude of the impulse exerted by the wall on the ball.

[2 marks]

Answer _____

2

Turn over for the next question

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10 A particle is moving between two points, *A* and *B*, on a line.

The speed of the particle can be found using the formula

$$v^2 = \omega^2\left(\left(\frac{d}{2}\right)^2 - x^2\right)$$

where

v is the speed of the particle in m s^{-1}

ω is a constant

d is the distance between the points *A* and *B* in metres

x is the distance of the particle from the midpoint of *AB* in metres.

Use dimensional analysis to find the units of ω

[3 marks]

Answer _____

3

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11 A boat is travelling at a speed of 6 m s^{-1} relative to the water.

The direction of the boat's motion relative to the water is due east.

The water is flowing towards the south west at a speed of 2 m s^{-1}

Find the resultant velocity of the boat, giving the direction as a bearing to the nearest degree.

[5 marks]

Answer _____

5

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12 Three spheres *A, B* and *C* of equal radius are at rest on a straight horizontal line.

The mass of *A* is 3 kg

The mass of *B* is 2 kg

The mass of *C* is 1 kg

Sphere *A* is set in motion so that it moves with speed 5 m s^{-1} towards sphere *B*

The two spheres collide directly.

The coefficient of restitution between *A* and *B* is 0.4

12 (a) Find the speed of *A* and the speed of *B* after their collision.

[5 marks]

Speed of *A* = _____ Speed of *B* = _____

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12 (b) The sphere *B* then collides directly with sphere *C*

The coefficient of restitution between *B* and *C* is e

The spheres *A* and *B* do not collide again.

Show that $e \leq \frac{m}{n}$ where m and n are integers.

[5 marks]

10

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