OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

January 2022

Version: 1.0 Final

221XMA03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	<table><tr><td>x</td><td>y</td></tr><tr><td>0</td><td>$e^{-0^2} = 1$</td></tr><tr><td>0.75</td><td>$e^{-0.75^2} = 0.569782825$</td></tr><tr><td>1.5</td><td>$e^{-1.5^2} = 0.105399225$</td></tr><tr><td>2.25</td><td>$e^{-2.25^2} = 0.006329715$</td></tr><tr><td>3.0</td><td>$e^{-3^2} = 0.000123410$</td></tr></table>	B1 M1	All five correct x values (and no extra used) **PI** by five correct y values At least four correct y values in exact form or decimals, rounded or truncated to three dp **or** better (in table or formula) (**PI** by **AWRT** correct answer)
	$\frac{1}{3}\times 0.75[1+0.000123...+4(0.56978 +0.0063297)+2\times 0.105399]$	m1	Correct sub into formula with $h = 0.75$ **oe** and at least four correct y values either listed, with + signs, or totalled. (**PI** by AWRT correct answer)
	$= 0.879$	A1	**CAO**, must see this value exactly and no error seen
		4	

Q	Answer	Marks	Comments
1(b)(i)	$f(x) = e^{-x^2} - 0.5x - 0.5$		Or reverse
	$f(0.5) = e^{-0.5^2} - 0.25 - 0.5 = 0.0288...$ $f(0.6) = e^{-0.6^2} - 0.3 - 0.5 = -0.102...$	M1	Both values rounded or truncated to at least 1sf
	Change of sign, $0.5 < x < 0.6$	A1	Must have both statement and interval in words or symbols **or** comparing 2 sides: at 0.5, $e^{-0.5^2} > 0.75$; at 0.6, $e^{-0.6^2} < 0.8 = 0.8(\cdots)$ **(M1)** Conclusion as before **(A1)**
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$-x^2 = \ln\left(\frac{1}{2}(x+1)\right)$	**M1**	
	$x^2 = \ln\left(\frac{2}{(x+1)}\right)$		Must see a middle line
	$x = \sqrt{\ln\left(\frac{2}{(x+1)}\right)}$	**A1**	**AG** Condone inclusion of ±
		2	

Q	Answer	Marks	Comments
1(b)(iii)	$x_2 = 0.536$	**B1**	
	$x_3 = 0.514$	**B1**	If 0 scored then **SC1** for 0.54 AND 0.51
		2	

	Question 1 Total	**10**	

Q	Answer	Marks	Comments
2(a)	$\frac{dy}{dx} = 8\times 2(2x+1)^7 \cos 3x + (2x+1)^8 \times (-3\sin 3x)$	M1	$p(2x+1)^7 \cos 3x + (2x+1)^8 \times (-q\sin 3x)$
	$= 16(2x+1)^7 \cos 3x - 3(2x+1)^8 \sin 3x$	A1	All correct
		2	

Q	Answer	Marks	Comments
2(b)	$\frac{dy}{dx} = \frac{(2x^3+5)9x^2 - (3x^3-1)6x^2}{(2x^3+5)^2}$	M1	$\frac{(2x^3+5)ax^2 - (3x^3-1)bx^2}{(2x^3+5)^2}$
	$= \frac{51x^2}{(2x^3+5)^2}$	A1	Must see use of differentiation
		2	

Q	Answer	Marks	Comments
2(c)	$2y^2 + 4xy\frac{dy}{dx} = 6xy + 3x^2\frac{dy}{dx}\left[+\frac{dy}{dx}[1]\right]$	M1 A1	LHS or RHS correct implicit differentiation Both correct
	$\frac{dy}{dx} = \frac{6xy - 2y^2}{4xy - 3x^2 - 1}$	A1	**oe**
		3	

	Question 2 Total	7	

Q	Answer	Marks	Comments
3(a)	$8[(0.5)^3]+a[(0.5)^2]+b[0.5]+6=6$ $8[(-0.5)^3]+a[(-0.5)^2]+b[-0.5]+6=9$	M1	One correct substitution **or** for **M1** use of long division
	$0.25a+0.5b=-1$ $0.25a-0.5b=4$	A1 m1	Attempt to solve
	$b=-5$ $a=6$	A1	Both answers correct
		4	

