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(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	H_0: $\sigma = 1$ H_1: $\sigma > 1$	B1	Both hypotheses, **oe**
	$\nu = 100$	B1	**PI** by correct critical value
	$\frac{(n-1)s^2}{\sigma^2} = \frac{(101-1)\times 1.2^2}{1^2}$	M1	**PI** Condone one slip
	$= 144$	A1	
	$X^2_{100}(0.99) = 135.807$	B1	Finds correct critical value or correct probability, 0.0026
	144 > 135.807	A1ft	Correctly compares their X^2 test statistic and their critical value or their probability and 0.01
	Reject H_0. Evidence to suggest that the standard deviation of the diameters of metal discs produced by the machine has increased	E1ft	States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a X^2 model
	Total	**7**	

Q	Answer	Marks	Comments
2(a)	$z = 1.8808$	**B1**	**AWRT** 1.88
	$1.8808 \times \frac{1.6}{\sqrt{n}} = 0.2$	**M1**	Set up an equation with their $z \times \frac{1.6}{\sqrt{n}}$ **oe**
	$n = 226$	**A1**	226 or 227
		3	
2(b)	$\overline{x} = 10.18$	**B1**	**AWRT**
	10.18 ± 0.2	**M1**	Follow through their $\overline{x}$
	(9.98, 10.38)	**A1**	**AWRT**
		3	
	Total	**6**	

Q	Answer	Marks	Comments
3	H_0: There is not an association between gender and survey response H_1: There is an association between gender and survey response	**B1**	Both hypotheses, variables must be stated in at least the null hypothesis **oe**
	<table><tr><td>Expected</td><td>S</td><td>N</td></tr><tr><td>M</td><td>24</td><td>16</td></tr><tr><td>F</td><td>24</td><td>16</td></tr></table>	**M1** **A1**	At least two expected values correct All correct
	$\sum \frac{(\lvert O-E\rvert-0.5)^2}{E} = \frac{(\lvert 21-24\rvert-0.5)^2}{24} + \frac{(\lvert 27-24\rvert-0.5)^2}{24} + \frac{(\lvert 19-16\rvert-0.5)^2}{16} + \frac{(\lvert 13-16\rvert-0.5)^2}{16}$	**M1**	Attempts to calculate test statistic Condone use of $\sum \frac{(O-E)^2}{E}$
	= 1.30	**A1**	**AWRT** 1.3
	$\nu = 1$	**B1**	**PI** by correct critical value
	Critical value = 2.71	**B1**	**AWRT** 2.7
	1.30 < 2.71	**A1ft**	Correctly compares their χ^2 test statistic and their critical value
	Accept H_0. Evidence to suggest that there is not an association between gender and survey response	**E1ft**	States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a χ^2 model
	Total	**9**	

Q	Answer	Marks	Comments
4(a)(i)	$P(X \leq 1) = 0.0266$ [< 0.03] $P(X \leq 2) = 0.0884$ [> 0.03]	**M1**	Both seen
	$P(X \geq 11) = 0.0253$ [< 0.03] $P(X \geq 10) = 0.0538$ [> 0.03]	**M1**	Both seen or $P(X \leq 10) = 0.9747$ and $P(X \leq 9) = 0.9462$
	Accept if $2 \leq X \leq 10$	**A1**	**oe, PI**
	X ~Po(5)	**M1**	Use of
	Type II probability = $P(2 \leq X \leq 10)$ $= P(X \leq 10) - P(X \leq 2)$ $= 0.9863 - 0.0404$	**M1**	Follow through their region
	$= 0.946$	**A1**	**AWRT**
		6	
4(a)(ii)	Power = $1 - 0.946$	**M1**	Applies power formula with their Type II probability
	$= 0.054$	**A1ft**	**AWRT** **ft** their probability of Type II error
		2	
4(b)	$X \leq 1$ or $X \geq 11$	**B1ft**	**ft** their acceptance region in part **(a)**
		1	
	Total	**9**	

