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MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

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2 2 1 X M A 0 4 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1 (a)	$H_0 : \mu = 27$ $H_1 : \mu \neq 27$	**B1**	Accept 27 as 27.0 throughout
	$\overline{X} \sim N\left(27, \frac{9.2^2}{50}\right)$	**B1**	Must be seen
	$z = \frac{25.3 - 27}{\left(\frac{9.2}{\sqrt{50}}\right)}$	**M1**	$z = \frac{25.3 - 27}{\text{their } \left(\frac{s}{\sqrt{n}}\right)}$ for $n \neq 1$ **PI**
	$z = -1.31$	**A1**	**AWRT** or exact value $-\frac{85\sqrt{2}}{92}$
	$z_{\text{critical}} = -1.96$	**B1**	**AWRT** or $P(z < -1.31) = 0.0951$ to 0.0957 or comparison of probability to 2.5%
	Do not reject H_0 as $z_{\text{critical}} < z$ or $-1.96 < -1.31$ or $\lvert z \rvert < 1.96$	**A1ft**	Allow 'accept H_0' Comment about H_0 and 0.0951 to $0.0957 > 0.025$ Correct conclusion based upon their z
	Evidence to support the claim that the mean age of subscribers to the course is 27 [at the 5% level of significance]	**E1**	Correct statement based on their comparison and given in context Must have been awarded at least **M1 A0 A1ft**
		7	

Q	Answer	Marks	Comments
1(b)	The central limit theorem states that when the sample size is large enough [e.g. $n \geq 30$], the sample mean will be (approximately) normally distributed	**B2**	**B1** for mention of the CLT **B1** for comment on the size of the sample
		2	

	Question 1 Total	**9**	

Q	Answer	Marks	Comments
2(a)(i)	$\left[\lambda = 3^2 =\right]\ 9$	**B1**	
		1	

Q	Answer	Marks	Comments
2(a)(ii)	$P(X < 8) = P(X \leqslant 7)$ 0.324	**M1** **A1**	**PI** **SC1** for $\lambda = 3$ Þ 0.988 Condone 0.3239 (from use of tables)
		2	

Q	Answer	Marks	Comments
2(a)(iii)	$E(X) = 9$ $\left[P(X = 9) =\right]\ \dfrac{e^{-9} \times 9^9}{9!}$ 0.132	**B1** **M1** **A1**	**PI** Allow ft with $\lambda = 3$ Using tables: $= 0.5874 - 0.4557$ **SC2** for $\lambda = 3$ Þ 0.224 Condone 0.1317 (from use of tables)
		3	

Q	Answer	Marks	Comments
2(b)(i)	Exponential $\left[\lambda =\right] 9$	**B1** **B1**	$T \sim \text{Exp}\left(9\right)$ is **B2** **PI** by subsequent working
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\text{Mean} = \frac{1}{9}$	**B1**	**SC1** only for $\lambda = 3$ in **part (b)(i)** leading to a mean of $\frac{1}{3}$
		1	

Q	Answer	Marks	Comments
2(b)(iii)	$\text{P}\left(T > 0.5\right) = 1 - \text{F}\left(0.5\right)$ $= 1 - \left(1 - \text{e}^{-9 \times 0.5}\right)$ 0.0111	**M1** **m1** **A1**	**oe PI** Attempts to find correct probability using cdf of exponential or integration of pdf **AWRT** **NMS** 3/3 **SC2** for 0.2231 from $\lambda = 3$
		3	

Q	Answer	Marks	Comments
2(b)(iv)	$1 - \text{e}^{-9t} = 0.9$ $t = 0.256$ [hours]	**M1** **A1**	Forms equation using a cdf of exponential or integration of pdf with their λ **SC1** for 0.768 from $\lambda = 3$
		2	

	Question 2 Total	**14**	

Q	Answer	Marks	Comments
3(a)	Correctly identifying Stage 2 **and** Stage 5 **Stage 2** $X \sim \mathrm{B}(14, 0.4)$ **Stage 5** [critical region is] $\{0, 1, 2, 10, 11, 12, 13, 14\}$	**B1** **B1** **B1**	 **oe**, such as critical region should not include 9
		3	

