OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Wednesday 12 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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11	
TOTAL	

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 The matrix **C** is defined by

$$\mathbf{C} = \begin{bmatrix} 4 & -3 & 0 \\ 3 & 0 & -k \\ 0 & k & -3 \end{bmatrix}$$

where k is a constant.

1 (a) Write down the matrix $\mathbf{C}^{\mathbf{T}}$

[1 mark]

Answer _____

1 (b) (i) Find $\mathbf{CC}^{\mathbf{T}}$

[2 marks]

_____

_____

_____

_____

_____

_____

Answer _____

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1 (b) (ii) The diagonal from the top left corner to the bottom right corner of a square matrix is called the leading diagonal.

It is given that the three elements of the leading diagonal of $\mathbf{CC^T}$ are equal.

Show that one possible value of k is 4 and find the other possible value of k

[2 marks]

Answer _____

1 (c) The matrix **D** is defined by $\mathbf{D} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 2 & -2 \end{bmatrix}$

1 (c) (i) Explain why the matrix **CD** does not exist.

[1 mark]

1 (c) (ii) In the case when $k = 4$ find the matrix **DC**

[2 marks]

Answer _____

8

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2 The equation $2.7^x = 2x + 5$ has two real roots α and β where $\alpha > 0$ and $\beta < 0$

2 (a) Show that α lies in the interval $2 < x < 3$

[2 marks]

2 (b) Use interval bisection to find the value of α to one decimal place.

[4 marks]

Answer _____

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2 (c) The root β is such that $n < \beta < n+1$ where n is an integer.

Find the value of n

[2 marks]

Answer _____

8

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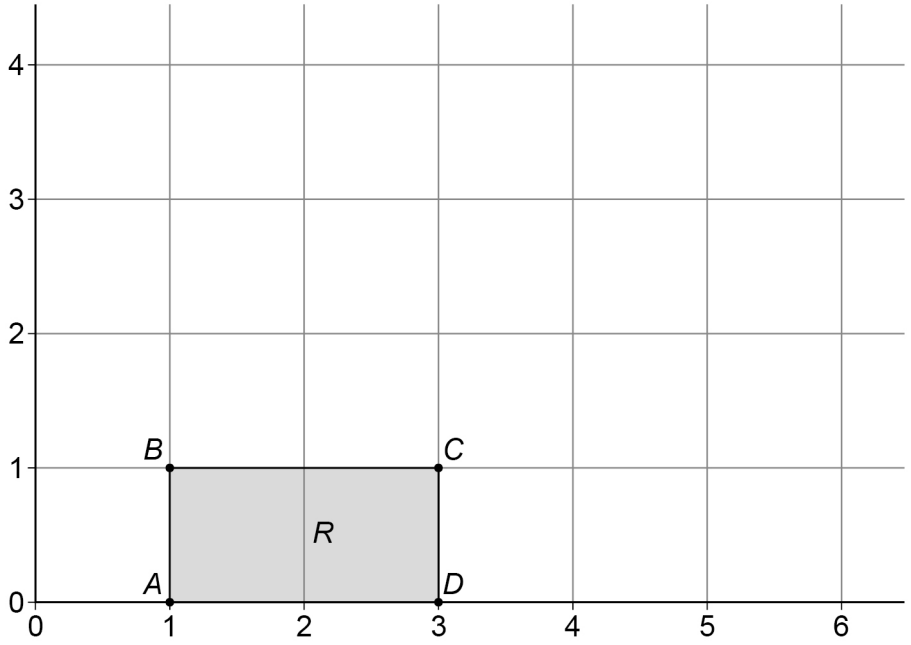
3 The rectangle R has vertices with coordinates A (1,0), B (1,1), C (3,1) and D (3,0) is shown in the diagram.