Q	Answer	Marks	Comments
5	H_0: $\mu_{2008} = \mu_{2018}$ H_1: $\mu_{2008} \neq \mu_{2018}$	**B1**	Both hypotheses
	Household: A, B, C, D, E Difference: 10, 3, 0, –2, 0 or –10, –3, 0, 2, 0	**B1**	All differences
	$\bar{x} = \frac{11}{5}$ or $-\frac{11}{5}$	**B1**	Sight of 2.2 or –2.2 Must be consistent with their differences
	$s^2 = \frac{1}{5-1}\left(113 - \frac{11^2}{5}\right)$	**M1**	Attempt at variance formula Allow one slip **PI**
	$= 22.2$	**A1**	Accept $s =$ **AWRT** 4.71
	$t = \frac{2.2}{\sqrt{\frac{22.2}{5}}}$ or $\frac{-2.2}{\sqrt{\frac{22.2}{5}}}$	**M1**	Using their mean and variance Condone use of $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}}$
	$= 1.04$ or $–1.04$	**A1**	**AWRT** Must be consistent with their differences
	$\nu = 4$	**B1**	**PI** by correct critical value
	t_4 critical value $= 2.13$ or $–2.13$	**B1**	**AWRT** Must be consistent with their differences unless changes -1.04 to 1.04 first
	$1.04 < 2.13$	**A1ft**	Correctly compares their t test statistic and their critical value
	Accept H_0. Evidence to suggest that average weekly household income has not changed between 2008 and 2018	**E1ft**	States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using either a t or z test statistic
	Total	**11**	

Q	Answer	Marks	Comments
6	H_0: $\mu_X = \mu_Y$ H_1: $\mu_X > \mu_Y$	B1	Both hypotheses
	$\bar{x} = \frac{65760}{400}$	B1	Implied by sight of 164.4
	$\bar{y} = \frac{49197}{300}$	B1	Implied by sight of 163.99
	$s_X^2 = \frac{1}{400-1}\left(10814020 - \frac{65760^2}{400}\right)$ or $s_Y^2 = \frac{1}{300-1}\left(8070022 - \frac{49197^2}{300}\right)$	M1	Attempt at one of the variance formulae Allow one slip Implied by one correct answer
	$s_X^2 = 7.71$	A1	**AWRT**
	$s_Y^2 = 7.38$	A1	**AWRT**
	$z = \frac{164.4 - 163.99}{\sqrt{\frac{7.7...}{400} + \frac{7.3...}{300}}}$	M1 M1	Correct numerator for their means, subtracted either way round Correct denominator for their variances
	$= 1.96$	A1	**AWRT**
	z critical value $= 1.64$	B1	Allow 1.645 **AWRT**
	1.96 > 1.64	A1ft	Correctly compares their z or t test statistic and critical value
	Reject H_0. Evidence to suggest that the average height of women who play basketball is larger than the average height of women who don't play basketball	E1ft	States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite **ft** their comparison using z or t test statistic
	Total	**12**	

Q	Answer	Marks	Comments
7(a)	$\mathrm{E}(R) = \frac{1}{n}\mathrm{E}(B_1) = \frac{np}{n}$	**M1**	Finds $\mathrm{E}(R)$ in terms of n and p
	$= p$ therefore unbiased	**A1**	Must see p and conclusion n must not be assigned a value
		2	
7(b)	$\mathrm{Var}(T) = \mathrm{Var}\left(\frac{\sum_{i=1}^{k} B_i}{kn}\right) = \frac{knp(1-p)}{k^2n^2}$	**M1**	Finds $\mathrm{Var}(T)$ in terms of k,n and p, condoning slips May be seen in part **(c)** if not attempted in this part
	$= \frac{p(1-p)}{kn}$ or $\frac{kp(1-p)}{k^2n}$	**A1**	Correct simplification May be seen in part **(c)** if not attempted in this part
	As $n \to \infty$, $\mathrm{Var}(T) \to 0$	**M1**	Applies limiting process to their $\mathrm{Var}(T)$
	Therefore consistent estimator	**A1**	**CSO**
		4	
7(c)	$\mathrm{Var}(R) = Var\left(\frac{B_1}{n}\right) = \frac{np(1-p)}{n^2}$	**M1**	Finds $\mathrm{Var}(R)$ in terms of n and p, condoning slips **PI** by correct simplified $\mathrm{Var}(R)$ or relative efficiency
	$= \frac{p(1-p)}{n}$	**A1**	Correct simplification **PI** by correct relative efficiency
	Relative Efficiency $= \frac{\frac{1}{Var(T)}}{\frac{1}{Var(R)}} = \frac{\frac{kn}{p(1-p)}}{\frac{n}{p(1-p)}}$	**M1**	Applies formula with either the correct $\mathrm{Var}(T)$ or their $\mathrm{Var}(T)$ from part **(b)** and their $\mathrm{Var}(R)$
	$= k$	**A1**	Correct simplification
	Estimator T is more efficient than estimator R	**E1**	**CSO**
		5	
	Total	**11**	

