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(9660/MA05) Unit M2 Mechanics

Mark scheme

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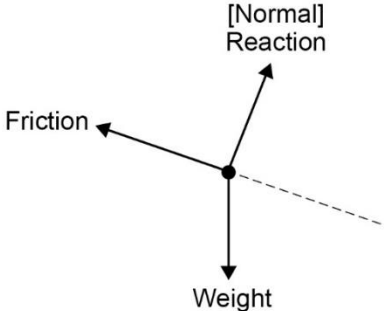
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$ \mathbf{r} ^2 = (\sin(2t) \cos(3t))^2$ $+ (\sin(2t) \sin(3t))^2$ $+ (\cos(2t))^2$	M1	Squaring each component in terms of t and adding
	$ \mathbf{r} ^2 = \sin^2(2t) (\cos^2(3t) + \sin^2(3t))$ $+ \cos^2(2t)$	M1	Use of $\cos^2(3t) + \sin^2(3t) = 1$
	$ \mathbf{r} ^2 = \sin^2(2t) + \cos^2(2t)$	A1	
	$ \mathbf{r} = 1$	A1	Shows that the distance or square of the distance is a constant, independent of t
		4	

Q	Answer	Marks	Comments
1(b)	$\mathbf{v} = \begin{bmatrix} 2 \cos(2t) \cos(3t) - 3 \sin(2t) \sin(3t) \\ 2 \cos(2t) \sin(3t) + 3 \sin(2t) \cos(3t) \\ -2 \sin(2t) \end{bmatrix}$	B1 M1	Correct \mathbf{k} component Use of product rule for \mathbf{i} or \mathbf{j} component
		A1	Both \mathbf{i} and \mathbf{j} components correct
	$\mathbf{v} = \begin{bmatrix} 2 \cos(0) \cos(0) - 3 \sin(0) \sin(0) \\ 2 \cos(0) \sin(0) + 3 \sin(0) \cos(0) \\ -2 \sin(0) \end{bmatrix}$	M1	PI ft their velocity evaluated at $t = 0$ provided at least one component is correct
	$\mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$	A1	ft their velocity evaluated correctly at $t = 0$ Do not ISW
		5	

	Question 1 Total	9	
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Q	Answer	Marks	Comments
2(a)		B1	<p>Must have names on the three arrows, not symbols unless the symbols are defined</p> <p>Allow gravitational force instead of weight but do not accept 'gravity' instead of weight</p> <p>These three forces and no others</p>
		1	

Q	Answer	Marks	Comments
2(b)	<p>Component of weight down the slope $\left[= mg \sin \alpha = 5 \times 9.8 \times \sin(10^\circ) \right]$ $= 8.51 \text{ [N]}$</p> <p>Friction force acting on the particle $\left[\leq \mu mg \cos \alpha = 0.25 \times 5 \times 9.8 \times \cos(10^\circ) \right]$ $\left[\leq \right] 12.1 \text{ [N]}$</p> <p>$8.51 \leq 12.1$</p> <p>[Resultant force = 0] therefore remains at rest</p>	<p>B1</p> <p>B1</p> <p>E1</p>	<p>If B0 B0 then allow SC1 for $\left[F = \right] mg \sin \alpha - \mu mg \cos \alpha$ or better</p> <p>Inequality and a correct conclusion that component of weight down the slope is less than the maximum friction force</p>
		3	

Q	Answer	Marks	Comments
2(c)	Resultant force down the slope $F = mg \sin \alpha - \mu mg \cos \alpha$ $\left[F = 5 \times 9.8 \sin \left(20^\circ \right) - 0.25 \times 5 \times 9.8 \cos \left(20^\circ \right) \right]$ $= 5.24775... \text{ [N]}$ or $\left[a = g \sin \alpha - \mu g \cos \alpha \right]$ $a = 9.8 \times \sin \left(20^\circ \right) - 0.25 \times 9.8 \times \cos \left(20^\circ \right)$ $a = 1.05 \text{ [m s}^{-2}\text{]}$	M1	Correct component of weight down the slope or correct friction force PI
		A1	Correct value for resultant force or correct calculation for acceleration PI by correct answer
			A1
		3	

