OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Tuesday 4 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

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1 Show that

$$\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2\left(an+b\right)\left(bn+1\right)$$

where a and b are integers.

[5 marks]

5

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2 (a) It is given that $z = \dfrac{7 - 3\mathrm{i}}{k - 5\mathrm{i}}$ where k is a real number.

Find, in terms of k, the real part of z and the imaginary part of z

[3 marks]

Re $z =$ _____ Im $z =$ _____

2 (b) Use your answer to **part (a)** to show that $\arg\left(\dfrac{7 - 3\mathrm{i}}{2 - 5\mathrm{i}}\right) = \dfrac{\pi}{4}$

[3 marks]

6

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3 A circular patch of water plants is floating on the surface of a lake.

The area of the circular patch increases at a rate of 3 square metres per day.

Find the rate at which the **radius** of the circular patch is increasing when the area of the circular patch is 36π square metres.

Give your answer in terms of π

[6 marks]

Answer _____

6

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4 (a) Find the general solution of the equation

$$\cos\left(2x-\frac{\pi}{2}\right)=-\frac{1}{2}$$

giving your answer in terms of π

[4 marks]

Answer _____

4 (b) Hence find the number of solutions of the equation

$$\cos\left(2x-\frac{\pi}{2}\right)=-\frac{1}{2}$$

which are between 0 and $(4k-1)\frac{\pi}{2}$ where k is an integer and $k \geq 1$

Give your answer in terms of k

[3 marks]

Answer _____

7

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5 The quadratic equation

$$x^2 + 5x + 9 = 0$$

has roots α and β

5 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

[2 marks]

$\alpha + \beta =$ _____ $\alpha\beta =$ _____

5 (b) Hence find the value of $\alpha^2 + \beta^2$

[2 marks]

Answer _____

5 (c) Show that $\alpha^3 + \beta^3 = 10$

[2 marks]

Do not write outside the box

5 (d) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{\beta}{\alpha} \quad \text{and} \quad \beta + \frac{\alpha}{\beta}$$

[6 marks]

Answer _____

12

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6 The function f is defined by

$$\mathrm{f}(x) = \frac{2x+1}{x^2}$$

6 (a) Write down the equations of the asymptotes of the graph of $y = \mathrm{f}(x)$

[2 marks]

Answer _____

6 (b) It is given that the line $y = k$, where k is a constant, intersects the graph of $y = \mathrm{f}(x)$

Prove that $k \geq -1$

[3 marks]

6 (c) The graph of $y = \mathrm{f}(x)$ has one stationary point.

Use the result given in **part (b)** to find the coordinates of this stationary point.

[2 marks]

Answer _____

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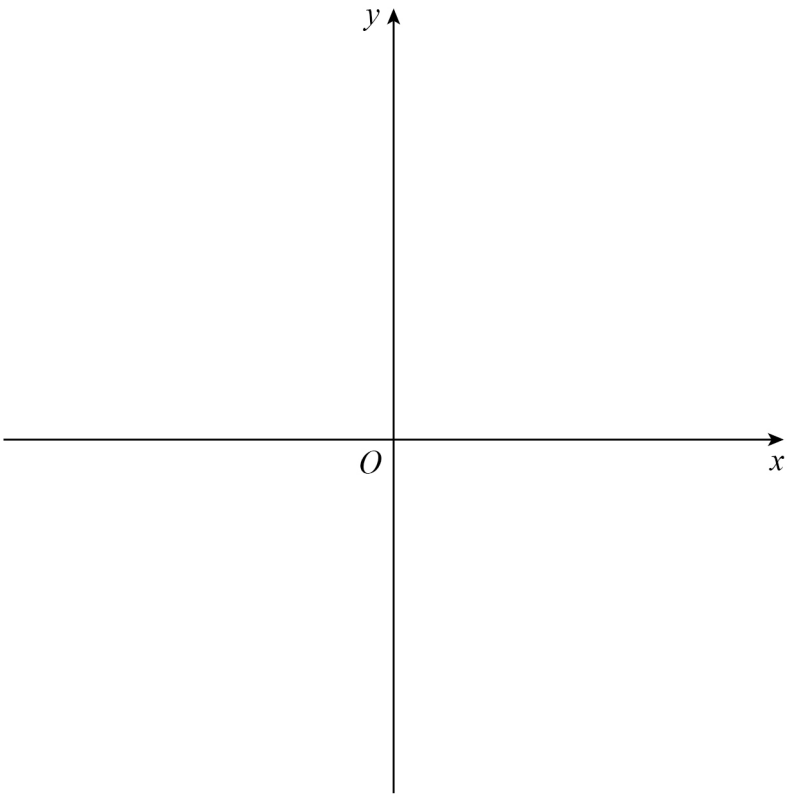
6 (d) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below.