3 (a) The matrix **M** is defined by $\mathbf{M} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

The transformation represented by the matrix **M** maps R onto the quadrilateral S

3 (a) (i) Find the coordinates of the vertices of S

[2 marks]

Answer _____

3 (a) (ii) Name the type of transformation represented by the matrix **M**

[2 marks]

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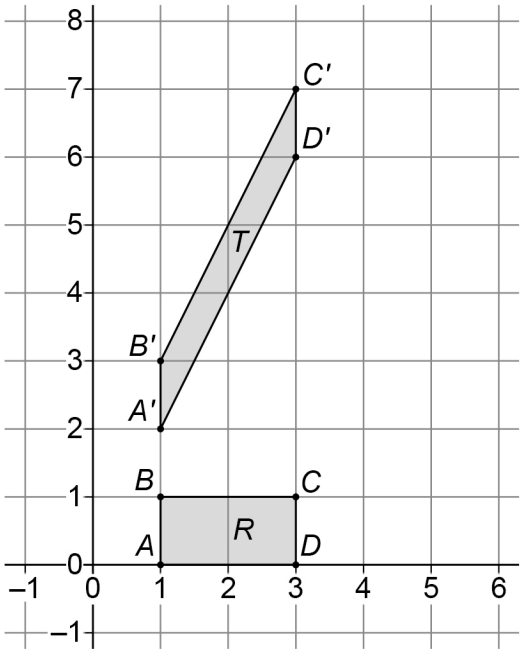
3 (b) The matrix **N** is defined by $\mathbf{N} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$ where a is a constant.

The transformation represented by the matrix **N** maps R onto quadrilateral T

The quadrilateral T has vertices with coordinates $A'(1, 2)$, $B'(1, 3)$, $C'(3, 7)$ and $D'(3, 6)$ and A maps to A', B maps to B', C maps to C' and D maps to D'.

R and T are shown on the diagram below.



Find the value of a

[2 marks]

Answer _____

3 (c) The transformation represented by the matrix **NM** maps a polygon P onto a polygon Q

Explain whether the area of Q is equal to the area of P

[3 marks]

9

Turn over ►

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4 A curve passes through the point (2,1.5) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - \frac{y^3}{x} \quad \text{where } x \neq 0$$

Use Euler's step-by-step method with a step length of 0.1 to estimate the value of y when $x = 2.2$

Give your answer to four decimal places.

[4 marks]

Answer _____

4

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5 A businesswoman believes that the total profit, P million dollars, made by her company can be modelled by the equation

$$P = a \times 10^{kt}$$

where t is the number of years since the company started, and where a and k are constants.

She collected data as shown in the table below.

t	1	1.25	1.5	2	2.75
P	2.97	4.80	7.73	20.11	84.32

5 (a) Show that $P = a \times 10^{kt}$ can be written as a linear equation of the form

$$\log_{10} P = \log_{10} a + kt$$

[1 mark]

Question 5 continues on the next page

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5 (b) Complete the table below to show the values of $\log_{10} P$

Give your values to two decimal places.

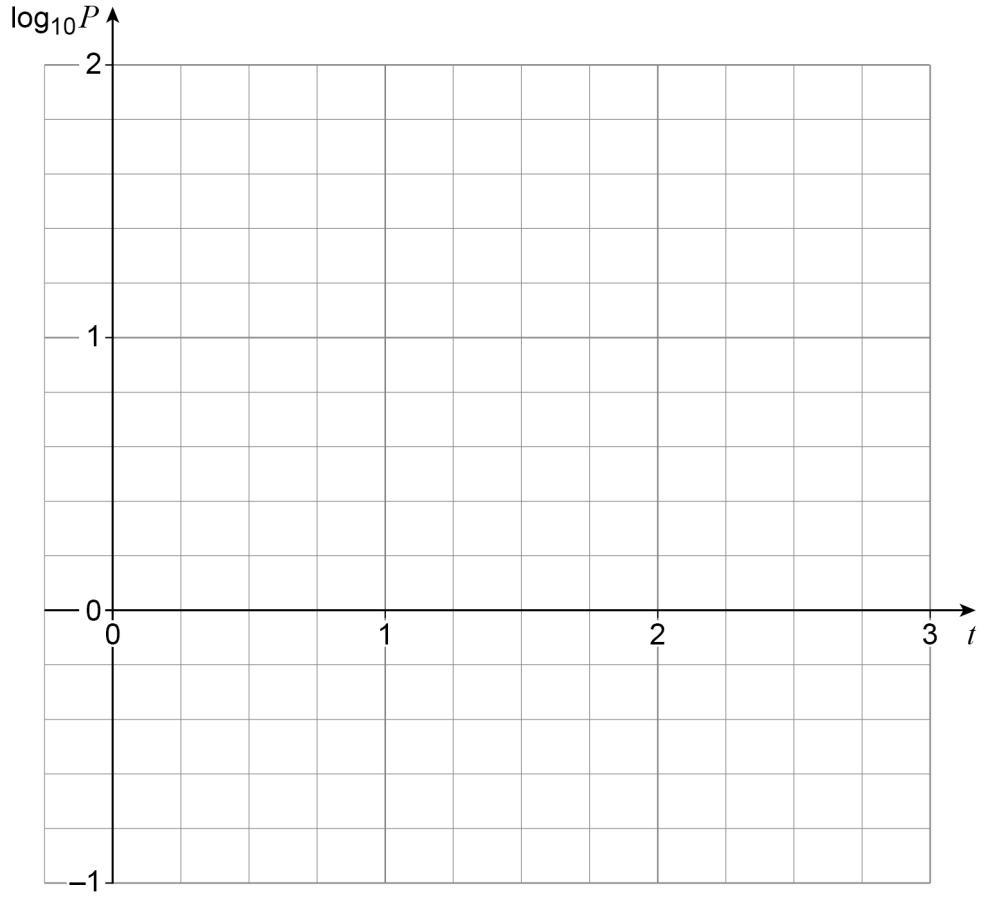
[1 mark]

