OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
(9665/FM03) Unit FP2 Pure Mathematics

Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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12	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 A curve C has equation

$$y = \tan^{-1}(x+1) + \tanh^{-1}\left(\frac{x}{2}\right) \quad \text{where} \quad -2 < x < 2$$

1 (a) Find $\frac{dy}{dx}$

[2 marks]

Answer ______________________________

1 (b) **Hence** find an equation of the normal to C at the point P on the curve given that the x-coordinate of P is 0

[3 marks]

Answer ______________________________

5

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2 The matrix $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2 (a) Describe fully the single transformation represented by the matrix $\mathbf{A}$

[2 marks]

2 (b) For this transformation, state the line of invariant points.

[1 mark]

Answer ______________________

3

Turn over for the next question

Do not write outside the box

3 (a) Express $\dfrac{6}{(r-1)(r+1)}$ in the form $\dfrac{A}{r-1}+\dfrac{B}{r+1}$, where A and B are integers.

[2 marks]

$\dfrac{6}{(r-1)(r+1)} =$ ______________________

3 (b) Use the method of differences to show that

$$\sum_{r=2}^{n}\frac{6}{(r-1)(r+1)}=\frac{an^2+bn+c}{2n(n+1)}$$

where a , b and c are integers.

[4 marks]

Do not write outside the box

6

Turn over for the next question

Do not write outside the box

4 Solve the differential equation

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$$

given that $y = 4$ and $\frac{dy}{dx} = 1$ when $x = 0$

[6 marks]

Answer ____________________

6

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5 (a) Explain why $\int_{0}^{e^2} \ln x \, dx$ is an improper integral.

[1 mark]

5 (b) Evaluate $\int_{0}^{e^2} \ln x \, dx$ showing the limiting process used.

[6 marks]

Answer

7

Turn over ►

6 (a) A student states that vectors **r**, **m** and **n** can be found such that

$$\mathbf{r} \times \mathbf{m} = \mathbf{n} \quad \text{and} \quad \mathbf{m} \cdot \mathbf{n} = 12$$

Explain why the student is **not** correct.

[2 marks]

6 (b) The points A, B and C have position vectors **a**, **b** and **c** respectively relative to an origin O, where

$$\mathbf{a} = 2\mathbf{i} + p\mathbf{j} - \mathbf{k} \qquad \mathbf{b} = -p\mathbf{i} + 4\mathbf{j} - 7\mathbf{k} \qquad \mathbf{c} = -4\mathbf{i} + 2p\mathbf{j} - 9\mathbf{k}$$

and p is real.

The position vectors **a**, **b** and **c** define the edges of a parallelepiped.

The volume of the parallelepiped is 17 cubic units.

Use a scalar triple product to find the four possible values of p

[6 marks]

Answer ____________________

8

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7 The matrix $\mathbf{M} = \begin{bmatrix} 3 & -2 \\ 5 & p \end{bmatrix}$ where p is a constant.

The matrix $\mathbf{M}$ has two distinct eigenvalues.

One of the eigenvalues is 1

7 (a) Find the other eigenvalue.

[4 marks]

Answer ____________________

7 (b) Find an eigenvector for each eigenvalue.

[3 marks]

Eigenvectors ____________________ and ____________________

7

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8 (a) By direct expansion, or otherwise, show that

$$\begin{vmatrix} k & 2 & k-4 \\ 2k-2 & 3k-2 & 4 \\ 2k+3 & 3k & 5 \end{vmatrix} = -8k^2 + pk + q$$

where p and q are positive integers.

[2 marks]

8 (b) A system of equations is given such that

$$kx + 2y + (k-4)z = a$$

$$(2k-2)x + (3k-2)y + 4z = b$$

$$(2k+3)x + 3ky + 5z = c$$

where k , a , b and c are real constants.

8 (b) (i) Find the two values of k for which the system of equations does **not** have a unique solution.

[2 marks]

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Answer ______________________

8 (b) (ii) For the integer value of k found in **part (b)(i)**, find an expression for b in terms of a and c such that the system of equations is consistent.