Show the coordinates of the stationary point and of any intersection with the axes.

[3 marks]



6 (e) Solve the inequality

$$\frac{2x+1}{x^2} > 3$$

[4 marks]

14

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7 The integrals I_1 and I_2 are defined below.

$$I_1 = \int_1^{\infty} \frac{1}{x^2}\,\mathrm{d}x \qquad \text{and} \qquad I_2 = \int_0^{64} \frac{1}{\left(\sqrt[3]{x}\right)^2}\,\mathrm{d}x$$

7 (a) Explain why I_1 is an improper integral.

[1 mark]

7 (b) Explain why I_2 is an improper integral.

[1 mark]

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7 (c) Find the value of I_2 clearly showing the limiting process.

[3 marks]

Answer _____

5

Turn over for the next question

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8 The circle C is the locus of points on an Argand diagram such that

$$|z - 4 - 6\mathrm{i}| = 4$$

The half-line L is the locus of points on an Argand diagram such that

$$\arg(z - 4) = \frac{\pi}{2}$$

P is the point on C with the smallest argument.

The line OP meets the half-line L at the point T

8 (a) Write down the complex number that represents the centre of C

[1 mark]

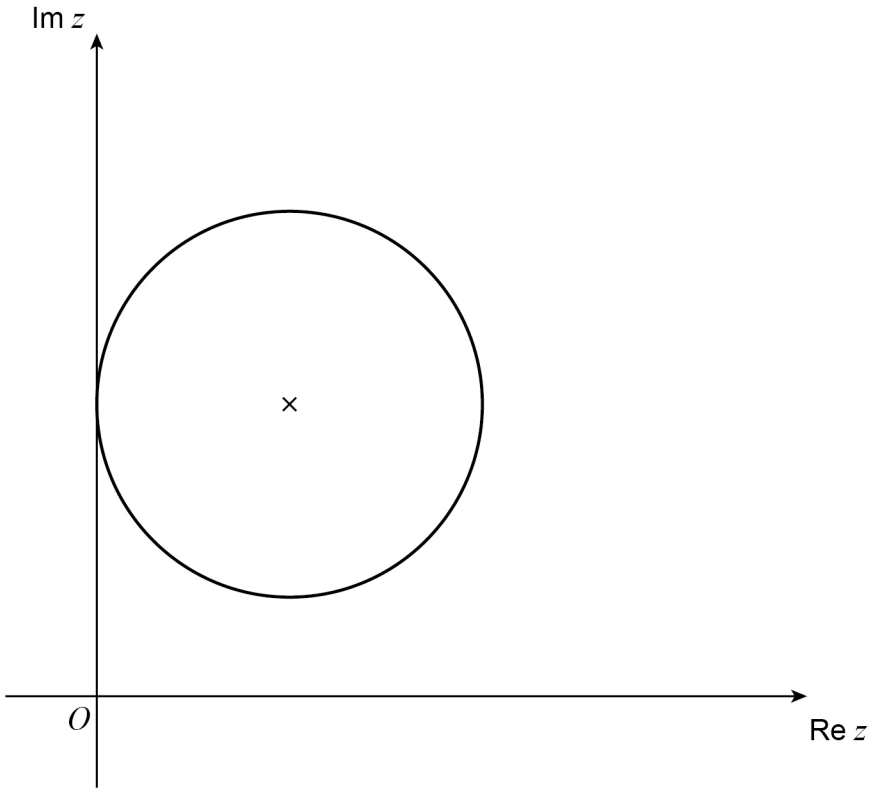
Answer _____

8 (b) The circle C is drawn on the Argand diagram below.

