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MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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2 2 6 X M A 0 5 / M S

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
2	$\bar{X} = \frac{1.25 \times 3 + 2.5 \times 4 + 3.75 \times 5 + 2.5 \times 1 + 1.25 \times 2}{1.25 + 2.5 + 3.75 + 2.5 + 1.25}$ $\bar{Y} = \frac{1.25 \times 1 + 2.5 \times 2 + 3.75 \times 3 + 2.5 \times 2 + 1.25 \times 1}{1.25 + 2.5 + 3.75 + 2.5 + 1.25}$ $\bar{X} = \frac{37.5}{11.25}, \quad \bar{Y} = \frac{23.75}{11.25}$ $\left(\frac{10}{3}, \frac{19}{9} \right)$	<p>M1</p> <p>A1 A1</p>	<p>Forming an equation for the x-coordinate or y-coordinate of the centre of mass. Condone one slip or omission</p> <p>oe for x-coordinate, AWRT 3.3 oe for y-coordinate, AWRT 2.1</p>
	Total	3	

Q	Answer	Marks	Comments
3(a)	$[\text{Resultant force} =] \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 7 \\ 3 \\ -3 \end{bmatrix}$ $= \begin{bmatrix} 12 \\ 2 \\ 4 \end{bmatrix}$ $\mathbf{a} = \frac{1}{2} \begin{bmatrix} 12 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix} [\text{m s}^{-2}]$	<p>M1</p> <p>A1</p> <p>A1ft</p>	<p>Adds together the three forces PI by resultant force <u>vector</u> with at least two correct components</p> <p>Finds the correct resultant force vector</p> <p>oe, finds the correct acceleration from their resultant force vector Do not ISW</p> <p>If M0 awarded, SC1 for AWRT 6.4, such as $\sqrt{41}$ [the magnitude of the acceleration]</p>
		3	
3(b)	$\begin{bmatrix} 12 \\ 2 \\ 4 \end{bmatrix} + \mathbf{F}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\mathbf{F}_4 = \begin{bmatrix} -12 \\ -2 \\ -4 \end{bmatrix}$	B1ft	<p>ft their resultant force vector from (a) Do not ISW</p>
		1	
	Total	4	

Q	Answer	Marks	Comments
4(a)	<p>Forces up the slope: $F + 60\cos(15^\circ)$</p> <p>Forces down the slope: $500\sin(15^\circ)$</p> <p>Equilibrium requires: $F = 500\sin(15^\circ) - 60\cos(15^\circ)$</p> $F = 500 \times \frac{\sqrt{6} - \sqrt{2}}{4} - 60 \times \frac{\sqrt{6} + \sqrt{2}}{4}$ $F = 110\sqrt{6} - 140\sqrt{2}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>At least one of the expressions for the 'forces up the slope' or 'forces down the slope' correct PI by correct equation for F</p> $125(\sqrt{6} - \sqrt{2})$ <p>Correct equation for F</p> <p>Uses both of the given sine and cosine relationships with their equation for F PI</p> <p>CAO</p>
		4	
4(b)	<p>Forces into the slope $500\cos(15^\circ) + 60\sin(15^\circ)$ $\left[= 125(\sqrt{6} + \sqrt{2}) + 15(\sqrt{6} - \sqrt{2}) \right]$</p> <p>Equilibrium requires: $R = 500\cos(15^\circ) + 60\sin(15^\circ)$</p> $\left[R = 140\sqrt{6} + 110\sqrt{2} \right]$ $110\sqrt{6} - 140\sqrt{2} \leq \mu(500\cos(15^\circ) + 60\sin(15^\circ))$ $\frac{110\sqrt{6} - 140\sqrt{2}}{140\sqrt{6} + 110\sqrt{2}} \leq \mu$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>At least one contribution correct PI by correct expression for normal reaction R</p> <p>Correct expression for normal reaction R Note $R = 498.49... \text{ [N]}$</p> <p>Use of $F \leq \mu R$ with their R and with their F of the form $a\sqrt{6} + b\sqrt{2}$ Note: $F = 71.45... \text{ [N]}$ Condone equality or strict inequality CAO such as $\frac{616 - 317\sqrt{3}}{467} \leq \mu, 0.14[334...] \leq \mu$ Must be weak inequality for μ and there must be no upper limit</p>
		4	
	Total	8	

