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FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\mathbf{C}^{\mathrm{T}} = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 0 & k \\ 0 & -k & -3 \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
1(b)(i)	$\mathbf{CC}^{\mathrm{T}} = \begin{bmatrix} 4 & -3 & 0 \\ 3 & 0 & -k \\ 0 & k & -3 \end{bmatrix}\begin{bmatrix} 4 & 3 & 0 \\ -3 & 0 & k \\ 0 & -k & -3 \end{bmatrix}$ $= \begin{bmatrix} 25 & 12 & -3k \\ 12 & 9+k^2 & 3k \\ -3k & 3k & k^2+9 \end{bmatrix}$	M1 A1	Attempt to multiply matrices with at least three elements correct
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$k^2+9=25$ $k=4$ **and** $k=-4$	M1 A1	Sets their k^2+9 equal to 25 **PI**
		2	

Q	Answer	Marks	Comments
1(c)(i)	The number of columns in the matrix **C** must be the same as the number of rows in the matrix **D**	E1	**oe**
		1	

Q	Answer	Marks	Comments
1(c)(ii)	$\mathbf{DC} = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -2 \end{bmatrix}$	B1 B1	For a 2 by 3 matrix For a fully correct answer
		2	

	Question 1 Total	**8**	

Q	Answer	Marks	Comments
2(a)	Lets $\mathrm{f}(x) = 2.7^x - 2x - 5$ $\mathrm{f}(2) = -1.71$ $\mathrm{f}(3) = 8.683$ Since change of sign between $x = 2$ and $x = 3$ and as the curve is continuous [on this interval] then there is a root in the interval $2 < x < 3$	**M1** **A1**	Correct evaluation of a suitable interval Must mention change of sign and continuous curve in conclusion
		2	

Q	Answer	Marks	Comments
2(b)	$\mathrm{f}(2) = -1.71$ $\mathrm{f}(3) = 8.683$ $\mathrm{f}(2.5) = 1.9786...$ $2 < x < 2.5$ $\mathrm{f}(2.25) = -0.1552...$ $2.25 < x < 2.5$ $\mathrm{f}(2.375) = 0.8300...$ $2.25 < x < 2.375$ $\mathrm{f}(2.3125) = 0.3182...$ $2.25 < x < 2.3125$ $[a] = 2.3$	**M1** **A1** **A1** **A1**	Attempting to calculate f (mid-point) of at least two sets of values Interval stated. **PI** Interval stated. **PI** **CSO** Must see $2.25 < x < 2.3125$ or better
		4	

Q	Answer	Marks	Comments
2(c)	$\mathrm{f}(-3) = 1.0508...$ $\mathrm{f}(-2) = -0.8628...$ $n = -3$	**M1** **A1**	**PI** by correct answer or by $\beta = -2.4564...$ Condone $-3 < \beta < -2$ with no incorrect work
		2	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)(i)	$\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 3 & 3\\0 & 1 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 5 & 7 & 3\\0 & 1 & 1 & 0\end{bmatrix}$ $(1,0),\ (5,1),\ (7,1),\ (3,0)$	M1 A1	Attempts to multiply matrices or multiply $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}1\\1\end{bmatrix}$, $\begin{bmatrix}3\\1\end{bmatrix}$ or $\begin{bmatrix}3\\0\end{bmatrix}$ by M **CAO**
		2	

Q	Answer	Marks	Comments
3a(ii)	Shear Parallel to the x-axis	B1 B1	**oe** for example 'leaving all the points on the x-axis as invariant' Condone 'x-axis is invariant'
		2	

Q	Answer	Marks	Comments
3(b)	$\begin{bmatrix}1 & 0\\a & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}\Rightarrow\ \begin{bmatrix}1\\a\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$ $a=2$	M1 A1	Multiplying matrices or using coordinates from the diagram to find a **PI** by correct answer
		2	