Q	Answer	Marks	Comments
3(b)	As 8 is **not** in the critical region Do not reject H_0 as we have evidence to suggest the germination rate is 40% for a cabbage seed [at the 10% level of significance]	**B1** **B1**	Comment about the test statistic and critical region (allow a comparison of probabilities) General conclusion Allow accept H_0
		2	

Q	Answer	Marks	Comments
3(c)(i)	$[0.0398 + 0.0175 =]$ 0.0573	**B1**	
		1	

Q	Answer	Marks	Comments
3(c)(ii)	Accepting that the germination rate is not 0.4 when it is.	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
3(d)(i)	The acceptance region may decrease [or may not change if k is close to 10]	E1	**oe** Condone a definitive statement
		1	

Q	Answer	Marks	Comments
3(d)(ii)	The acceptance region may increase [or may not change if k is close to 10]	E1	**oe** Cannot be definitive
		1	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)	$\int_0^1 ct^2\,\mathrm{d}t \;+\; \int_1^2 \tfrac{1}{2}t\,\mathrm{d}t = 1$	M1	Correct integrals set equal to 1 at some point in working **oe**
	$\left[\frac{ct^3}{3}\right]_0^1 + \left[\frac{1}{4}t^2\right]_1^2 = 1$	M1	Correct integration
	$\frac{c}{3} + \left(1 - \frac{1}{4}\right) = 1$		**oe eg** $\frac{c}{3} = \frac{1}{4}$
	$c = \frac{3}{4}$	A1	**AG**
		3	

Q	Answer	Marks	Comments
4(b)(i)	$\int_0^1 \tfrac{3}{4}t^3\,\mathrm{d}t \;+\; \int_1^2 \tfrac{1}{2}t^2\,\mathrm{d}t$	M1	At least one correct integral with correct limits **PI**
	$= \left[\frac{3t^4}{16}\right]_0^1 + \left[\frac{1}{6}t^3\right]_1^2$	A1	Sum of two correct integrations **PI** by correct answer
	$= \left(\frac{3}{16} - 0\right) + \left(\frac{8}{6} - \frac{1}{6}\right)$		
	$= \frac{65}{48}$	A1	**oe eg** $1.3541\dot{6}$
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$\mathrm{Var}(T) = \mathrm{E}(T^2) - \left(\mathrm{E}(T)\right)^2 = \frac{81}{40} - \left(\frac{65}{48}\right)^2$	M1	Allow their $\mathrm{E}(T)$ from **part (b)(i)** **PI**
	$\frac{2203}{11520}$	A1	**CAO**
		2	

Q	Answer	Marks	Comments
4(c)(i)	$=3\mathrm{E}\left(T\right)+1$ $=3\times\frac{65}{48}+1$ $\frac{81}{16}$	**M1** **A1ft**	Allow their $\mathrm{E}\left(T\right)$ from **part (b)(i)** **PI** 5.0625
		2	

Q	Answer	Marks	Comments
4(c)(ii)	$=9\mathrm{Var}\left(T\right)$ $=9\times\frac{2203}{11520}$ $\frac{2203}{1280}$	**M1** **A1ft**	Allow their $\mathrm{Var}\left(T\right)$ from **part (b)(ii)** **PI** 1.72109375
		2	

	Question 4 Total	**12**	

Q	Answer	Marks	Comments
5(a)	$\mathrm{P}(X < 6) = \mathrm{P}\left(z < \frac{6 - 8}{1.3}\right)$	**M1**	Standardises
	$\mathrm{P}\left(z < -1.54\right)$		
	$\left[= 1 - \mathrm{F}(1.54)\right] = 1 - 0.93822$ [using tables]	**m1**	Use of 1 – their p, **PI**
	0.0618 [using tables]	**A1**	Accept 0.0620 Condone 0.062 (i.e. omitting trailing zero)
		3	

