OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS
FURTHER MATHEMATICS
FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

June 2022

Version 1.0 Final

226XFM02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

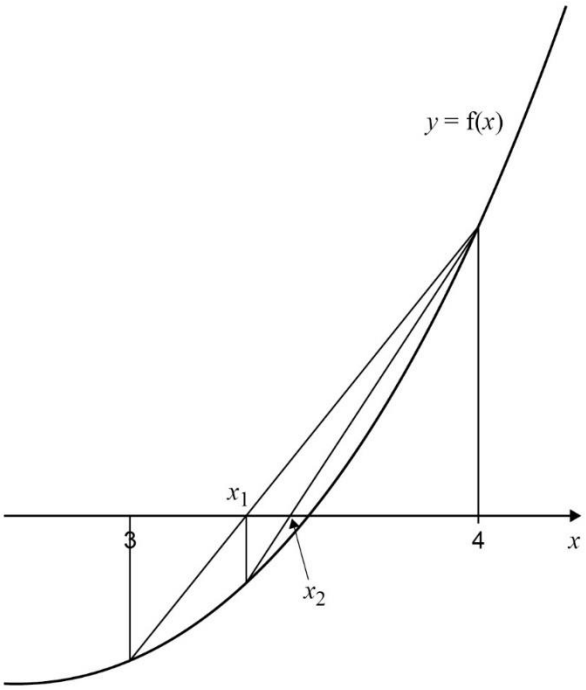
M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$h\,\mathrm{f}(x,y) = 0.2\left(\dfrac{\sqrt{3+2}}{4.8\times(3+1)}\right)$ [= 0.023292375]	M1	PI
	$y_2 = 4.8 + 0.023...$	m1	4.8 + their $h\,\mathrm{f}(x,y)$
	$y_2 = 4.823292375$	A1	**AWRT** 4.82
	$y_3 = 4.823292375 + 0.2\left(\dfrac{\sqrt{3.2+2}}{4.823292375\times(3.2+1)}\right)$	m1	**ft** their y_2
	$y_3 = 4.846$	A1	**CAO**
	Total	**5**	

Q	Answer	Marks	Comments
2(a)	Rotation about O	**B1**	Allow 'rotate'
	60° clockwise	**B1**	**oe**
		2	
2(b)	Shear	**B1**	Ignore additional information about shears and no other transformations mentioned
		1	
2(c)	$\mathbf{AB} = \begin{bmatrix} \frac{1}{2}+\frac{\sqrt{3}}{2}p & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2}+\frac{1}{2}p & \frac{1}{2} \end{bmatrix}$	**B1**	$\left[\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix}\right]$
	$\mathbf{BA} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2}p-\frac{\sqrt{3}}{2} & \frac{1}{2}+\frac{\sqrt{3}}{2}p \end{bmatrix}$	**B1**	$\left[\begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\right]$
	$\mathbf{AB}-\mathbf{BA} = \begin{bmatrix} \frac{1}{2}+\frac{\sqrt{3}}{2}p & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2}+\frac{1}{2}p & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2}p-\frac{\sqrt{3}}{2} & \frac{1}{2}+\frac{\sqrt{3}}{2}p \end{bmatrix}$	**M1**	**ft** their **AB** and **BA**
	$\mathbf{AB}-\mathbf{BA} = \frac{\sqrt{3}}{2}p\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	**A1**	Allow $\begin{bmatrix} \frac{\sqrt{3}}{2}p & 0 \\ 0 & -\frac{\sqrt{3}}{2}p \end{bmatrix}$ and $k = \frac{\sqrt{3}}{2}p$ **CSO**
		4	
2(d)	det **C** $= -1$	**B1**	
	det **A** = det **B** = 1 and det **CAB** = (det **A**)(det **B**)(det **C**)	**M1**	or det **AB** = 1 and det **CAB** = (det **AB**)(det **C**)
	det **CAB** $= 1 \times 1 \times -1 = -1$	**A1**	**CSO**
		3	