On the Argand diagram:

- draw the half-line L
- draw the line OP
- label the point T

[3 marks]



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8 (c) Find the complex number that represents the point T

[6 marks]

Answer _____

10

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9 The locus of a point P is such that the distance from P to the point (12, 0) is equal to the distance from P to the line $x = -12$

The locus of P is the curve C_1

9 (a) Show that the equation of C_1 is

$$y^2 = 48x$$

[2 marks]

9 (b) The translation by the vector $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ maps the curve C_1 onto the curve C_2

Find the equation of C_2

[2 marks]

Answer _____

Question 9 continues on the next page

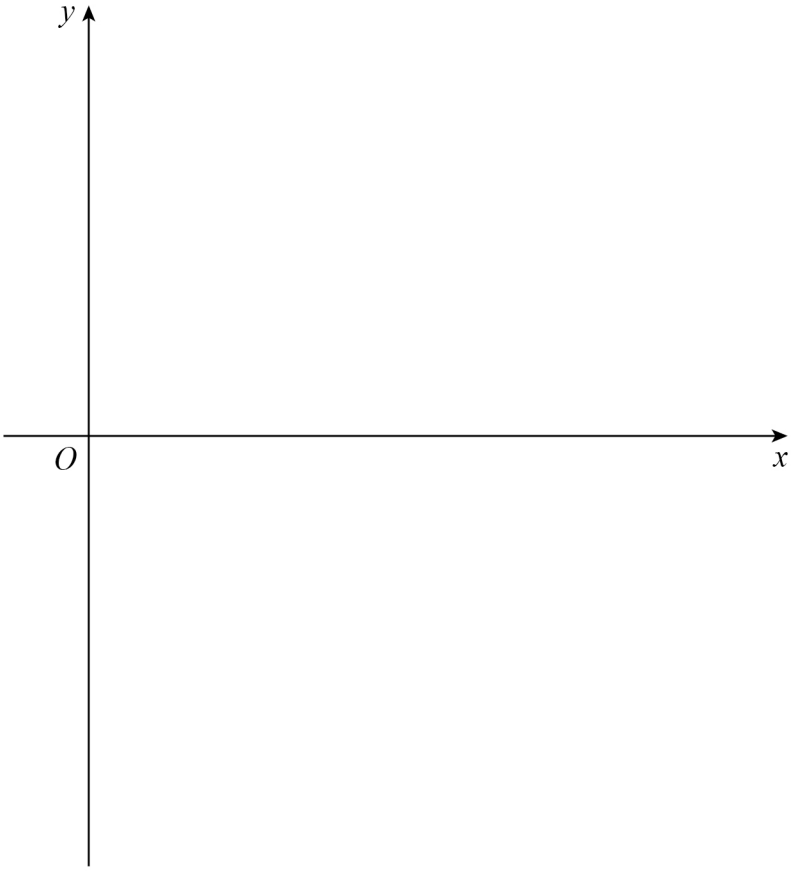
9 (c) The points Q and R both lie on C_2 such that

- the y-coordinate of Q is positive
- the y-coordinate of R is negative
- the line OQ is a tangent to C_2
- the line OR is a tangent to C_2

9 (c) (i) Sketch the curve C_2 and the lines OQ and OR

[You do **not** need to sketch the curve C_1]

[3 marks]



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9 (c) (ii) Find the coordinates of Q and the coordinates of R

[8 marks]

Q _____ R _____

END OF QUESTIONS

15

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