Q	Answer	Marks	Comments
3(b)	$\mathrm{f}(-1.5)=8(-1.5)^3+6(-1.5)^2-5(-1.5)+6$ $=-27+13.5+7.5+6$ $=0$ As equal to 0, $(2x+3)$ is a factor	E1	Must see working Condone omission of statement
		1	

Q	Answer	Marks	Comments
3(c)	$\dfrac{8x^3+6x^2-5x+6}{4x^2+4x-3}=\dfrac{(2x+3)(4x^2-3x+2)}{(2x-1)(2x+3)}$	B1ft B1	Numerator correct PI Denominator correct Accept long division, or other equivalent methods eg $4x^2+4x-3\,\overline{)\,8x^3+6x^2-5x+6}$ with quotient $2x-0.5$ $8x^3+8x^2-6x$ $-2x^2+x+6$ $-2x^2-2x+1.5$ $3x+4.5$
	$=\dfrac{2x(2x-1)-0.5(2x-1)+1.5}{(2x-1)}$	M1	
	$=2x-\dfrac{1}{2}+\dfrac{3}{2(2x-1)}$	A1	
		4	

	Question 3 Total	9	

Q	Answer	Marks	Comments
4(a)	$\int y^2 \mathrm{d}y = \int 2x \,\mathrm{d}x$	M1	For attempt at integration after separating variables
	At (2, 3) $\frac{1}{3}y^3 = x^2 + 5$	A1	ACF
		2	

Q	Answer	Marks	Comments
4(b)	$\int 2y \,\mathrm{d}y = \int x^2 \,\mathrm{d}x$	M1	For attempt at integration after separating variables
	At (2, 3) $y^2 = \frac{1}{3}x^3 + \frac{19}{3}$	A1	ACF
		2	

Q	Answer	Marks	Comments
4(c)	C_1 $(2,3)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{9}$	M1	Either gradient correct
	C_2 $(2,3)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{6}$ $\left[= \frac{2}{3}\right]$		
	$\tan\theta = \dfrac{\frac{4}{6} - \frac{4}{9}}{1 + \frac{4}{6} \times \frac{4}{9}}$	M1	Correct use of trig identity oe
	$\tan\theta = \frac{6}{35}$	A1	
		3	

	Question 4 Total	7	

Q	Answer	Marks	Comments
5(a)(i)	$[12\cos\theta - 5\sin\theta =]$ $R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	M1	PI
	$R = 13$	A1	
	$\alpha = 0.395$	A1	
		3	

Q	Answer	Marks	Comments
5(a)(ii)	$\cos(x + 0.4 + 0.395) = \dfrac{6.5}{13}$ $\left[x + 0.795 = \pm\dfrac{\pi}{3} \quad \text{oe}\right]$	M1	Ft *their* **part (a)**
	$x = -1.84$	A1	One correct answer
	$x = 0.25$	A1	2nd correct answer and no extras Ignore answers outside range
		3	

Q	Answer	Marks	Comments
5(b)	$8\cot^2 y = 8\operatorname{cosec}^2 y - 8 \quad [= 2\operatorname{cosec} y + 7]$	M1	Correct use of trig identity **PI**
	$8\operatorname{cosec}^2 y - 8 = 2\operatorname{cosec} y + 7$ $8\operatorname{cosec}^2 y - 2\operatorname{cosec} y - 15 = 0$ $(4\operatorname{cosec} y + 5)(2\operatorname{cosec} y - 3)[= 0]$	m1	Factorisation or correct use of formula **PI**
	$\operatorname{cosec} y = -\dfrac{5}{4}, \dfrac{3}{2}$ or $\sin y = \dfrac{2}{3}, -0.8$	A1	Both correct and no errors seen (May use cos/sin for first **M1, m1, A1)**
	$y = -127°, -53°,$ $42°, 138°$	B1	Sight of **at least two** of these values correct
		B1	All 4 correct and no extras in interval (ignore answers outside interval)
		5	