	Question 2 Total	7	
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Q	Answer	Marks	Comments
3(a)	(4, 2.5) The centre of mass of the uniform rectangular lamina is at the geometric centre	B1 E1	Condone 'middle' for geometric centre
		2	

Q	Answer	Marks	Comments
3(b)	$(2 + 3 + 4 + 5)\bar{X}$ $= 2 \times 4 + 3 \times 2 + 4 \times 4 + 5 \times 7$ $\bar{X} = \frac{65}{14}$ $(2 + 3 + 4 + 5)\bar{Y}$ $= 2 \times 2.5 + 3 \times 2.5 + 4 \times 4 + 5 \times 1$ $\bar{Y} = \frac{67}{28}$	M1 A1 M1 A1	M1: Forming equation for x -coordinate of the centre of mass. Condone one slip A1: Correct and in an exact form, oe M1: Forming equation for y -coordinate of the centre of mass. Condone one slip A1: Correct and in an exact form, oe
		4	

Q	Answer	Marks	Comments
3(c)	<p>Angle AC makes with edge of length 5:</p> $\tan^{-1}\left(\frac{8 - \frac{65}{14}}{5 - \frac{67}{28}}\right) = \tan^{-1}\left(\frac{47/14}{73/28}\right) = 52.167\dots^\circ$ <p>Angle AR makes with edge of length 5:</p> $\tan^{-1}\left(\frac{1}{4}\right) = 14.036\dots^\circ$ <p>$52.167\dots^\circ - 14.036\dots^\circ$</p> <p>$38^\circ$</p>	B1ft B1 M1 A1	<p>[Note that C represents the centre of mass of the system] or Angle AC makes with edge of length 8:</p> $\tan^{-1}\left(\frac{73/28}{47/14}\right) = 37.832\dots^\circ$ <p>or Angle AR makes with edge of length 8: $\tan^{-1}(4) = 75.963\dots^\circ$</p> <p>Subtracting their angles – must have scored at least one of the B1 marks</p> <p>CAO given to the nearest degree</p>
		4	

	Question 3 Total	10	
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Q	Answer	Marks	Comments
4(a)	<p>No resultant force in vertical direction gives:</p> $T \cos \theta = mg$ $\cos \theta = \frac{3 \times 9.8}{60}$ $\theta = 60.659...^\circ$ $r = l \sin \theta$ $l = \frac{0.6}{\sin(60.659...^\circ)}$ $l = 0.688$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Forming a correct trigonometric equation</p> <p>Finding the correct angle from their trigonometric equation, PI oe</p> <p>Forming a correct equation for l</p> <p>CAO Ignore inclusion of units on final answer</p>
		4	

Q	Answer	Marks	Comments
4(b)	<p>Magnitude of resultant force</p> $F = 60 \sin(60.659\dots^\circ)$ $F = 52.303\dots \text{ [N]}$ <p>Magnitude of acceleration</p> $a = \frac{F}{m} = \frac{52.303\dots}{3}$ $a = 17 \text{ [m s}^{-2}\text{]}$ <p>Direction: Towards C</p>	<p>M1</p> <p>A1</p> <p>B1</p>	<p>PI Calculating resultant force</p> <p>Answer to 3 sf is 17.4 [m s⁻²]</p> <p>Condone 'to centre'</p>
		3	

Q	Answer	Marks	Comments
4(c)	$a = r\omega^2 \Rightarrow \omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{17.434...}{0.6}}$ $\omega = 5.4 \left[\text{rad s}^{-1} \right]$	<p>M1</p> <p>A1</p>	<p>ft their acceleration from part (b) or for linear speed = 3.23... [m s⁻¹]</p> <p>CAO Answer to 3 sf is 5.39 [rad s⁻¹]</p>
		2	