Q	Answer	Marks	Comments
2(d) ALT	**CAB** $=$ $\begin{bmatrix} \left(\frac{1}{2}+\frac{\sqrt{3}}{2}p\right)\cos 2\theta+\left(\frac{1}{2}p-\frac{\sqrt{3}}{2}\right)\sin 2\theta & \frac{\sqrt{3}}{2}\cos 2\theta+\frac{1}{2}\sin 2\theta \\ \left(\frac{\sqrt{3}}{2}-\frac{1}{2}p\right)\cos 2\theta+\left(\frac{1}{2}+\frac{\sqrt{3}}{2}p\right)\sin 2\theta & -\frac{1}{2}\cos 2\theta+\frac{\sqrt{3}}{2}\sin 2\theta \end{bmatrix}$	**B1**	
	det **CAB** $=$ $\left(\left(\frac{1}{2}+\frac{\sqrt{3}}{2}p\right)\cos 2\theta+\left(\frac{1}{2}p-\frac{\sqrt{3}}{2}\right)\sin 2\theta\right)\left(-\frac{1}{2}\cos 2\theta+\frac{\sqrt{3}}{2}\sin 2\theta\right)$ $-\left(\left(\frac{\sqrt{3}}{2}-\frac{1}{2}p\right)\cos 2\theta+\left(\frac{1}{2}+\frac{\sqrt{3}}{2}p\right)\sin 2\theta\right)\left(\frac{\sqrt{3}}{2}\cos 2\theta+\frac{1}{2}\sin 2\theta\right)$	**M1**	
	det **CAB** $=-\frac{1}{4}\left(\cos^2 2\theta+\sin^2 2\theta\right)-\frac{3}{4}\left(\cos^2 2\theta+\sin^2 2\theta\right)$		
	det **CAB** $=-1$	**A1**	**CSO**
		3	
	Total	**10**	

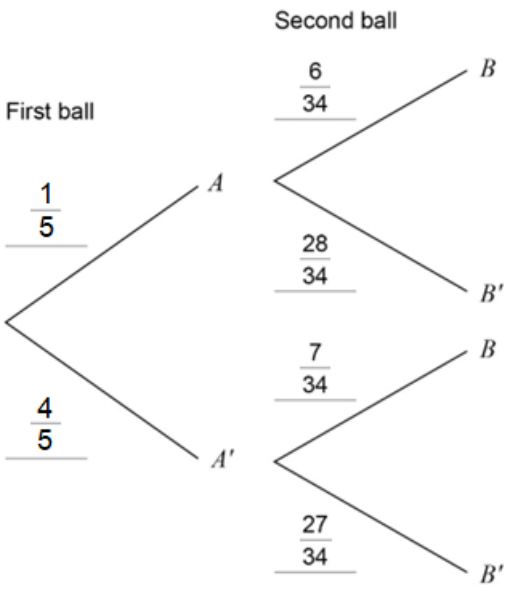
Q	Answer	Marks	Comments
3(a)	$\log_{10} y = n\log_{10} x + b$ $\log_{10} y = \log_{10} x^n + \log_{10} 10^b$ $\log_{10} y = \log_{10} 10^b x^n$ $y = 10^b x^n$	**M1** **A1**	Apply at least one law of logarithms correctly **CSO**
		2	
3(b)	X: 0.30, 0.60, 1, 1.30, 1.60 Y: 0.66, 0.48, 0.26, 0.08, –0.10	**B1** **B1**	X values – at least 1 dp or exact value Y values – at least 2 dp
		2	
3(c)(i)	All their points plotted ± 1 square Line of best fit drawn	**B1** **B1**	Must have linear scales
		2	
3(c)(ii)	$a = 10^{(\text{their } y\text{-intercept})}$ $a = 6.8$ $n =$ their gradient $n = -0.58$	**M1** **A1** **M1** **A1**	**PI** Between 6 and 8 Between –0.7 and –0.5
		4	
	Total	**10**	

Q	Answer	Marks	Comments
4(a)	$f(3) = -3$ and $f(4) = 6$	**M1**	Correct evaluation of a suitable interval
	Change of sign and f is continuous on the interval so α is in the interval (3, 4)	**A1**	Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	
4(b)	Vertical lines to curve at 3 and 4	**M1**	See diagram
	Correct points on graph joined by line that meets x-axis at x_1	**A1**	
	Vertical line to curve at their x_1	**M1**	
	Correct points on graph joined by line that meets x-axis at x_2	**A1**	
			