Q	Answer	Marks	Comments
3(c)	$\mathbf{NM}=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 4\\2 & 9\end{bmatrix}$ $\det(\mathbf{NM})=9-8=1$ Area of the shape remains the same	M1 A1 E1ft	Calculating **NM** using their a **PI** by correct value for det(**NM**) Concluding statement: **ft** if det(**NM**) = 1
		3	
3(c) ALT	det **N**=1 det **M**=1 det **NM**=det **N** x det **M**=1 Area of the shape remains the same	M1 A1 E1	Calculation of det **N** and det **M** using their a **PI** by correct value for det(**NM**) Concluding statement
		3	

	Question 3 Total	**9**	

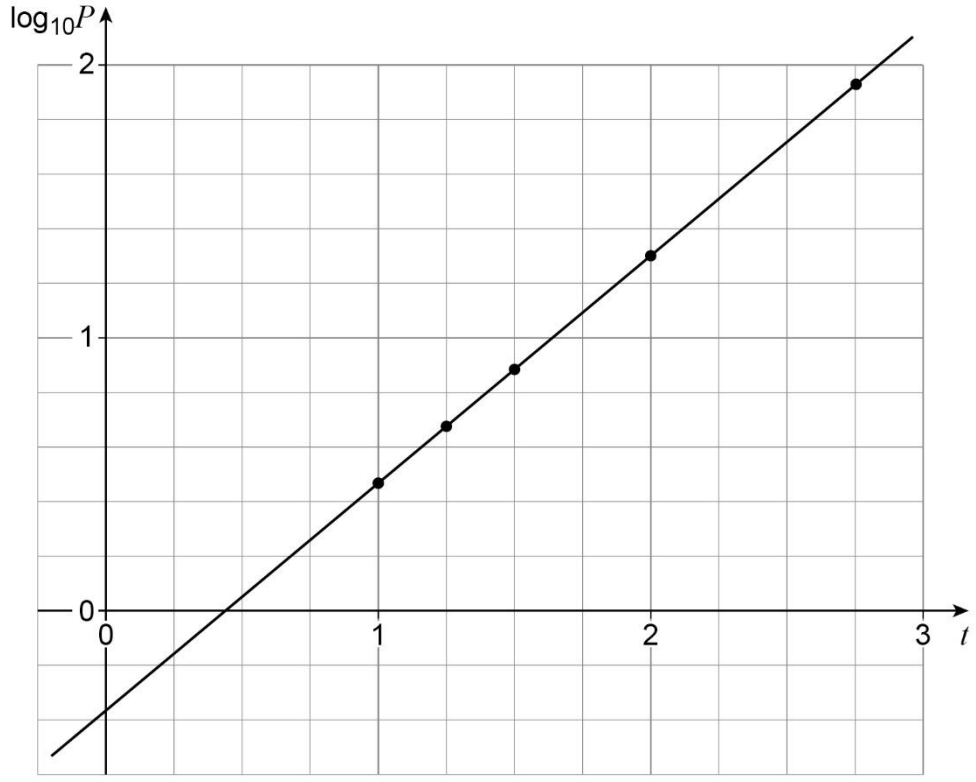
Q	Answer	Marks	Comments
4	$hf(x,y) = 0.1\left(2 - \frac{1.5^3}{2}\right)$	M1	Condone slip on substitution **PI**
	$y_1 = 1.5 + 0.1 \times 0.3125 = \frac{49}{32} = 1.53125$	A1	**AWRT** 1.53 **PI**
	$y_2 = 1.53125 + 0.1\left(2.1 - \frac{1.53125^3}{2.1}\right)$	M1	Correct use of formula. **PI**
	$y = 1.5703 \quad (\text{to 4 dp})$	A1	**CAO**
		4	

	Question 4 Total	4	

Q	Answer	Marks	Comments
5(a)	$P = a \times 10^{kt}$ $\log_{10} P = \log_{10}\left(a \times 10^{kt}\right)$ $= \log_{10} a + \log_{10}\left(10^{kt}\right)$ $= \log_{10} a + kt$	**B1**	Using the rules for logs correctly **AG** Full steps must be shown
		1	