t	1	1.25	1.5	2	2.75
P	2.97	4.80	7.73	20.11	84.32
$\log_{10} P$	0.47				

5 (c) On the grid plot $\log_{10} P$ against t

Draw a line of best fit for the points you have plotted.

[2 marks]



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5 (d) (i) Use your line of best fit to find estimates for a and k

Give your values to two decimal places.

[3 marks]

$a =$ _____ $k =$ _____

5 (d) (ii) Hence write down an equation for P in terms of t

[1 mark]

Answer _____

5 (d) (iii) Use your answer to **part (d)(ii)** to forecast the value of the total profit 4 years after the company started.

Give your answer to two significant figures.

[2 marks]

Answer _____

5 (d) (iv) Explain why your answer to **part (d)(iii)** may be unreliable.

[1 mark]

11

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The discrete random variable X has probability distribution function

$$\mathrm{P}(X = x) = \begin{cases} 0.25 & x = 1,\ 2,\ 3,\ 4 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Name the distribution of X

[1 mark]

6 (b) Find $\mathrm{P}(X \geq 2)$

[1 mark]

Answer _____

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6 (c) The discrete random variable Y has a geometric distribution with parameter p and is independent of X

The mean of Y is 5

6 (c) (i) Find the value of p

[1 mark]

Answer _____

6 (c) (ii) Find $\mathrm{E}(2X - 5Y)$

[3 marks]

Answer _____

6

Turn over ►

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7 Basia is the best player on her sports team.

The team is about to play a game.

The probability that the team wins the game is $\frac{33}{50}$

The probability that Basia plays in the game, given that the team wins, is $\frac{26}{33}$

The probability that Basia plays in the game, given that the team does not win, is $\frac{13}{34}$

7 (a) Find the probability that Basia plays in the game.

[3 marks]

Answer _____

7 (b) Show that the probability that the team wins, given that Basia plays in the game, is $\frac{4}{5}$

[2 marks]