		4	
4(c)	$x_1 = \frac{3\times 4 + 6\times 3}{3+6}$	**M1**	Uses values from **(a)** to find x_1
	$x_1 = \frac{10}{3}$ and $f(x_1) = -\frac{38}{27}$	**A1**	**AWRT** 3.33 and -1.41
	$x_2 = 3.46$ and $f(x_2) = -0.464664$	**A1**	**AWRT** 3.46 (if earlier values rounded) and -0.465
	$x_3 = 3.50$	**A1**	**CSO**, accept 3.5
		4	
	Total	**10**	

Q	Answer	Marks	Comments
5	$\mathbf{P}(4\mathbf{I}+\mathbf{Q})=\mathbf{R}$	**M1**	Condone 4 for $4\mathbf{I}$ if $\mathbf{I}$ implied by later work **PI**
	$4\mathbf{I}+\mathbf{Q}=\begin{bmatrix}-2 & 4\\ -2 & 5\end{bmatrix}$ and $\det(4\mathbf{I}+\mathbf{Q})=-2$	**A1**	**PI**
	$(4\mathbf{I}+\mathbf{Q})^{-1}=\begin{bmatrix}-2.5 & 2\\ -1 & 1\end{bmatrix}$	**M1**	
	$\mathbf{P}=\mathbf{R}(4\mathbf{I}+\mathbf{Q})^{-1}$	**M1**	
	$\mathbf{P}=\begin{bmatrix}-2 & 3\\ -16 & 37\end{bmatrix}\begin{bmatrix}-2.5 & 2\\ -1 & 1\end{bmatrix}=\begin{bmatrix}2 & -1\\ 3 & 5\end{bmatrix}$	**A1**	
5 ALT	$\mathbf{P}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $4\mathbf{P}+\mathbf{PQ}=\begin{bmatrix}-2a-2b & 4a+5b\\ -2c-2d & 4c+5d\end{bmatrix}$	**M1** **A1**	**PI** At least two correct elements **PI** Fully correct
	$-2a-2b=-2$ $4a+5b=3$	**M1**	Form simultaneous equations in a and b
	$-2c-2d=-16$ $4c+5d=37$	**M1**	Form simultaneous equations in c and d
	$\mathbf{P}=\begin{bmatrix}2 & -1\\ 3 & 5\end{bmatrix}$	**A1**	
	Total	**5**	

Q	Answer	Marks	Comments
6(a)	0.121032	**B1**	**CAO**
		1	
6(b)	P($R > 6$) = $(1 - 0.18)^6$ 0.304	**M1** **A1**	**PI** Applies formula (**oe**) or finds P($R \leq 6$) [= 0.6959…] **AWRT**
		2	
6(c)	E(R) = 5.56	**B1**	**AWRT**
		1	
6(d)	Var(R) = 25.3	**B1**	**AWRT**
		1	
6(e)	Var($R - S$) = 28.3	**B1ft**	**AWRT** **ft** their Var(R) + 3
		1	
	Total	**6**	

Q	Answer	Marks	Comments
7(a)	$G_Y(t) = (0.22 + 0.78t)^2$	**B1**	Applies formula for $G_Y(t)$
	$G_Y(t) = 0.6084t^2 + 0.3432t + 0.0484$	**M1**	Multiplies out
	$G'_Y(t) = 1.2168t + 0.3432$	**M1**	Differentiates $G'_Y(t) = 2 \times 0.78(0.22 + 0.78t)$ gains both M marks
	$G''_Y(t) = 1.2168$ or $\frac{1521}{1250}$	**A1**	**CAO**
		4	
7(b)	$G'_Y(1) = 1.2168 \times 1 + 0.3432$	**M1**	**PI** Accept sight of 1.56 Their $G'_Y(t)$ must have non-constant terms
	$\text{Var}(Y) = G''_Y(1) + G'_Y(1) - (G'_Y(1))^2$ $= 1.2168 + 1.56 - 1.56^2$	**M1**	Uses variance formula and substitutes in their values
	$= 0.3432$ or $\frac{429}{1250}$	**A1**	**AWFW** [0.34, 0.35]
		3	
7(b) ALT	$Y \sim B(2, 0.78)$	**M1**	**PI** Identifies binomial distribution
	$\text{Var}(Y) = 2 \times 0.78 \times 0.22$	**M1**	Applies variance formula
	$= 0.3432$ or $\frac{429}{1250}$	**A1**	
		3	
	Total	**7**	