Q	Answer	Marks	Comments
5(b)	$\log_{10} P$ | 0.47 | **0.68** | **0.89** | **1.30** | **1.93**	**B1**	**CAO**
		1	

Q	Answer	Marks	Comments
5(c)	'Their' points correctly plotted Line of best fit drawn	**B1 ft** **B1 ft**	All points plotted ± 0.25 square See image below



		2	

Q	Answer	Marks	Comments
5(d)(i)	Their intercept $= \log_{10} a$ $\log_{10} a = -0.35$ $a = 10^{-0.35} = 0.45$ Their gradient $= k$ $k = 0.83$	**M1** **A1** **B1**	Allow answers $-0.4 \leq \log_{10} a \leq -0.3$ Allow answers in the range $0.40 \leq a \leq 0.50$ Allow answers in the range $0.70 \leq k \leq 0.90$
		3	

Q	Answer	Marks	Comments
5(d)(ii)	$P = 0.45 \times 10^{0.83t}$	**B1ft**	**ft** their values of a and k
		1	

Q	Answer	Marks	Comments
5(d)(iii)	$P = 0.45 \times 10^{0.83 \times 4}$ $= 940$ Total profit is 940 million dollars	**M1** **A1**	Substitute t = 4 into their formula Must include units Allow answers in the range 250 million < P < 2000 million
		2	

Q	Answer	Marks	Comments
5d(iv)	This total profit is extrapolated [so therefore may be unreliable]	**E1**	Reference to extrapolation
		1	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	Discrete Uniform (Distribution)	**B1**	Condone uniform distribution
		1	

Q	Answer	Marks	Comments
6(b)	0.75	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
6(c)(i)	$p = 0.2$	**B1**	**oe**
		1	

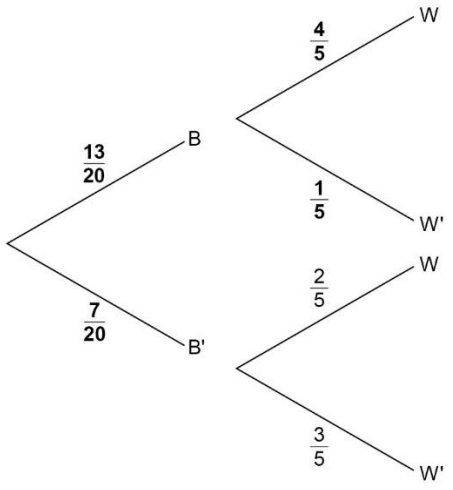
Q	Answer	Marks	Comments
6(c)(ii)	$\mathrm{E}(X) = 2.5$	**B1**	**oe** may not be evaluated
	$\mathrm{E}(2X - 5Y) = 2\mathrm{E}(X) - 5\mathrm{E}(Y)$	**M1**	Applies formula Implied by sight of $2 \times 2.5 - 5 \times 5$ for their 2.5 **PI**
	$= -20$	**A1**	
		3	

	Question 6 Total	**6**	

Q	Answer	Marks	Comments
7(a)	$\text{P(not win)} = \frac{17}{50}$	**B1**	**oe**, seen or used
	$\text{P(Basia plays)} = \frac{33}{50} \times \frac{26}{33} + \frac{17}{50} \times \frac{13}{34}$	**M1**	
	$= \frac{13}{20}$	**A1**	**oe**
		3	

Q	Answer	Marks	Comments
7(b)	$\text{P(win} \mid \text{Basia plays)} = \frac{\frac{33}{50} \times \frac{26}{33}}{\frac{13}{20}}$	**M1**	**ft** their P(Basia plays) provided between 0 and 1
	$= \frac{4}{5}$	**A1**	**AG**, be convinced
		2	

Q	Answer	Marks	Comments
7(c)	See image below	**B1ft**	**ft** their P(B) provided between 0 and 1 **oe**



		1	
	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	$G_X(t) = 0.4 + 0.25t + 0.35t^2$	B1	**oe** eg $\frac{2}{5} + \frac{1}{4}t + \frac{7}{20}t^2$
		1	