	Question 5 Total	**11**	

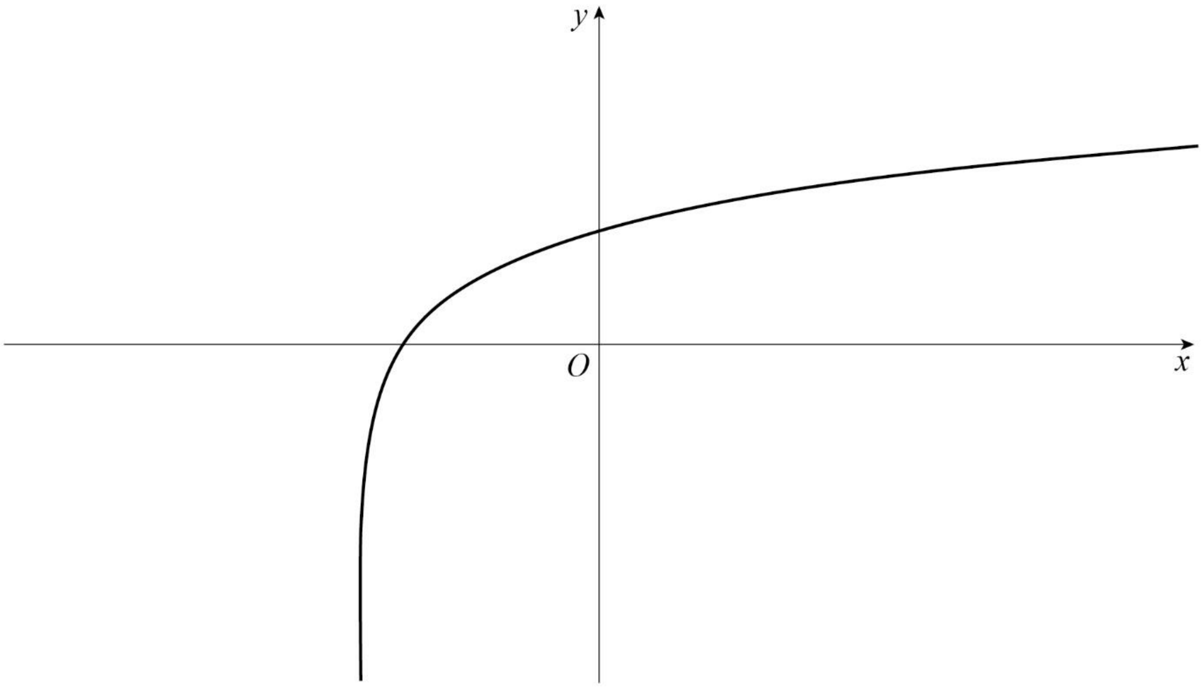
Q	Answer	Marks	Comments
6(a)	Translation $\begin{bmatrix} a \\ b \end{bmatrix}$	**M1**	Where at least one of a or b is non-zero
	$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$	**A1**	Correct transformation and vector If **M0** scored **SC1** for $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
		2	

Q	Answer	Marks	Comments
6(b)(i)	$y = \ln(x+2)+1$		
	$x-1 = \ln(y+2)$	**M1**	Swap y and x
	$y+2 = e^{x-1}$	**M1**	Attempt to isolate
	$f^{-1}(x) = e^{x-1} - 2$	**A1**	Correct answer and no errors seen
		3	

Q	Answer	Marks	Comments
6(b)(ii)	Reflection in $y = x$	**B1**	
		1	

Q	Answer	Marks	Comments
6(b)(iii)	$[f^{-1}(x)] > -2$	**B1**	Do not allow $x > -2$
		1	

Q	Answer	Marks	Comments
6(c)(i)	See diagram below	B1 B1 B1	Correct shape and position $(0, 1+\ln 2)$ stated or marked $(e^{-1}-2, 0)$ stated or marked
		3	



Q	Answer	Marks	Comments
6(c)(ii)	$\frac{dy}{dx}=\frac{1}{x+2}$ At $(-1,1)$ $y-1=1(x-(-1))$ $[y=x+2]$	M1 A1	
		2	