	Question 4 Total	9	
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Q	Answer	Marks	Comments
5(a)	<p>Resultant force:</p> $T - \mu mg - 31\sqrt{v} = ma$ $T = ma + \mu mg + 31\sqrt{v}$ $T = 20 \times 2 + 0.5 \times 20 \times 9.8 + 31\sqrt{4}$ $T = 200$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>Forming equation of motion Allow one slip/omission PI</p> <p>Substitution into their equation for T</p> <p>Correct value for T Ignore inclusion of units</p>
		3	

Q	Answer	Marks	Comments
5(b)	$P = Fv$ $= 200 \times 8$ $= 1600 \text{ W}$	M1 A1	Use of $P = Fv$ with their T from part (a) CAO and must include units Allow J s^{-1}
		2	

Q	Answer	Marks	Comments
5(c)	<p>When in equilibrium:</p> $T - \mu mg - 31\sqrt{v} = 0$ $\Rightarrow v = \left(\frac{T - \mu mg}{31} \right)^2$ $v = \left(\frac{200 - 0.5 \times 20 \times 9.8}{31} \right)^2$ $v = 10.8$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>Forming a three-term equation for v using equilibrium and making v the subject. Allow one slip PI</p> <p>ft substituting their T into their formula for v</p> <p>CAO Ignore inclusion of units</p>
		3	

	Question 5 Total	8	
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Q	Answer	Marks	Comments
6(a)	<p>Initial GPE (relative to horizontal ground):</p> $mgh = 12 \times 9.8 \times 8$ $= 940.8 \text{ J}$ <p>Work done against the resistive force:</p> $Fd = 16 \times 20$ $= 320 \text{ J}$ <p>Conservation of Energy:</p> $940.8 - 12 \times 9.8 \times 3 = 320 + \frac{1}{2}mv^2$ $v = \sqrt{\frac{2 \times 268}{12}} \quad \text{or} \quad v = 6.683\dots$ <p>Therefore $v = 6.7$ [to 2 sf]</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>May be seen in the calculation for the change in GPE between A and B</p> <p>v or v^2 seen in an exact form or to at least 3 sf. Be convinced</p> <p>Accept $\frac{\sqrt{402}}{3}$</p>
		4	

Q	Answer	Marks	Comments
6(b)	<p>At maximum height, vertical component of velocity is zero:</p> $v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$ $s = \frac{0 - (6.7 \sin 40^\circ)^2}{2 \times -9.8}$ $s = 0.946\dots \text{ [m]}$ <p>Maximum height above the horizontal ground is:</p> $0.946\dots + 3 = 3.9 \text{ [m]}$	<p>M1</p> <p>A1</p> <p>A1ft</p>	<p>Use of SUVAT equation with $v = 0$ or use of conservation of energy, such as</p> $mg\Delta h = \frac{1}{2}m(6.7 \sin 40^\circ)^2$ <p>Correct value for s, PI (the height above B) Value is 0.941... m if 6.683... is used</p> <p>their $s + 3$ AWFW [3.9, 4.0] from correct working</p>
		3	

Q	Answer	Marks	Comments
6(c)	Time at which the particle reaches C: $s = ut + \frac{1}{2}at^2$ $-3 = 6.7 \sin(40^\circ) \times t + \frac{1}{2} \times -9.8 \times t^2$ $t = 1.336... \quad [, -0.457...]$ $x = 6.7 \cos(40^\circ) \times 1.336...$ $x = 6.9$	M1 m1 A1 M1 A1	Use of SUVAT equation with $s = -3$ Forming correct quadratic equation in t Correct value for t $t = 1.335...$ if $v = 6.683...$ is used Must use cosine and their $t > 0$ CAO Answer is: 6.9 or 6.86 if $v = 6.7$ is used throughout 6.8 or 6.84 if $v = 6.683...$ is used throughout
		5	

Q	Answer	Marks	Comments
6(d)	Vertical component of velocity at C $v = u + at$ $v = 6.7 \sin(40^\circ) + (-9.8) \times 1.336...$ $= -8.794... \quad [\text{m s}^{-1}]$ Using Pythagoras' theorem $\text{speed} = \sqrt{(6.7 \cos(40^\circ))^2 + (-8.794...)^2}$ $= 10 \quad [\text{m s}^{-1}]$	M1 A1	May use energy considerations Value is $-8.789...$ if $v = 6.683...$ is used PI by correct final answer AWRT 10 from correct working Answer to 3 sf is $10.2 \text{ [m s}^{-1}]$
		2	