Q	Answer	Marks	Comments
8(b)(i)	$G_{X+Y}(t) = G_X(t)G_Y(t)$	M1	Applies formula
	$0.19t + 0.32875t^2 + 0.2975t^3 + 0.18375t^4$	A1	**oe** eg $\frac{19}{100}t + \frac{263}{800}t^2 + \frac{119}{400}t^3 + \frac{147}{800}t^4$
		2	

Q	Answer	Marks	Comments
8(b)(ii)	$G'_{X+Y}(t) = 0.19 + 0.6575t + 0.8925t^2 + 0.735t^3$	M1	Differentiates once **oe** eg $\frac{19}{100} + \frac{263}{400}t + \frac{357}{400}t^2 + \frac{147}{200}t^3$ Condone one slip
	$G''_{X+Y}(t) = 0.6575 + 1.785t + 2.205t^2$	M1	Differentiates twice **oe** eg $\frac{263}{400} + \frac{357}{200}t + \frac{441}{200}t^2$ Condone one slip **ft** their $G'_{X+Y}(t)$
	$G'_{X+Y}(1) = 2.475$ or $G''_{X+Y}(1) = 4.6475$	A1	Finds one of $G'_{X+Y}(1)$ or $G''_{X+Y}(1)$ **oe** eg $\frac{99}{40}$ **or** $\frac{1859}{400}$ **PI**
	$\sigma^2 = G''_{X+Y}(1) + G'_{X+Y}(1) - \left(G'_{X+Y}(1)\right)^2$	M1	Applies variance formula to their $G_{X+Y}(t)$
	$= 0.996875$	A1	**oe**, eg $\frac{319}{320}$ **SC** a fully correct solution that does not use the result obtained in part **(b)(i)** scores 3/5
		5	

	Question 8 Total	8	

Q	Answer	Marks	Comments
9	$[s] = L$ $[vt] = LT^{-1} \times T = L$ $\left[\frac{1}{2}at^2\right] = LT^{-2} \times T^2 = L$ $\therefore$ Dimensionally consistent	**B1** **B1** **B1**	For correct dimensions of at least two terms (condone use of units) All three dimensions correct Obtains same dimensions and states conclusion
		3	

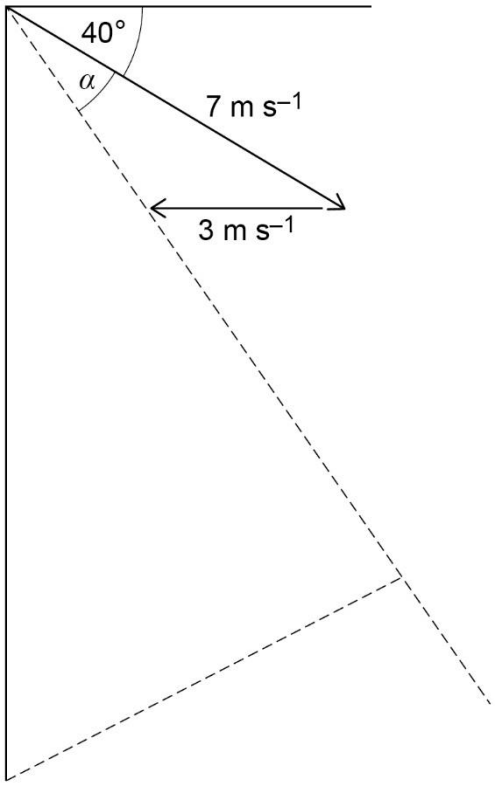
	Question 9 Total	**3**	

Q	Answer	Marks	Comments
10(a)	$7\times4+3\times(-5)=7v_A+3v_B$ $13=7v_A+3v_B$	**M1**	Conservation of momentum equation Condone sign errors
	$v_A-v_B=-0.9(4-(-5))$ $v_A-v_B=-8.1$	**M1** **A1**	Coefficient of restitution equation Condone sign errors Both equations correct
	$v_A=-1.13$ Speed of $A=1.13\text{ m s}^{-1}$	**A1**	Correct speed for *A*
	$v_B=6.97$ Speed of $B=6.97\text{ m s}^{-1}$	**A1**	Correct speed for *B*
		5	

