OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Tuesday 19 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The means and variances of the independent continuous random variables X_1, X_2 and X_3 are given in the table below where f, g and h are constants.

	Mean	**Variance**
X_1	f	g
X_2	h	g
X_3	g	2

1 (a) Find in terms of f, g and h

1 (a) (i) $\mathrm{E}(3X_1 + 2X_2 - X_3)$

[1 mark]

Answer _____

1 (a) (ii) $\mathrm{Var}(4X_1 - 3X_2)$

[1 mark]

Answer _____

Do not write outside the box

1 (b) Given that

$$\mathrm{E}(3X_2) = \mathrm{E}(X_1)$$

$$\mathrm{E}\left(\sum_{n=1}^{3} X_n\right) = 7.1$$

$$\mathrm{Var}\left(\sum_{n=1}^{3} X_n\right) = 9$$

find the value of f, the value of g and the value of h

[4 marks]

$f =$ _____ $g =$ _____ $h =$ _____

6

Turn over ►

Do not write outside the box

2 Speedy Delivery Services claim to have a 90% 'at least satisfactory' rating.

They hire a new delivery driver called Steve.

After his first 1000 deliveries, a random sample of 25 of his deliveries is selected. Of these 25 deliveries, 24 are rated as 'at least satisfactory'.

It is assumed that 90% of all deliveries of Speedy Delivery Services achieve a 'at least satisfactory' rating.

2 (a) Test at the 10% level of significance whether Steve is achieving a 'at least satisfactory' rating on more than 90% of his deliveries.

[7 marks]

Do not write outside the box

2 (b) (i) Find the probability of making a Type I error in this test.

[1 mark]

Answer _____

2 (b) (ii) Describe, in the context of **part (a)**, a Type I error.

[1 mark]

9

Turn over for the next question

Do not write outside the box

3 The continuous random variable T has a cumulative distribution function $\mathrm{F}(t)$ defined by

$$\mathrm{F}(t) = \begin{cases} 0 & t < 4 \\ kt^2 - \frac{1}{3} & 4 \le t \le 8 \\ 1 & t > 8 \end{cases}$$

where k is a constant.

3 (a) Find the value of k

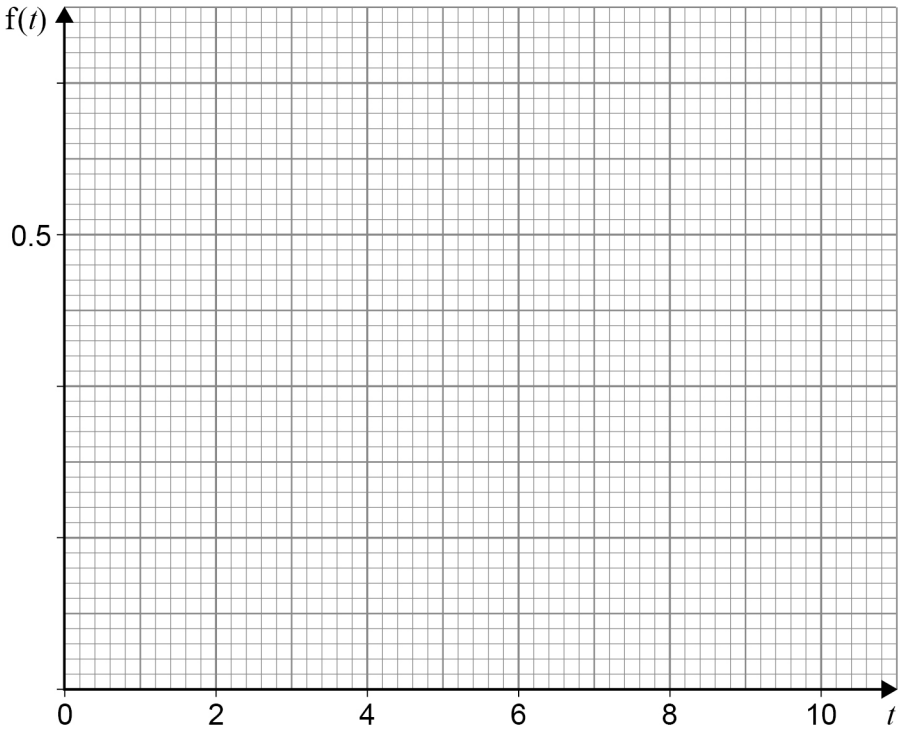
[2 marks]