Q	Answer	Marks	Comments
8(a)	$\frac{1}{5}$	B1	**oe** such as $\frac{7}{35}$
		1	
8(b)(i)		B1ft B1ft B1ft	1/5 and 4/5 **oe** **ft** their part (a) 6/34 and 28/34 **oe** **ft** their 1/5 and 4/5 7/34 and 27/34 **oe** **ft** their 1/5 and 4/5
		3	
8(b)(ii)	$\frac{1}{5}\times\frac{28}{34}$ $\frac{1}{5}\times\frac{28}{34}+\frac{4}{5}\times\frac{27}{34}$ $\frac{\frac{14}{85}}{\frac{4}{5}}=\frac{7}{34}$	M1 M1 A1	Finds numerator using Bayes Theorem Sight of 14/85 **oe** Find denominator using Bayes Theorem Sight of 4/5 **oe** **oe** **AWRT** 0.206
		3	
	Total	7	

Q	Answer	Marks	Comments
9	$I = 0.3\times 7 - 0.3\times(-5)$	**M1**	Applies impulse equation. Condone sign errors for the velocities.
	$I = 3.6$ N s	**A1**	If units stated, must be correct.
	Total	**2**	

Q	Answer	Marks	Comments
10	$\left(LT^{-1}\right)^2 = [\omega]^2 L^2$ $[\omega]^2 = \dfrac{L^2T^{-2}}{L^2} = T^{-2}$	**M1**	Forms correct dimensional analysis equation. Condone $[\omega]^2\left(L^2 - L^2\right)$ for RHS
	$[\omega] = T^{-1}$	**A1**	
	Units of ω are s^{-1}	**B1**	
	Total	**3**	

Q	Answer	Marks	Comments
11	$v^2 = 2^2 + 6^2 - 2\times 2\times 6\cos 45°$	**M1**	Uses cosine rule for a valid triangle **oe**
	$v = 4.7989$ $\left[= 4.80 \text{ m s}^{-1} \text{ to 3sf}\right]$	**A1**	Finds correct resultant velocity
	$\dfrac{\sin\alpha}{2} = \dfrac{\sin 45°}{4.7989}$ or $\dfrac{\sin\beta}{6} = \dfrac{\sin 45°}{4.7989}$	**M1**	Uses sine rule with correct sides **oe** Condone 135 instead of 45
	$\alpha = 17.14$ or $\beta = 62.14$	**A1**	**AWRT** 17 or 62
	Bearing = 107° to the nearest degree	**A1ft**	**ft** 90 + their α or 45 + their β
	Total	**5**	

Q	Answer	Marks	Comments
12(a)	$3\times 5 = 3v_A + 2v_B$	**B1**	Correct conservation of momentum equation
	$v_B - v_A = 0.4\times 5$	**B1**	Correct coefficient of restitution equation
	$v_A = \frac{11}{5} = 2.2$, $v_B = \frac{21}{5} = 4.2$	**M1**	Attempt at solving simultaneous equations to find a value for v_A or v_B
	Speed for A = 2.2 m s^{-1}	**A1**	Correct speed for A or B
	Speed for B = 4.2 m s^{-1}	**A1**	Correct speeds for A and B including units
		5	
12(b)	$2\times 4.2 = 2w_B + w_C$	**B1ft**	Correct conservation of momentum equation, **ft** their v_B
	$w_C - w_B = 4.2e$	**B1ft**	Correct coefficient of restitution equation, **ft** their v_B
	$w_B = \frac{8.4 - 4.2e}{3}$	**M1**	Solves simultaneous equations to find an expression for w_B
	$\frac{8.4 - 4.2e}{3} \geq 2.2$	**M1**	Forms an inequality using their $w_B \geq v_A$ Condone $w_B > v_A$
	$e \leq \frac{3}{7}$	**A1**	Correct simplified inequality. Accept any equivalent fraction.
		5	
	Total	**10**	