[3 marks]

$b=$ ______________________

7

Do not write outside the box

9 (a) Explain why the cubic equation

$$ax^3 + bx^2 + cx + 8 = 0$$

where a , b and c are real numbers, cannot have exactly one non-real root.

[1 mark]

9 (b) The equation

$$2z^3 + pz^2 + 4z - 6\mathrm{i} = 0$$

where p is a constant, has roots α , β and γ

9 (b) (i) Show that

$$(\alpha\beta + 2)(\alpha\gamma + 2)(\beta\gamma + 2) = k - 3\mathrm{i}p$$

where k is an integer.

[4 marks]

Do not write outside the box

9 (b) (ii) Find a cubic equation which has roots $\alpha\beta + 2$, $\alpha\gamma + 2$ and $\beta\gamma + 2$

[3 marks]

Answer

8

Turn over ►

Do not write outside the box

10 (a) Prove by induction that, for all integers $n \geq 1$

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

[5 marks]

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10 (b) Find, in terms of π, the two smallest positive values of θ that satisfy the equation

$$2(\cos\theta + \mathrm{i}\sin\theta)^3 = 1 - \mathrm{i}\sqrt{3}$$

[4 marks]

Answer

9

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11 The plane Π_1 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

The plane Π_2 has equation $\mathbf{r} \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = 5$

11 (a) Find an equation for the plane Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = d$

[4 marks]

Answer ______________________________

11 (b) Find the acute angle between the planes Π_1 and Π_2 giving your answer to the nearest $0.1°$

[4 marks]

Do not write outside the box

Answer

11 (c) Write down a Cartesian equation of the plane Π_2

[1 mark]

Answer

11 (d) Find a vector equation for the line of intersection of the planes Π_1 and Π_2 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

[5 marks]

Answer

14

Turn over ►

Do not write outside the box

12 It is given that $y = \ln\left[e^{2x}\left(1 + \tan^2 x\right)\right]$

12 (a) (i) Show that $\dfrac{dy}{dx} = 2\left(1 + \tan x\right)$

[2 marks]

12 (a) (ii) Find $\dfrac{d^4 y}{dx^4}$ in terms of x

[3 marks]

Answer

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12 (b) Hence, show that the first three non-zero terms in ascending powers of x in the Maclaurin series of $\ln\left[e^{2x}\left(1+\tan^2 x\right)\right]$ are

$$2x + x^2 + \frac{1}{6}x^4$$

[3 marks]

12 (c) Show that

$$\lim_{x \to 0}\left[\frac{2\ln(\cos x) + x\sin x}{2\sqrt{x^8 + x^{10}}}\right]$$

exists and state its value.

[4 marks]

Answer ________________

12

Turn over ►

13 (a) Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of e^{θ} and $e^{-\theta}$ to show that

$$1+\sinh^2\theta=\cosh^2\theta$$

[3 marks]

13 (b) Use an integrating factor to find the general solution of the differential equation

$$\frac{dy}{dx}+\frac{x}{1+x^2}y=2$$

Give your answer in the form $y=f(x)$

[11 marks]

Do not write outside the box

Answer

14

Turn over ►

14 A curve C_1 is given parametrically by the equations

$$x = 2e^{0.5\theta}\cos\theta \quad \text{and} \quad y = 2e^{0.5\theta}\sin\theta$$

The point P on C_1 is where $\theta = 0$

The point Q on C_1 is where $\theta = \pi$

14 (a) Find the length of the arc PQ of the curve C_1

Give your answer in an exact form.

[7 marks]

Answer

14 (b) A curve C_2 has polar equation

$$r = 2e^{0.5\theta} - 1 \quad \text{where} \quad 0 \le \theta \le \pi$$

The point D on C_2 is where $\theta = 0$

The point E on C_2 is where $\theta = \pi$

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14 (b) (i) Sketch the curve C_2

[2 marks]

14 (b) (ii) By finding the polar equation of the curve C_1, or otherwise, show that the area of the region bounded by C_1 and C_2 and the line segments PD and QE is

$$\frac{1}{2}\left(a\,\mathrm{e}^{\frac{\pi}{2}}+b+c\,\pi\right)$$

where a, b and c are integers.

[5 marks]

14

END OF QUESTIONS

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