	Question 6 Total	**12**	

Q	Answer	Marks	Comments
7(a)	$[\frac{du}{dx}=]\ 4e^{4x}$ $dx = \frac{du}{4(u-1)}$	B1	
	$[\int \frac{1}{e^{4x}+1}dx = \int \frac{dx}{u}]$	M1	All in terms of u, condone omission of du
	$= \int \frac{du}{4u(u-1)}$	A1	Must see du here, or earlier
	$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$	M1	Use of partial fractions
	$1 = A(u-1) + Bu$ $A = -1,\quad B = 1$	A1	
	$\int \frac{du}{4u(u-1)} = \frac{1}{4}\left(\ln(u-1) - \ln u\right)$	m1	Correct integration
	$[x]_0^{\ln 2} = [u]_2^{17}$	B1	Change of limits, maybe seen earlier (may change back to x and not change limits)
	$\int_0^{\ln 2} \frac{1}{e^{4x}+1}dx = \frac{1}{4}[\ln\frac{16}{17} - \ln\frac{1}{2}]$		
	$= \frac{1}{4}\ln\frac{32}{17}$	A1	
		8	

Q	Answer	Marks	Comments
7(b)	$\left[\int \frac{e^{4x}}{1+2e^{4x}}dx = \right] k\ln(1+2e^{4x})\ [+c]$	M1	
	$= \frac{1}{8}\ln(1+2e^{4x}) + c$	A1	
		2	

	Question 7 Total	**10**	

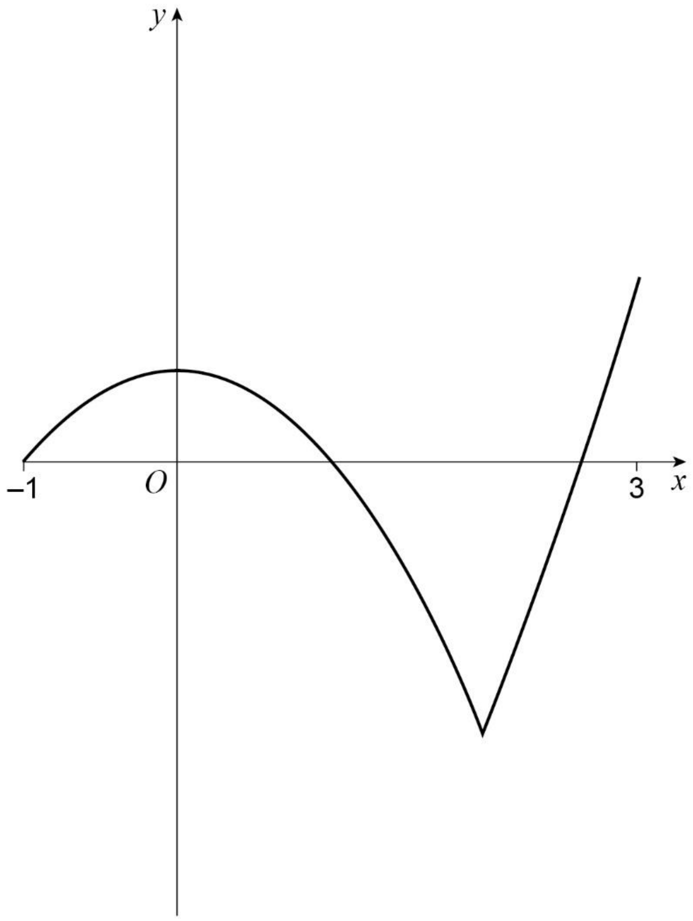
Q	Answer	Marks	Comments
8(a)	$\sec\theta = \frac{x}{a} \quad \tan\theta = \frac{y}{b}$	M1	
	$\left(\frac{x}{a}\right)^2 = 1 + \left(\frac{y}{b}\right)^2$	A1	ACF
		2	

Q	Answer	Marks	Comments
8(b)	When $\theta = \frac{\pi}{4}$: $x = a\sqrt{2}, \quad y = b$	B1	PI
	Either		
	$\frac{dx}{d\theta} = a\sec\theta\tan\theta \quad \frac{dy}{d\theta} = b\sec^2\theta$	M1	Either derivative correct
	$\frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta} = \left[\frac{b}{a}\text{cosec}\,\theta\right]$	A1	
	or		
	$\frac{dy}{dx} = p(\frac{x^2}{a^2} - 1)^{-\frac{1}{2}} \times qx$	(M1)	Attempt at isolating *y* and chain rule
	$= b\frac{1}{2}(\frac{x^2}{a^2} - 1)^{-\frac{1}{2}} \times \frac{2x}{a^2}$	(A1)	
	or		
	$\frac{2x}{a^2} = \frac{2y}{b^2}\frac{dy}{dx}$	(M1)	Attempt at implicit differentiation
	$\frac{dy}{dx} = \frac{xb^2}{ya^2}$	(A1)	
	When $\theta = \frac{\pi}{4}$: $\frac{dy}{dx} = \frac{b\sqrt{2}}{a}$ oe	A1	
	Equation of normal		
	$y - b = -\frac{a}{b\sqrt{2}}(x - a\sqrt{2})$	A1	All correct **ACF** eg $y = -\frac{\sqrt{2}a}{2b}x + \frac{a^2+b^2}{b}$
		5	