Q	Answer	Marks	Comments
5(a)(i)	Initial vertical velocity $= 15 \sin(39^\circ) \quad [= 9.4398... \text{ m s}^{-1}]$ Time taken to reach maximum height $= \frac{0 - 15 \sin(39^\circ)}{-9.8} \quad [= 0.9632... \text{ s}]$ Time of flight $= 2 \times \frac{15 \sin(39^\circ)}{9.8}$ $= 1.93 \text{ [s]}$	M1 m1 A1	PI by correct time to maximum height PI by correct time of flight oe CAO to 3 sf
		3	
5(a)(ii)	Horizontal velocity $= 15 \cos(39^\circ) \quad [= 11.657... \text{ m s}^{-1}]$ Range $= 15 \cos(39^\circ) \times \left(2 \times \frac{15 \sin(39^\circ)}{9.8} \right)$ $= 22 \text{ [metres]}$	M1 A1ft	Finds the horizontal component of velocity ft their (a)(i) to correctly find their range to 2 or more sf Note: correct unrounded answer is 22.457... metres
		2	
5(a)(iii)	Air resistance on the pebble is negligible	E1	Any plausible assumption
		1	
5(b)(i)	Maximum height $= 15 \sin(39^\circ) \times \left(\frac{15 \sin(39^\circ)}{9.8} \right)$ $+ \frac{1}{2} \times (-9.8) \times \left(\frac{15 \sin(39^\circ)}{9.8} \right)^2$ $= 4.5 \text{ [metres]}$	M1 A1	Use of $s = ut + \frac{1}{2}at^2$ or $v^2 = u^2 + 2as$ with $v = 0$, $u = 15 \sin(39^\circ)$ and $a = -9.8$ Condone one slip AWRT 4.5 or 4.55 metres Note: correct unrounded answer is 4.546... metres
		2	
5(b)(ii)	11 [metres]	B1ft	ft half their answer to (a)(ii) to 2 sf
		1	
	Total	9	

Q	Answer	Marks	Comments
6(a)	Kinetic energy of cyclist and bicycle $= \frac{1}{2} \times 70 \times 6.0^2$ $= 1260 \text{ [J]}$	M1 A1	Use of $E = \frac{1}{2}mv^2$ Accept 1300 [J]
		2	
6(b)(i)	Driving force provided by cyclist $= \frac{150}{6} = 25 \text{ [N]}$ As the cyclist and bicycle are not accelerating, the total resistive force must be equal to the driving force.	B1 E1	Must show calculation leading to 25 Explanation based upon acceleration or resultant force being zero
		2	
6(b)(ii)	150 J As the cyclist is not gaining/losing kinetic energy or gravitational potential energy or All the work she does each second is against the resistive forces.	B1 E1	Must include units Explanation based on energy argument
		2	