Q	Answer	Marks	Comments
5(b)	$\left[\mathrm{F}^{-1}\left(0.8\right) = z = \right]$ 0.8416	**B1**	Condone a z-value of 0.84 or 0.842 Seen or used
	$\frac{26.3 - m}{1.5} = 0.8416$	**M1**	**oe** Standardising with their z-value
	$\left[m = \right]$ $26.3 - 1.5 \times 0.8416 = 25.037...$ $= 25$ to 2 sf	**A1**	**CSO** Be convinced
		3	

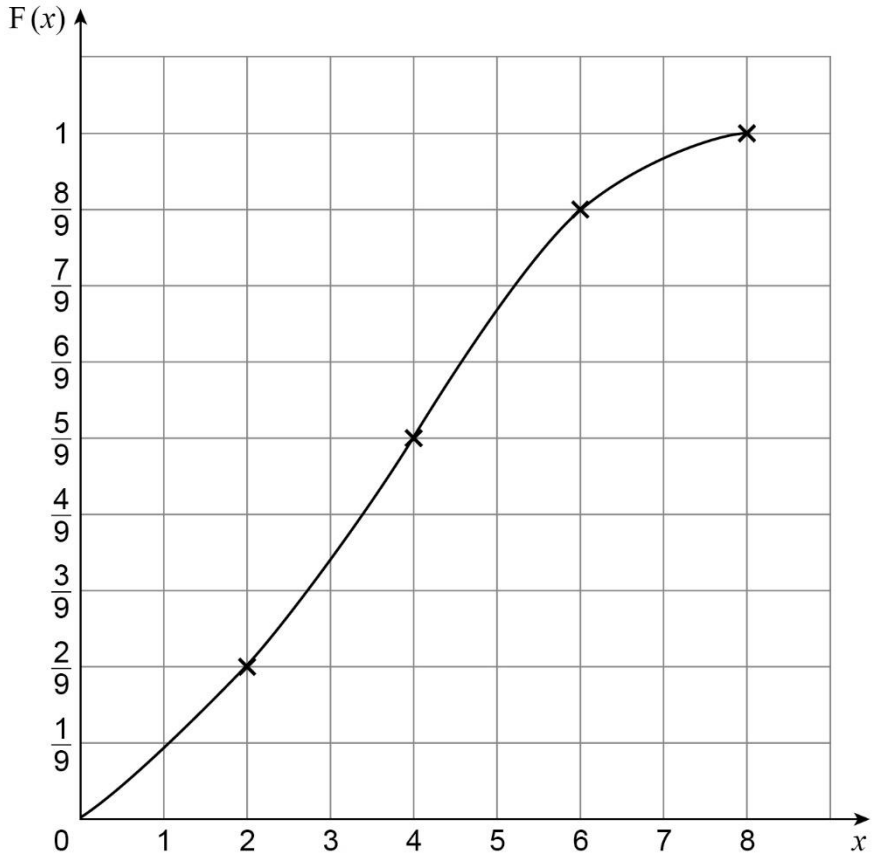
Q	Answer	Marks	Comments
5(c)(i)	$X_{\mathrm{M}} + X_{\mathrm{B}} + X_{\mathrm{D}} \sim \mathrm{N}\left(53,\ 10.19\right)$	**B2**	**B1** for $m = 53$ seen or used **B1** for $s^2 = 10.19$ seen or used
	$\mathrm{P}\left(z < 2.19\right)$	**M1**	**PI** standardises with their m and s^2
	= 0.986	**A1**	**AWRT** z-value is 2.18… and answer is 0.985 if $m = 25.037...$ is used
		4	

Q	Answer	Marks	Comments
5(c)(ii)	$X \sim \mathrm{B}(8, 0.986)$	**B1**	**PI**
	$\mathrm{P}(X = 8) = 0.986^8$	**M1**	**ft** their p from **(c)(i)**, $\mathrm{P}(X = 8) = p^8$
	= 0.89	**A1**	**AWRT**, **NMS** 3/3
		3	

	Question 5 Total	**13**	

Q	Answer	Marks	Comments
6(a)	$2k+\frac{(k+2k)}{2}\times 2+\frac{2k\times 4}{2}=1$ **or** $9k=1$ so $k=\frac{1}{9}$	**M1** **A1**	Attempt to find the area and making equal to 1 Could be seen on the diagram **AG** Be convinced
		2	