Q	Answer	Marks	Comments
8(a)	$\mathrm{M}_{X_i}(t) = \mathrm{E}\left(e^{tX_i}\right) = \int_0^{\infty} e^{tx}\lambda e^{-\lambda x}\ dx$	M1	Applies mgf formula with any or missing limits Condone spurious notation
	$= \left[\frac{\lambda e^{(t-\lambda)x}}{t-\lambda}\right]_0^{\infty}$	M1	Integrates to $Ae^{(t-\lambda)x}$
	$= -\frac{\lambda}{t-\lambda}$	A1	Applies limits to obtain a correct form **oe**
	$= \left(\frac{\lambda - t}{\lambda}\right)^{-1}$ $= \left(1 - \frac{t}{\lambda}\right)^{-1}$	A1	Reaches correct form with no errors and at least one intermediate line Condone interchangeable x and x_i **AG CSO**
		4	
8(b)	$\mathrm{M}_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-n}$	B1	
	$\mathrm{M}_X'(t) = \frac{n}{\lambda}\left(1 - \frac{t}{\lambda}\right)^{-n-1}$	M1	Differentiates to $A\left(1 - \frac{t}{\lambda}\right)^{-n-1}$
		A1	$\frac{n}{\lambda}\left(1 - \frac{t}{\lambda}\right)^{-n-1}$ **oe**
	$\mathrm{M}_X''(t) = \frac{n(n+1)}{\lambda^2}\left(1 - \frac{t}{\lambda}\right)^{-n-2}$	M1	Differentiates to $A\left(1 - \frac{t}{\lambda}\right)^{-n-2}$
		A1	$\frac{n(n+1)}{\lambda^2}\left(1 - \frac{t}{\lambda}\right)^{-n-2}$ **oe**
	$\mathrm{Var}(X) = \mathrm{M}_X''(0) - \left(\mathrm{M}_X'(0)\right)^2$ $= \frac{n(n+1)}{\lambda^2}\left(1 - \frac{0}{\lambda}\right)^{-n-2} - \left(\frac{n}{\lambda}\left(1 - \frac{0}{\lambda}\right)^{-n-1}\right)^2$	M1	Applies variance formula with their $\mathrm{M}_X''(0)$ and $\mathrm{M}_X'(0)$
	$= \frac{n}{\lambda^2}$	A1	**CAO**
		7	

Q	Answer	Marks	Comments
8(b) ALT	$\mathrm{M}_{X_i}'(t) = \frac{1}{\lambda}\left(1-\frac{t}{\lambda}\right)^{-2}$	**B1**	Condone X for X_i **oe**
		M1	Differentiates to $A\left(1-\frac{t}{\lambda}\right)^{-3}$
	$\mathrm{M}_{X_i}''(t) = \frac{2}{\lambda^2}\left(1-\frac{t}{\lambda}\right)^{-3}$		Condone X for X_i
		A1	$\frac{2}{\lambda^2}\left(1-\frac{t}{\lambda}\right)^{-3}$ **oe**
	$\mathrm{M}_{X_i}'(0) = \frac{1}{\lambda}$ and $\mathrm{M}_{X_i}''(t) = \frac{2}{\lambda^2}$	**M1**	Condone X for X_i X_i must be seen and used
	$\mathrm{Var}(X_i) = \frac{1}{\lambda^2}$	**A1**	Must be $\mathrm{Var}(X_i) = \frac{1}{\lambda^2}$ not $\mathrm{Var}(X) = \frac{1}{\lambda^2}$
	$\mathrm{Var}(X) = n\mathrm{Var}(X_i) = n \times \frac{1}{\lambda^2}$	**M1**	Multiplies their $\mathrm{Var}(X_i)$ by n
	$= \frac{n}{\lambda^2}$	**A1**	**CAO**
		7	
8(c)(i)	$\mathrm{E}(Y) = 2\lambda\mathrm{E}(X) = 2\lambda \times \frac{n}{\lambda}$	**M1**	2λ × their $\mathrm{E}(X)$
	$= 2n$	**A1**	**CAO**
		2	
8(c)(ii)	$\mathrm{Var}(Y) = 4\lambda^2\,\mathrm{Var}(X) = 4\lambda^2 \times \frac{n}{\lambda^2}$	**M1**	$4\lambda^2$ × their $\mathrm{Var}(X)$
	$= 4n$	**A1**	**CAO**
		2	
	Total	**15**	