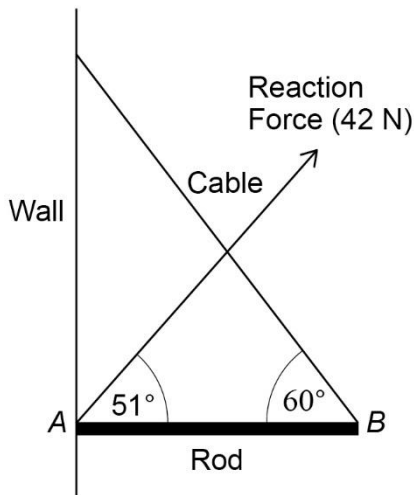
Q	Answer	Marks	Comments
6(c)(i)	<p>Force down the road $= 70 \times 9.8 \times \sin(1.5^\circ) + 25$</p> <p>$= 42.957... \text{ [N]}$</p> <p>Resultant force on cyclist and bicycle $70a = 40 - 42.957...$</p> <p>$70a = -2.957...$</p> <p>Constant acceleration of cyclist and bicycle $a = \frac{-2.957...}{70}$</p> <p>$a = -0.042 \text{ [} 2... \text{ m s}^{-2} \text{]}$</p>	<p>M1 A1</p> <p>m1</p> <p>A1</p>	<p>M1: Sum of 25 and component of weight, condone trigonometric error A1: All correct</p> <p>Forms an equation for the resultant force ft their forces down the road</p> <p>AWFW [-0.043, -0.042] Must be negative</p>
		4	
6(c)(ii)	<p>$v^2 = u^2 + 2as$</p> <p>$= 6^2 + 2 \times (-0.0422...) \times 100$</p> <p>$[\Rightarrow v =] 5.2 \text{ [} 488... \text{ m s}^{-1} \text{]}$</p>	<p>M1</p> <p>A1</p>	<p>ft their (c)(i) even if positive</p> <p>CAO</p>
		2	
6(c)(iii)	<p>$E = mg\Delta h$</p> <p>$= 70 \times 9.8 \times 100 \sin(1.5^\circ)$</p> <p>$= 1800 \text{ [J]}$</p>	<p>M1</p> <p>A1</p>	<p>Unrounded answer is 1795.73... [J]</p>
		2	
	Total	14	

Q	Answer	Marks	Comments
7(a)(i)	$[x =] (u \cos \alpha)t$	B1	
		1	
7(a)(ii)	$[y =] (u \sin \alpha)t - \frac{1}{2}gt^2$	B1	
		1	
7(b)	$t = \frac{x}{u \cos \alpha}$ $y = (u \sin \alpha) \times \frac{x}{u \cos \alpha} - \frac{1}{2}g \times \left(\frac{x}{u \cos \alpha} \right)^2$ $(u \sin \alpha) \times \frac{x}{u \cos \alpha} = x \tan \alpha$	B1 M1 A1	Seen or used M1: Eliminates t in both terms with at least one term correct A1: All correct
	$-\frac{1}{2}g \times \left(\frac{x}{u \cos \alpha} \right)^2 = -\frac{gx^2}{2u^2} \times \frac{1}{\cos^2 \alpha}$		
	$-\frac{gx^2}{2u^2} \times \frac{1}{\cos^2 \alpha} = -\frac{gx^2}{2u^2} \sec^2 \alpha$		
	$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha$	A1	AG Be convinced
		4	

Q	Answer	Marks	Comments
7(c)(i)	<p>At maximum height, the vertical component of velocity is zero.</p> $0 = u \sin \alpha - gt$ $t = \frac{u \sin \alpha}{g}$ $y = (u \sin \alpha)t - \frac{1}{2}gt^2$ $y = (u \sin \alpha) \times \frac{u \sin \alpha}{g} - \frac{1}{2}g \left(\frac{u \sin \alpha}{g} \right)^2$ $y = \frac{u^2 \sin^2 \alpha}{2g}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>or $u \sin \alpha$ and $0 = u^2 + 2as$</p> <p>Substitutes in correct expression for time to reach maximum height</p> <p>or $s = \frac{0 - (u \sin \alpha)^2}{2 \times -g}$ or better</p> <p>CAO</p>
		3	
7(c)(ii)	$16 = \frac{20^2 \sin^2 \alpha}{2 \times 9.8}$ $\sin \alpha = [\pm] \frac{7\sqrt{10}}{25}$	M1	PI by correct answer ft their (c)(i)
	$\alpha = 62$	A1	AWRT 62
		2	
	Total	11	