Answer _____

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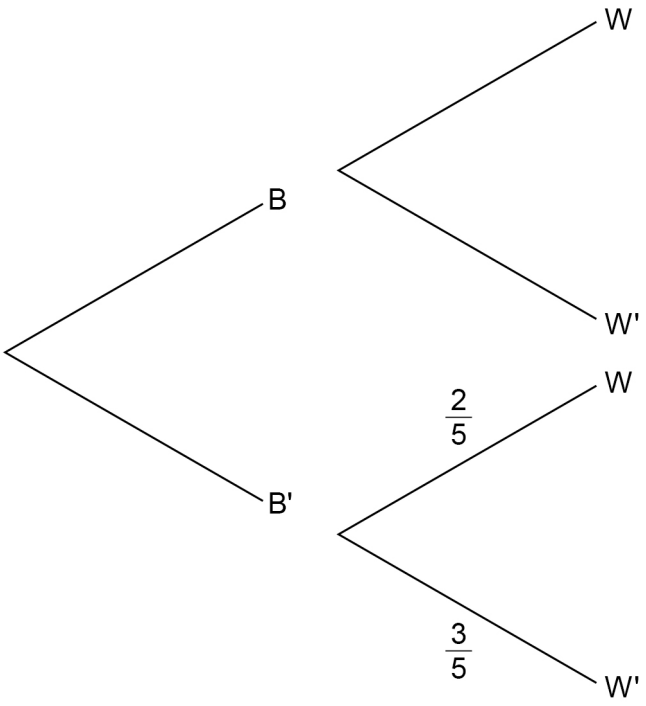
7 (c) B represents the event that Basia plays in the game.

W represents the event that the team wins the game.

Complete the tree diagram by writing the probability for each branch.

[1 mark]



6

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8 The discrete random variable X has the probability distribution

x	0	1	2
$P(X = x)$	0.4	0.25	0.35

8 (a) Find the probability generating function $G_X(t)$

[1 mark]

Answer _____

8 (b) The discrete random variable Y is independent of X and has probability generating function

$$G_Y(t) = 0.475t + 0.525t^2$$

8 (b) (i) Find $G_{X+Y}(t)$

Give your answer in the form $at + bt^2 + ct^3 + dt^4$ where a, b, c and d are constants.

[2 marks]

Answer _____

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8 (b) (ii) Hence find the variance of $X + Y$

[5 marks]

Answer _____

8

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 The equation $s = vt - \frac{1}{2}at^2$ is used to calculate displacement.

In this equation

s is the displacement in metres

v is the final velocity in m s^{-1}

t is the time in seconds

a is the acceleration in m s^{-2}

Show that the equation $s = vt - \frac{1}{2}at^2$ is dimensionally consistent.

[3 marks]

3

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10 Two spheres, A and B, have the same radius and move on a smooth horizontal surface.

They are moving directly towards each other on a straight line when they collide.

Sphere A has mass 7 kg and before the collision has speed $4\ \text{m s}^{-1}$

Sphere B has mass 3 kg and before the collision has speed $5\ \text{m s}^{-1}$

The coefficient of restitution between the spheres is 0.9

10 (a) Find the speed of sphere A and the speed of sphere B after the collision.

[5 marks]

Speed of sphere A _____

Speed of sphere B _____

10 (b) For each sphere, state whether its direction of motion changes during the collision.

[1 mark]

Sphere A _____

Sphere B _____

Do not write outside the box

10 (c) The spheres are in contact for 0.02 seconds during the collision.

The magnitude of the force, F newtons, exerted by *A* on *B* during the collision is given by

$$F = kt(0.02 - t) \qquad \text{for } 0 \leq t \leq 0.02$$

where k is a constant.

Find the value of k giving your answer to three significant figures.

[5 marks]

Answer _____

11

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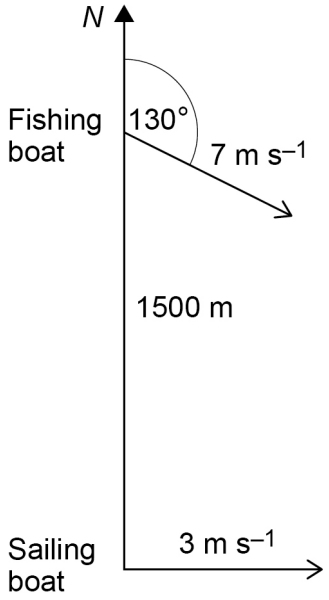
11 A sailing boat is moving with a constant velocity of $3\,\text{m s}^{-1}$ due east.

A fishing boat is moving with a constant velocity of $7\,\text{m s}^{-1}$ on a bearing of 130°

The fishing boat is initially 1500 metres due north of the sailing boat.

The diagram shows the initial positions and the velocities of the two boats.



Find the minimum distance between the two boats, giving your answer to the nearest metre.

[6 marks]

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Answer _____

6

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