Answer _____

3 (b) The probability density function of T is $\mathrm{f}(t)$

Draw the graph of $\mathrm{f}(t)$ for $0 \le t \le 10$

[3 marks]



5

Do not write outside the box

4 A weed treatment prevents weeds growing for a mean of 70 days in areas to which it is applied.

A new chemical is added to the treatment. The new treatment is applied to a random sample of 100 different areas and the number of days, x, for which the new treatment prevents weeds growing is recorded. The results are summarised below.

$$\sum x = 7060 \qquad \text{and} \qquad \sum x^2 = 499\,000$$

Test whether the mean number of days for which the weed treatment prevents weeds growing has increased from 70 days, using the 1% level of significance.

[10 marks]

10

Do not write outside the box

5 (a) For a Poisson distribution with $\lambda = 4$ it is given that

$$\mathrm{P}(X = 7) = k\,\mathrm{P}(X = 5)$$

Find the exact value of k

[3 marks]

Answer _____

5 (b) When active, a Venus flytrap plant captures flies.

The leaves of different plants are either red or green.

The number of flies captured per day by a **red** plant is modelled by a Poisson distribution with mean 2

5 (b) (i) Find the probability of a red plant capturing less than 3 flies on 1 day.

Give your answer to four significant figures.

[2 marks]

Answer _____

5 (b) (ii) Find the probability of a red plant capturing more than 8 and fewer than 17 flies over 7 days.

Give your answer to four significant figures.

[3 marks]

Answer _____

Do not write outside the box

5 (c) The number of flies captured per day by a **green** plant is also modelled by a Poisson distribution, but with mean 5

Dave owns one red plant and one green plant. He records the total number of flies captured by his two plants each day for 64 days.

The data recorded by Dave is shown below.

Number of flies captured	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	3	3	6	6	7	9	9	8	7	3	2

5 (c) (i) Use this data to find unbiased estimates of the mean and variance of the distribution of the total number of flies captured by his two plants each day.

Give your answers to three significant figures.

[3 marks]

Mean _____ Variance _____

5 (c) (ii) Dave claims that the number of flies captured by the two plants are independent of each other.

Using your answer in **part (c)(i)**, make **two** statements that support Dave's claim.

[2 marks]

Statement 1 _____

Statement 2 _____

13

Turn over ►

Do not write outside the box

6 A vehicle breakdown recovery service records the time taken, T minutes, between telephone calls requesting vehicle recovery.

It is assumed that T has an exponential distribution with $\lambda = 0.2$

6 (a) Find the variance for the time between telephone calls.

[1 mark]

Answer _____

6 (b) Katherine answers the telephone calls requesting vehicle recovery.
She begins work at 8.00 am

Find the probability that Katherine receives a telephone call before 8.15 am

Give your answer to three significant figures.

[2 marks]

Answer _____

Do not write outside the box

6 (c) Katherine does not receive a telephone call before 8.20 am

Find the probability that she does not receive a telephone call before 8.30 am

Give your answer to three significant figures.

[3 marks]

Answer _____

6 (d) Find the value of t for which $\mathrm{P}(T > t) = 0.6$

Give your answer to three significant figures.

[2 marks]

Answer _____

8

Turn over ►

Do not write outside the box

7 The continuous random variable X has a probability density function $\mathrm{f}(x)$ defined by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{60}\left(10x + x^3\right) & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the exact value of $\mathrm{E}\left(\frac{1}{X}\right)$

[3 marks]

Answer _____

3

Do not write outside the box

8 A machine produces metal spheres whose masses are normally distributed with mean 200 grams.

Following a power cut the machine is reset. A random sample of 10 spheres is taken and found to have a mean mass of 200.3 grams and a standard deviation of 0.55 grams.

