OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Tuesday 18 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Answer **all** questions in the spaces provided.

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1 The mean age of subscribers to a gym course is claimed to be 27.0 years by the owner of the course.

The standard deviation of the ages of subscribers to the course is 9.2 years.

A random sample of 50 subscribers is found to have a mean age of 25.3 years.

1 (a) Test the owner's claim using the 5% level of significance.

[7 marks]

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1 (b) Explain why it is not necessary to assume that the ages of the subscribers are normally distributed in order to carry out the test in **part (a)**.

[2 marks]

9

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2 (a) The random variable X has a Poisson distribution with parameter λ

The standard deviation of X is 3

The random variable X represents the number of times in one hour a road crossing is used.

2 (a) (i) State the value of λ

[1 mark]

Answer _____

2 (a) (ii) Find $P(X < 8)$, giving your answer to three significant figures.

[2 marks]

Answer _____

2 (a) (iii) Find $P(X = E[X])$, giving your answer to three significant figures.

[3 marks]

Answer _____

Do not write outside the box

2 (b) The random variable T represents the time in hours between two successive people using the road crossing.

2 (b) (i) State the distribution and parameter for T

[2 marks]

Distribution _____

Parameter _____

2 (b) (ii) State the mean of T

[1 mark]

_____

Answer _____

2 (b) (iii) Find the probability that the time between two successive crossings is more than 30 minutes, giving your answer to four decimal places.

[3 marks]

_____

_____

_____

_____

_____

_____

Answer _____

2 (b) (iv) Find the value of t such that $P(T < t) = 0.9$, giving your answer in hours to three significant figures.

[2 marks]

_____

_____

_____

_____

_____

Answer _____

14

Turn over ►

Do not write outside the box

3 A supplier of cabbage seeds claims that 40% of their cabbage seeds germinate after being planted.

A student carries out a hypothesis test at the 10% level of significance to determine whether there is evidence to support the supplier's claim.

A random sample of 14 cabbage seeds is used for the test.

The number of cabbage seeds which germinate after being planted is assumed to be binomially distributed.

The first five stages of the student's hypothesis test are shown below.
There are errors in **two** of these stages.

	Statement
Stage 1	$H_0 : p = 0.4$ $H_1 : p \neq 0.4$ where p is the probability that a cabbage seed germinates after being planted
Stage 2	Under H_0 $X \sim B(0.4,14)$ where X is the number of cabbage seeds which germinate
Stage 3	$P(X \leq 2) = 0.0398$ (< 0.05) $P(X \leq 3) = 0.1243$ (> 0.05) (to four decimal places)
Stage 4	$P(X \geq 9) = 0.0583$ (> 0.05) $P(X \geq 10) = 0.0175$ (< 0.05) (to four decimal places)
Stage 5	The critical region is {0, 1, 2, 9, 10, 11, 12, 13, 14}

3 (a) Identify the **two** stages which have errors.

Write down a correct statement for each of these stages.

[3 marks]

Error 1 Stage _____

Correct statement _____

_____

_____

Error 2 Stage _____

Correct statement _____

_____

_____

Do not write outside the box

3 (b) Of the 14 cabbage seeds used for the test, 8 germinate after being planted.

Determine the conclusion of the hypothesis test.

[2 marks]

3 (c) (i) Find the probability of a Type I error for this hypothesis test.

[1 mark]

Answer

3 (c) (ii) For this hypothesis test describe, in context, the meaning of a Type I error.

[1 mark]

3 (d) The level of significance used for this hypothesis test is now changed to k %

3 (d) (i) In the case where $k > 10$ explain how the acceptance region for this hypothesis test may change.

[1 mark]

3 (d) (ii) In the case where $k < 10$ explain how the acceptance region for this hypothesis test may change.

[1 mark]

9

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4 The continuous random variable T has probability density function

$$f(t) = \begin{cases} ct^2 & 0 < t \leq 1 \\ \frac{1}{2}t & 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

4 (a) Show that $c = \frac{3}{4}$

[3 marks]

4 (b) (i) Find the exact value of $\mathrm{E}(T)$

[3 marks]

Answer _____

Do not write outside the box

4 (b) (ii) It is given that $\mathrm{E}\left(T^2\right) = \frac{81}{40}$

Find the exact value of $\mathrm{Var}\left(T\right)$

[2 marks]

Answer _____

4 (c) (i) Find the exact value of $\mathrm{E}\left(3T + 1\right)$

[2 marks]

Answer _____

4 (c) (ii) Find the exact value of $\mathrm{Var}\left(3T + 1\right)$

[2 marks]

Answer _____

12

Turn over ►

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5 To make a cake there are three processes: mixing, baking and decorating.

The times taken, in minutes, for these processes can be modelled as independent normal distributions.

Mixing	$X_{\text{M}} \sim \text{N}\left(8,\ 1.3^2\right)$
Baking	$X_{\text{B}} \sim \text{N}\left(\mu,\ 1.5^2\right)$
Decorating	$X_{\text{D}} \sim \text{N}\left(20,\ 2.5^2\right)$

5 (a) Find the probability that it takes less than 6 minutes to complete the mixing process, giving your answer to three significant figures.

[3 marks]

Answer _____

5 (b) It is given that $\text{P}(X_{\text{B}} < 26.3) = 0.8$

Show that $\mu = 25$ correct to two significant figures.

[3 marks]

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5 (c) (i) Find the probability that the total amount of time it takes to mix, bake and decorate a cake is less than 1 hour, giving your answer to three significant figures.

[4 marks]

Answer _____

5 (c) (ii) On a given day, a baker is expected to make 8 cakes.
All three processes for each cake are completed independently.

Find the probability that all 8 cakes each take less than 1 hour to make, giving your answer to two significant figures.

[3 marks]

Answer _____

13

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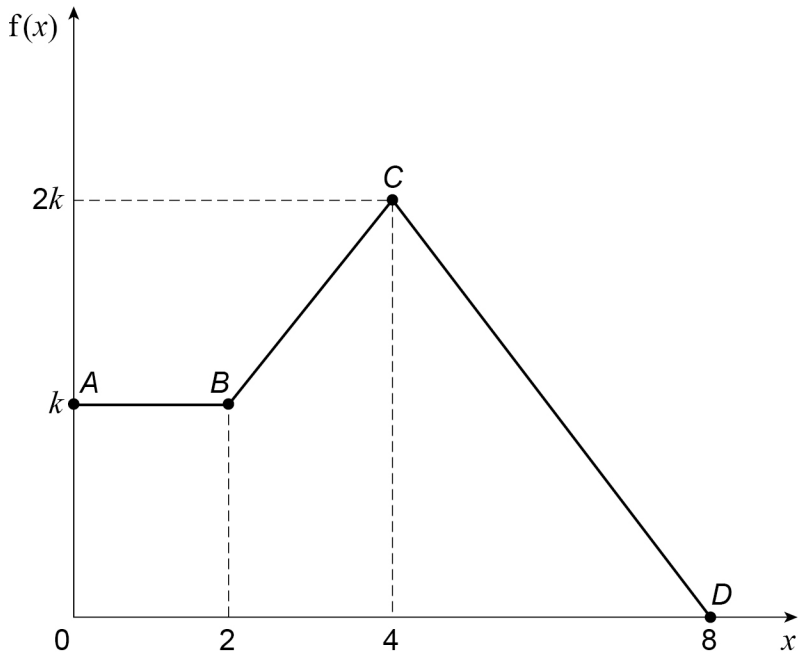
6 A continuous random variable X has probability density function f

The graph of $\mathrm{f}(x)$ consists of three line segments: *AB*, *BC* and *CD* for $0 \leq x \leq 8$ where k is a constant. For all other values of x, $\mathrm{f}(x) = 0$ as shown in **Figure 1**

Figure 1



6 (a) Show that $k = \frac{1}{9}$

[2 marks]

6 (b) Given that $\mathrm{P}(X < c) = 0.5$ show that $2 < c < 4$

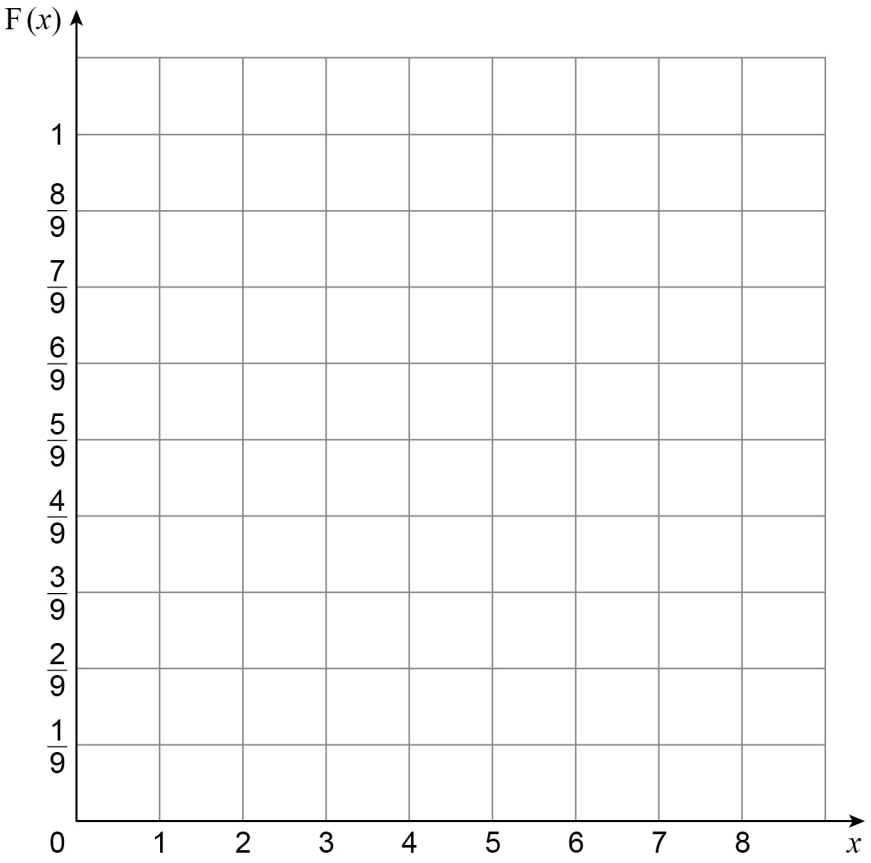
[2 marks]

Do not write outside the box

6 (c) Draw the cumulative distribution function F, stating the values of $\mathrm{F}(x)$ when $x = 2$, $x = 4$, $x = 6$, and $x = 8$ on **Figure 2**

[4 marks]

Figure 2



8

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7 Lottie is a classroom teacher of students in a junior school.

The height of students is known to be a random variable with a normal distribution.

Lottie measures the heights of a random sample of 10 students.

The sample mean height of these 10 students is 125 cm

The sample variance of the heights of these 10 students is 12.9 cm^2 correct to three significant figures.

7 (a) Lottie claims that the mean height of her students is less than the population mean of 127 cm

Test Lottie's claim using the 10% level of significance.

[7 marks]

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7 (b) Lottie wants to do further calculations with the sample collected for her hypothesis test in **part (a)** but two of these heights are now missing.

The heights in cm of the 10 students in her sample are

123 125 128 124 122 129 126 119 q r

where q and r are the missing heights.

7 (b) (i) Show that $q + r = 254$

[2 marks]

7 (b) (ii) It is given that $q > r$

Find the value of q and the value of r, giving your answers to three significant figures.

[6 marks]

Working space for this question continues on the next page

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$q =$ _____ $r =$ _____

15

END OF QUESTIONS

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ANSWER IN THE SPACES PROVIDED

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