Q	Answer	Marks	Comments
8(c)	$x=0,\quad y-b=-\dfrac{a}{b\sqrt{2}}(-a\sqrt{2})$ $y=\dfrac{a^2+b^2}{b}$ $y=0,\quad -b=-\dfrac{a}{b\sqrt{2}}(x-a\sqrt{2})$ $x=\dfrac{\left(a^2+b^2\right)\sqrt{2}}{a}$ Area $=\dfrac{\left(a^2+b^2\right)^2}{\sqrt{2}\,ab}$	**M1** **A1** **A1**	Attempt to find *A* and *B* Both correct **oe**
		3	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)(i)	See diagram below	**B1** **B1** **B1**	y-intercept at a max point [1 or (0, 1)] Correct for $-1 \le x \le 2$ Correct for $2 \le x \le 3$,curvature at $x = 2$ and nothing for $x > 3$
		3	



Q	Answer	Marks	Comments
9(a)(ii)	$-3 \le \mathrm{f}(x) \le 2$	**B1**	Condone use of y, f, etc
		1	

Q	Answer	Marks	Comments
9(a)(iii)	$\lvert 4-x^2 \rvert = 1$	**M1**	PI
	$x=\sqrt{5}$ $\quad$ $x=\sqrt{3}$	**A1**	Both correct values and no extras
		2	

Q	Answer	Marks	Comments
9(b)	$\left\lvert 4-\dfrac{1}{(x-1)^2} \right\rvert -3=-2$	**M1**	**PI**
	$x=1\pm\dfrac{1}{\sqrt{3}}$, $\quad x=1\pm\dfrac{1}{\sqrt{5}}$	**A2,1,0**	Answers must be in exact form **A1**: at least two correct **oe** **A2**: all correct and no extras **oe**
		3	

	Question 9 Total	**9**	

Q	Answer	Marks	Comments
10(a)	$\cos(2\theta+\theta)=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	B1	
	$=\left(2\cos^2\theta-1\right)\cos\theta-2\sin\theta\cos\theta\sin\theta$	M1	Correct use of double angle formulae and $\sin^2\theta=1-\cos^2\theta$
	$=2\cos^3\theta-\cos\theta-2\cos\theta\left(1-\cos^2\theta\right)$		
	$=4\cos^3\theta-3\cos\theta$	A1	**AG**, no errors seen
		3	

Q	Answer	Marks	Comments
10(b)	$\cos^3 2x=\frac{1}{4}\left(3\cos 2x+\cos 6x\right)$	B1	PI
	$\int x\cos^3 2x\ \mathrm{d}x$		
	$=\frac{1}{4}x\left(\frac{3}{2}\sin 2x+\frac{1}{6}\sin 6x\right)$	M1 A1	Correct use of parts formula Correct integral of $\cos 2x+\cos 6x$
	$-\frac{1}{4}\int\left(\frac{3}{2}\sin 2x+\frac{1}{6}\sin 6x\right)\mathrm{d}x$	A1	All correct
	$=\frac{3}{8}x\sin 2x+\frac{1}{24}x\sin 6x$	M1	Correct integration
	$+\frac{3}{16}\cos 2x+\frac{1}{144}\cos 6x\quad[+c]$	A1	All correct
		6	