Q	Answer	Marks	Comments
6(b)	$\mathrm{P}(X<2)=\frac{2}{9}$ or $\mathrm{F}(2)=\frac{2}{9}$ and $\left[\mathrm{P}(X>4)=\frac{4}{9}\text{ so }\right]\ \mathrm{P}(X<4)=\frac{5}{9}$ or $\mathrm{F}(4)=\frac{5}{9}$ As $\mathrm{P}(X<2)<0.5$ and $\mathrm{P}(X<4)>0.5$ [and $\mathrm{P}(X<c)=0.5$] then $2<c<4$	**M1** **A1**	Finds two probabilities or finds two areas **AG** Be convinced or correct working leading to $c=\sqrt{14}$
		2	

Q	Answer	Marks	Comments
6(c)	See image below	B1	Any two of $\left(2,\frac{2}{9}\right),\left(4,\frac{5}{9}\right),\left(8,1\right)$ plotted or stated
		B1	Straight line for $0 \leqslant x \leqslant 2$
		B1	(Increasing +ve gradient) Curve for $2 \leqslant x \leqslant 4$ where $x=3,\ \mathrm{F}\left(x\right) \approx \frac{3}{9}$
		B1	(Decreasing +ve gradient) Curve for $4 \leqslant x \leqslant 8$ where $x=6,\ \mathrm{F}\left(x\right) \approx \frac{8}{9}$ and no curve for $x>8$ or $\mathrm{F}\left(x\right)>1$
			
		4	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7(a)	$H_0 : \mu = 127$ $H_1 : \mu < 127$	**B1**	Both hypotheses
	d.o.f $n = 9$	**M1**	**PI** by correct t_{crit}
	$t_{crit} = -1.383$	**A1**	**AWRT** Seen or used Allow $+1.383$
	$t = \dfrac{125 - 127}{\sqrt{\dfrac{12.9}{10}}}$	**M1**	
	$t = -1.76$	**A1**	**AWRT** Allow $p = 0.05605$ Allow 'reject H_0'
	Do not accept H_0 as $-1.76 < -1.38\left[3\right]$ **or** $t < t_{crit}$ or $\lvert t \rvert > t_{crit}$	**A1ft**	Must be consistent with their conclusion on whether to accept H_0 or not **or** their t and t_{crit} if not explicitly stated Correct conclusion based upon ft their t
	Evidence to suggest that Lottie's claim is true **or** evidence to suggest that the mean height is less than 127 cm [at the 10% level of significance]	**E1**	Must be in context, must not be definite and all the previous 6 marks must have been awarded
		7	

Q	Answer	Marks	Comments
7(b)(i)	$\left[\frac{1}{n}\sum x =\right]$ $\frac{1}{10}\times\left(123+125+128+124+122+129\right.$ $\left.+126+119+q+r\right)=125$ $1250 - 996 = 254$ $254 = q + r$	M1 A1	Allow $\frac{996+q+r}{10}=125$ or $996+q+r=1250$ **AG** Be convinced
		2	

Q	Answer	Marks	Comments
7(b)(ii)	$\sum x^2 = 123^2+125^2+128^2+124^2+129^2+126^2$ $+119^2+q^2+r^2$ or $\sum x^2 = 124076+q^2+r^2$	B1	**oe PI**
	$\left[s^2=\frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)\right]$ $=\frac{1}{9}\left(124076+q^2+r^2 - \frac{(1250)^2}{10}\right)=12.9$	M1	Attempt to substitute into s^2 Allow 1 slip
	$0 = q^2+r^2 - 32290.1$	m1	Simplifies to an equation of the form $0=q^2+r^2+c$ or $0=(125-q)^2+(125-r)^2+c$ **oe**
	$0=(254-x)^2+x^2-32290.1$ $0=2x^2-508x+32225.9$	M1	Use of $q+r=254$ to form a quadratic equation in one variable (e.g. $x=q$ or $x=r$)
	Hence $x=131$ or $x=123$	A1	x, q or r
	$\left[q>r\Rightarrow\right]\ q=131,\ r=123$	A1	**CAO**
		6	

	Question 7 Total	**15**	