Q	Answer	Marks	Comments
8(a)	The string has zero mass.	E1	oe
		1	
8(b)	<p>Forces in the vertical plane $T \cos \theta = mg$</p> <p>Forces in the horizontal plane $T \sin \theta = \frac{mv^2}{r}$</p> <p>Radius of circle particle is moving on $r = 0.5 \sin \theta$</p> <p>$\tan \theta = \frac{v^2}{gr}$</p> <p>$\tan \theta = \frac{v^2}{g \times 0.5 \sin \theta}$</p> <p>$g \sin \theta \tan \theta = 2v^2$</p> <p>$g \sin^2 \theta = 2v^2 \cos \theta$</p>	<p>M1 A1</p> <p>B1</p> <p>B1</p> <p>m1</p> <p>A1</p>	<p>M1 for $T \cos \theta$</p> <p>Seen or used</p> <p>Eliminates T or r by combining two of the equations</p> <p>AG Be convinced</p>
		6	
8(c)(i)	<p>$9.8 \sin^2 \theta = 2 \times 4.0^2 \times \cos \theta$</p> <p>$9.8(1 - \cos^2 \theta) = 2 \times 4.0^2 \times \cos \theta$</p> <p>$9.8 \cos^2 \theta + 32 \cos \theta - 9.8 = 0$</p> <p>$\cos \theta = 0.2819...$</p> <p>$\theta = 74$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>Uses $\sin^2 \theta + \cos^2 \theta = 1$</p> <p>PI by correct quadratic equation in $\cos \theta$</p> <p>Forms a correct quadratic equation in $\cos \theta$</p> <p>Correct value for $\cos \theta$</p> <p>PI by correct answer</p> <p>CAO, AWRT 74</p>
		4	
8(c)(ii)	<p>$\omega = \frac{v}{r} = \frac{4.0}{0.5 \sin(73.6...^\circ)}$</p> <p>$\omega = 8.3 \text{ rad s}^{-1}$</p>	<p>M1</p> <p>A1</p>	<p>ft their (c)(i)</p> <p>Must use $r = 0.5 \sin \theta$</p> <p>CAO</p>
		2	
	Total	13	

Q	Answer	Marks	Comments
9(a)	No resultant force	E1	oe
	No resultant moment	E1	oe
		2	
9(b)	Anticlockwise moments about A $1.2 \times T \sin(60^\circ)$	M1	M1 for one moment calculation.
	Clockwise moments about A $0.7 \times 8.0 \times 9.8$		54.88 [N m]
	Equilibrium $1.2 \times T \sin(60^\circ) = 0.7 \times 8.0 \times 9.8$	A1	Both moments correct and used with the equilibrium condition
	$T = \frac{0.7 \times 8.0 \times 9.8}{1.2 \sin(60^\circ)}$ $T = 52.8 \text{ [N]}$	A1	CAO to 3 sf
		3	

Q	Answer	Marks	Comments
9(c)	<p>Vertical component of reaction force (via equilibrium of forces)</p> $= 8.0 \times 9.8 - 52.8 \dots \times \sin(60^\circ)$ $= 32.6 [66 \dots \text{N}]$ <p>Horizontal component of reaction force (via equilibrium of forces)</p> $[52.8 \dots \times \cos(60^\circ) =] 26.4 [04 \dots \text{N}]$ <p>Magnitude of reaction force</p> $= \sqrt{(26.404 \dots)^2 + (32.666 \dots)^2}$ $= 42 \text{ [N]}$ <p>Direction of reaction force relative to horizontal</p> $\tan \alpha = \frac{32.666 \dots}{26.404 \dots}$ $\alpha = 51^\circ \text{ above the horizontal}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>ft their (b)</p> <p>Correct vertical component of reaction force</p> <p>ft their (b)</p> <p>AWRT 42 [N]</p> <p>PI</p> <p>Angle consistent with diagram AWRT 51° Allow AWRT 39° to the vertical</p>
	 <p>The diagram shows a horizontal rod AB. At point A, the rod is attached to a vertical wall. A cable is attached to the wall at a point above A and to the rod at point B. The angle between the wall and the rod is labeled 51°. The angle between the cable and the rod is labeled 60°. An arrow labeled 'Reaction Force (42 N)' points upwards from point A. The rod is labeled 'Rod' and the wall is labeled 'Wall'.</p>		
		6	
	Total	11	