Q	Answer	Marks	Comments
10(b)	Sphere *A* changes direction Sphere *B* changes direction	**B1**	Both statements correct
		1	

Q	Answer	Marks	Comments
10(c)	$\int_0^{0.02}kt(0.02-t)dt$	**M1**	Forms integral
	$=\dfrac{k}{750000}$	**A1**	Evaluates integral correctly
	$I=7\times(-1.13)-7\times4$ or $I=3\times6.97-3\times(-5)$	**M1**	Uses impulse equation Condone sign errors
	$=-35.91$ or 35.91	**A1**	Correct impulse
	$k=750000\times35.91$ $=26900000$	**A1**	**CAO**
		5	

	Question 10 Total	**11**	

Q	Answer	Marks	Comments
11	$v_{SF}{}^2 = 7^2 + 3^2 - 2\times 7\times 3\cos 40°$ $v_{SF} = \sqrt{58 - 42\cos 40°} = 5.082$	**M1** **A1**	Correct method to find relative speed Correct relative speed
	$\dfrac{\sin\alpha}{3} = \dfrac{\sin 40°}{\sqrt{58 - 42\cos 40°}}$ $\alpha = 22.3°$	**M1** **A1**	Correct method to find unknown angle Correct angle. **PI** by sight of 27.7°
	Minimum Distance $= 1500\sin(90 - 50 - 22.3)$ $= 697$ metres	**M1** **A1**	Correct method to find minimum distance, eg $1500\sin(27.7)$ Correct minimum distance
			
		6	

Q	Answer	Marks	Comments
11 ALT 1	$\boldsymbol{v}_{SF} = \begin{bmatrix} 7\cos 40° \\ 7\sin 40° \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \end{bmatrix}$	**M1**	Correct method to find relative velocity
	$\boldsymbol{v}_{SF} = \begin{bmatrix} 7\cos 40° - 3 \\ 7\sin 40° \end{bmatrix}$	**A1**	Correct relative velocity
	$\tan\theta = \dfrac{7\sin 40°}{7\cos 40° - 3}$	**M1**	Correct method to find unknown angle
	$\theta = 62.3°$	**A1**	Correct angle. **PI** by sight of 27.7°
	Minimum Distance $= 1500\sin(90 - 62.3)$	**M1**	Correct method to find minimum distance
	$= 697$ metres	**A1**	Correct minimum distance
11 ALT 2	$\boldsymbol{r}_{SF} = \begin{bmatrix} 7t\cos 40° \\ 1500 - 7t\sin 40° \end{bmatrix} - \begin{bmatrix} 3t \\ 0 \end{bmatrix}$	**M1**	Correct method to find relative position vector
	$\boldsymbol{r}_{SF} = \begin{bmatrix} t(7\cos 40° - 3) \\ 1500 - 7t\sin 40° \end{bmatrix}$	**A1**	Correct relative position vector
	$t^2(7\cos 40° - 3)^2 + (1500 - 7t\sin 40°)^2$ $= t^2\left((7\cos 40° - 3)^2 + 49\sin^2 40°\right) - 21000t\sin 40° + 1500^2$	**M1**	Correct method to find unknown time
	$t_{\min} = \dfrac{21000\sin 40°}{2\left((7\cos 40° - 3)^2 + 49\sin^2 40°\right)} = 261.3...$	**A1**	Correct time. **AWRT** 261
	Minimum Distance $= \sqrt{261.3...^2(7\cos 40° - 3)^2 + (1500 - 7\times 261.3...\sin 40°)^2}$	**M1**	Correct method to find minimum distance
	$= 697$ metres	**A1**	Correct minimum distance

	Question 11 Total	**6**	