	Question 10 Total	9	

Q	Answer	Marks	Comments
11(a)	$\mathrm{f}(x)=\frac{A}{2-x}+\frac{B}{(1-2x)}+\frac{C}{(1-2x)^2}$		
	$12=A(1-2x)^2+B(2-x)(1-2x)+C(2-x)$	B1	Correctly eliminating fractions
	$x=2,\ \ 12=9A,\ \ A=\frac{4}{3}$	M1	Attempt at finding one constant
	$x=0.5,\ \ 12=\frac{3}{2}C,\ \ C=8$	A1	Two constants correct
	$x=0,\ \ 12=\frac{4}{3}+2B+16,\ \ B=-\frac{8}{3}$		
	$\mathrm{f}(x)=\frac{4}{3(2-x)}-\frac{8}{3(1-2x)}+\frac{8}{(1-2x)^2}$	A1	All correct Allow equivalent methods
		4	

Q	Answer	Marks	Comments
11(b)	$(2-x)^{-1}=\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}=\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2$	B1	
		1	

Q	Answer	Marks	Comments
11(c)	$\mathrm{f}(x):$ $\frac{4}{3}(2-x)^{-1}=\frac{4}{3}\left(\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2\right)$		
	$(1-2x)^{-1}=1+2x+4x^2$ $(1-2x)^{-2}=1+4x+12x^2$	M1 A1 A1	Expansion in the form $1+ax+bx^2$ One mark for each correct expansion
	$\mathrm{f}(x):$ $\frac{4}{3}\left(\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2\right)-\frac{8}{3}(1+2x+4x^2)$ $+8(1+4x+12x^2)$	M1	
	$\mathrm{f}(x)=6+27x+85.5x^2$	A1	Allow equivalent methods
		5	

	Question 11 Total	**10**	

Q	Answer	Marks	Comments
12	Coords of $P\,(-2+3p, 3+4p, -1-5p)$	B1	
	Direction AP $\begin{bmatrix} 3p \\ 5+4p \\ -4-5p \end{bmatrix}$	M1	Seen or used
	$\begin{bmatrix} 3p \\ 5+4p \\ -4-5p \end{bmatrix} . \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$ [= 0]	m1	OR $AP^2 = (3p)^2 + (5+4p)^2 + (4+5p)^2$ $(X) = 50p^2 + 80p + 41$
	$50p = -40$ $p = -0.8$	A1	$\frac{\mathrm{d}X}{\mathrm{d}p} = 100p + 80, \quad p = -0.8$
	Dist = $\sqrt{(-4.4+2)^2 + (-0.2+2)^2 + (3-3)^2}$	M1	$\frac{\mathrm{d}^2X}{\mathrm{d}p^2} = 100 > 0$ MIN
	$= 3$	A1	CSO
		6	

	Question 12 Total	**6**	

Q	Answer	Marks	Comments
13(a)	$t = 0, M = 10 \quad 10 = \dfrac{A}{1+2}$		
	$A = 30$	B1	
	$t = 1, M = 15 \quad 15 = \dfrac{30}{1+2e^k}$		
	$1 + 2e^k = 2$	M1	Attempt to find *k*
	$e^k = 0.5$		
	$k = -\ln 2$	A1	**oe**
		3	

Q	Answer	Marks	Comments
13(b)	$M = \dfrac{30}{1+2e^{-5\ln 2}}$	M1	
	$M = 28$	A1	
		2	

Q	Answer	Marks	Comments
13(c)	$18 = \dfrac{30}{1+2e^{-t\ln 2}}$		
	$1 + 2e^{-t\ln 2} = \dfrac{5}{3}$		
	$e^{-t\ln 2} = \dfrac{1}{3}$	M1	
	$-t\ln 2 = -\ln 3$		
	$t = \dfrac{\ln 3}{\ln 2}$	A1	**oe**
		2	

Q	Answer	Marks	Comments
13(d)	$M = A\left(1+2e^{kt}\right)^{-1}$ $\frac{dM}{dt} = -A\left(1+2e^{kt}\right)^{-2} \times 2ke^{kt}$ When $t = 4$ $\frac{dM}{dt} = -30\left(1+2e^{-4\ln 2}\right)^{-2} \times 2 \times (-\ln 2) \times e^{-4\ln 2}$ $\frac{dM}{dt} = \frac{80}{27}\ln 2$	 **M1 A1ft** **A1ft**	 ft their A and k Note that $e^k = \frac{1}{2}$ for the correct value of k **oe**
		3	

	Question 13 Total	**10**	