	Question 6 Total	14	
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Q	Answer	Marks	Comments
7(a)	Frictionless	B1	oe
		1	

Q	Answer	Marks	Comments
7(b)(i)	Taking moments about the base of the ladder $4 \times 25g \cos(65^\circ) + 6 \times 75g \cos(65^\circ)$ $= 8 \times R_W \sin(65^\circ)$ $R_W = \frac{(4 \times 25 + 6 \times 75) \times 9.8 \times \cos(65^\circ)}{8 \times \sin(65^\circ)}$ $R_W = 310 \text{ [N]}$	M1 m1 A1 A1	Forming equation using moments m1: At least one side of equation correct A1: Both sides of equation correct R_W = normal reaction on ladder from wall CAO Answer is 314 [N] to 3 sf
		4	

Q	Answer	Marks	Comments
7(b)(ii)	310 [N]	B1ft	
	Newton's 3rd Law	E1	Any reference to Newton's 3rd Law
		2	

Q	Answer	Marks	Comments
7(b)(iii)	310 [N]	B1ft	
	Newton's 1st Law	E1	Allow any reference to the ladder being in equilibrium in the horizontal direction, eg forces on the ladder to the left have the same magnitude as the forces on the ladder to the right
		2	

Q	Answer	Marks	Comments
7(c)	Equilibrium of forces in the vertical direction: $R_G = 100 \times 9.8 = 980 \text{ N}$ Equilibrium of forces in the horizontal direction: $f_G = 314.174... \text{ N}$ $f_G \leq 0.8\mu R_G$ $\mu \geq \frac{314.174...}{0.8 \times 980} = 0.40$	M1 m1 A1	R_G = normal reaction on ladder from ground f_G = friction on ladder from ground Forms inequality for the coefficient of friction, including 0.8 CAO, AWRT 0.40 Condone 0.4
		3	

Q	Answer	Marks	Comments
7(d)	[As] $0.35 < 0.40$ [the coefficient of friction is now less than the minimum coefficient of friction allowed by the safety reasons] It is not safe for the person to use the ladder	B1F E1F	Comparison of 0.35 with their minimum value for μ from part (c) using the $0.8F$ condition Statement must be consistent with their comparison
		2	

	Question 7 Total	14	
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Q	Answer	Marks	Comments
8(a)	$v = \frac{2\pi r}{T}$ or $v = \omega r$ $v = \frac{2\pi \times 6000 \times 10^3}{10 \times 60 \times 60}$ $v = 1047.197... \text{ [m s}^{-1}\text{]}$ Therefore $v = 1050 \text{ [m s}^{-1}\text{]} \text{ [to 3 sf]}$	M1 m1 A1	PI by a calculation Value seen to at least 4 sf or better. Be convinced
		3	

Q	Answer	Marks	Comments
8(b)	As spacecraft A is at the North pole, it is not travelling around a circle and so its speed is zero	E1	Any correct explanation based on $r = 0$
		1	

Q	Answer	Marks	Comments
8(c)	$a = \frac{v^2}{r} = \frac{1050^2}{6000 \times 10^3}$ $a = 0.18 \text{ [m s}^{-2}\text{]}$	M1 A1	PI AWRT $0.18 \text{ [m s}^{-2}\text{]}$, accept $\frac{147}{800}$
		2	

Q	Answer	Marks	Comments
8(d)	Radius of circle traversed by Spacecraft C $r = 6000 \times 10^3 \times \cos(45^\circ)$ $r = 4.242[640687] \times 10^6 \text{ [m]}$ $F = m\omega^2 r$ $= 185 \times \left(\frac{2\pi}{10 \times 60 \times 60} \right)^2 \times 4.242[...] \times 10^6$ $F = 24 \text{ [N]}$	B1 M1 A1	PI by correct answer Use of $r = 6000 \text{ km}$ is M0 AWRT 24 [N]
		3	

	Question 8 Total	9	
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