Test whether or not the mean mass has changed, using the 5% level of significance.

[7 marks]

7

Turn over ►

Do not write outside the box

9 (a) The continuous random variable $X \sim \mathrm{N}\left(\mu, 15^2\right)$ is such that

$$\mathrm{P}(X < 200) = \mathrm{P}(X > 170)$$

9 (a) (i) Explain why $\mu = 185$

[2 marks]

9 (a) (ii) Find $\mathrm{P}(\mu - 30 < X < \mu + 30)$

Give your answer to four significant figures.

[2 marks]

Answer _____

Do not write outside the box

9 (b) The continuous random variable $Q \sim \mathrm{N}\left(200, \sigma^2\right)$ is such that

$$\mathrm{P}(Q < 190) = 0.45$$

9 (b) (i) Find σ

Give your answer to three significant figures.

[3 marks]

Answer _____

9 (b) (ii) State the value of $\mathrm{P}(Q \neq 200)$

[1 mark]

Answer _____

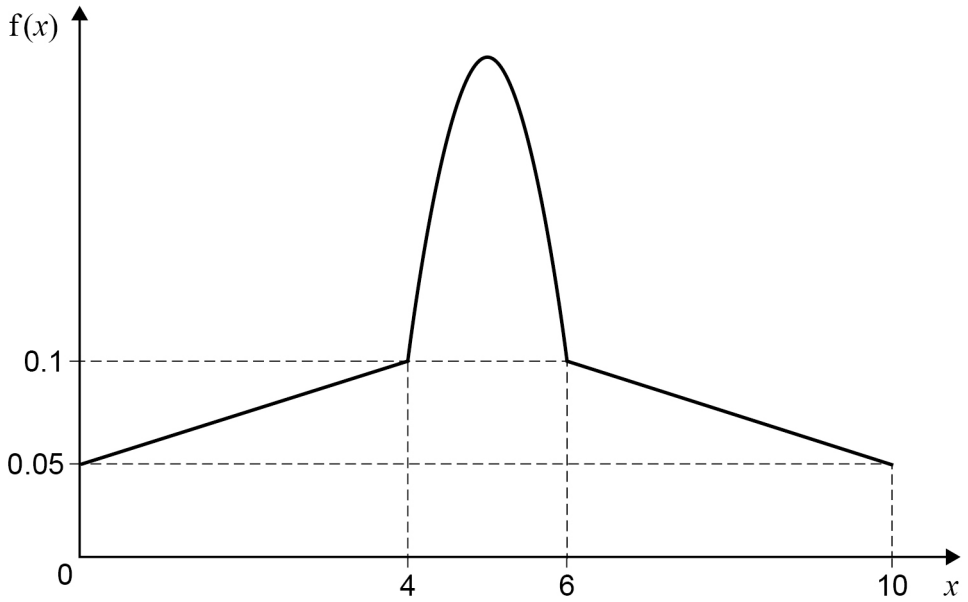
8

Turn over ►

Do not write outside the box

10 The continuous random variable X is defined by the probability density function $\mathrm{f}(x)$ and the graph shown below where a, q, n and b are constants.

$$\mathrm{f}(x) = \begin{cases} ax + 0.05 & 0 \le x \le 4 \\ q - n(x-5)^2 & 4 < x \le 6 \\ bx + 0.175 & 6 < x \le 10 \\ 0 & \text{otherwise} \end{cases}$$



The graph is symmetrical

10 (a) State the median of X

[1 mark]

Answer _____

10 (b) Find the value of a and the value of b

[3 marks]

Do not write outside the box

$a =$ _____ $b =$ _____

10 (c) Find the value of n and the value of q

[5 marks]

$n =$ _____ $q =$ _____

10 (d) Find $\mathrm{P}(0 < X < 4.5)$

[2 marks]

Answer _____

11

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED