OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL AS
MATHEMATICS

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Wednesday 6 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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11	
12	
TOTAL	

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Section A

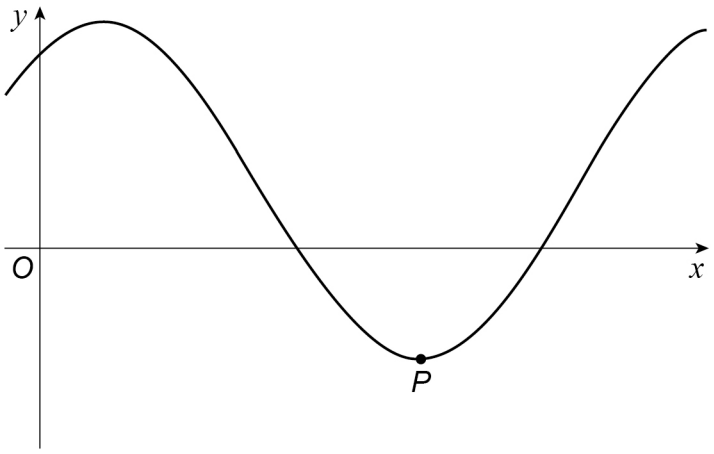
Pure Mathematics

Answer **all** questions in the spaces provided.

1 The diagram shows part of the curve with equation $y = \mathrm{f}(x)$ such that

$$\mathrm{f}(x) = \cos(x - a) + b$$

where a and b are constants such that $0^\circ < a < 90^\circ$ and $0 < b < 1$



1 (a) State the period of $\mathrm{f}(x)$

[1 mark]

Answer _____

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1 (b) The curve has a minimum at the point *P*, as shown in the diagram.

Find in terms of a and b the coordinates of *P*

[2 marks]

Answer _____

1 (c) The curve has rotational symmetry about a point *Q*

Find in terms of a or b the smallest positive value for the x-coordinate of *Q*

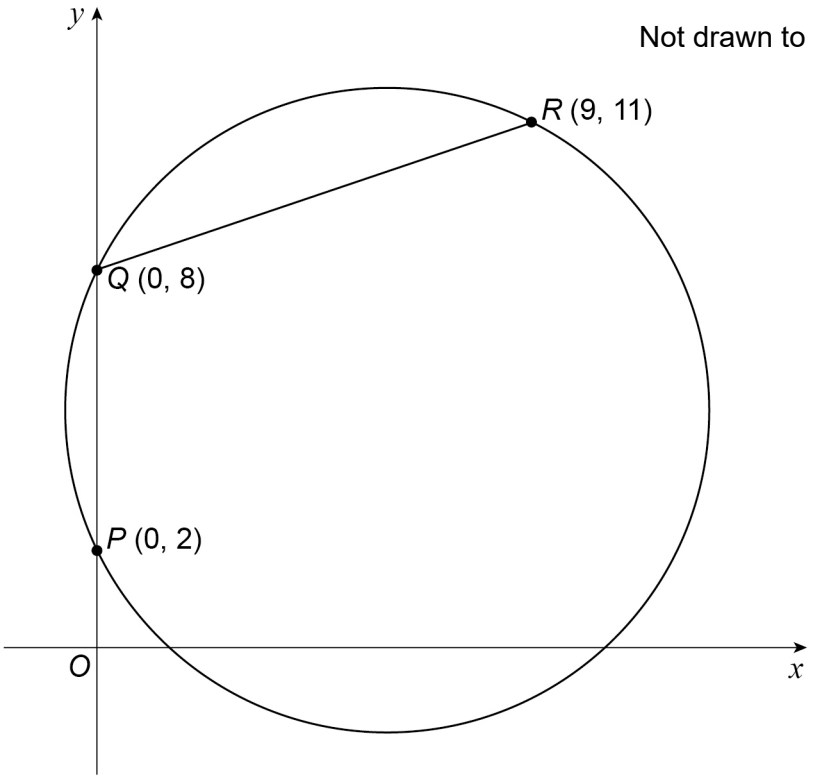
[1 mark]

Answer _____

4

Turn over for the next question

2 A circle passes through the points $P(0, 2)$, $Q(0, 8)$ and $R(9, 11)$ as shown in the diagram.



2 (a) The perpendicular bisector of the chord QR has equation $y = 23 - 3x$

Find the coordinates of the centre of the circle.

[2 marks]

Answer _____

Do not write outside the box

2 (b) Find the equation of the circle, giving your answer in the form

$$(x-d)^2+(y-e)^2=k$$

where d, e and k are positive integers.

[3 marks]

Answer _____

2 (c) Find the equation of the tangent to the circle at *R*

Give your answer in the form $x+by=c$ where b and c are integers.

[3 marks]

Answer _____

8

Turn over ►

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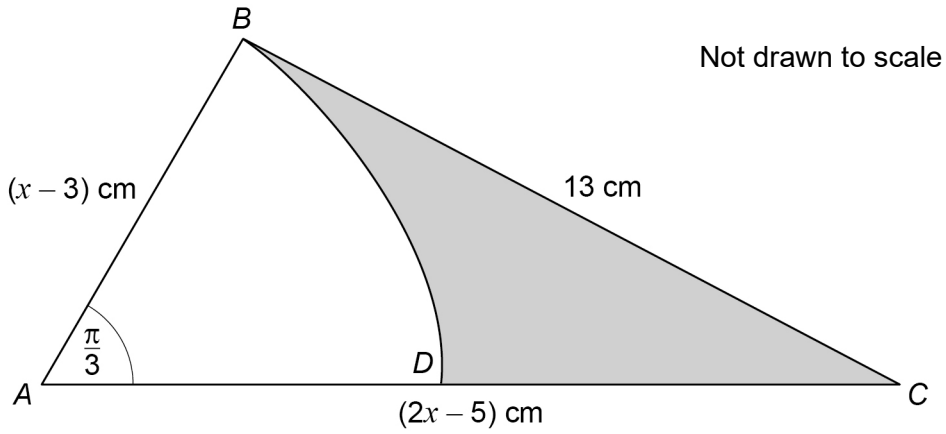
3 The diagram shows a triangle ABC and an arc BD of a circle with centre A



The lengths $AB = (x-3)$ cm, $AC = (2x-5)$ cm and $BC = 13$ cm

The angle $BAC = \frac{\pi}{3}$ radians.

3 (a) Show that x satisfies the equation

$$x^2 - 5x - 50 = 0$$

[3 marks]

Do not write outside the box

3 (b) Hence find the perimeter of the shaded region *BCD*

Give your answer to three significant figures.

[5 marks]

Answer _____

8

Turn over for the next question

Do not write outside the box

4 Find all of the possible values of the constant k for which

$$x^2 + y^2 + 10x - 14y + k^2 + 10 = 0$$

is the equation of a circle.

[4 marks]

Answer _____

4

Do not write outside the box

5 It is given that $y = \log_{16} x$

5 (a) Find an expression in terms of y for $\log_{16} x^3$

[1 mark]

Answer _____

5 (b) Find an expression in terms of y for $\log_2 x$

[3 marks]

Answer _____

5 (c) Using your answers to **parts (a)** and **(b)**, find the value of x for which

$$4\log_{16} x^3 + 5\log_2 x - \log_3 81 = 60$$

[3 marks]

Answer _____

7

Do not write outside the box

6 (a) Given that

$$6\tan x \sin x = 5\left(1+\tan^2 x\right)\cos^2 x$$

where $-90^\circ < x < 90^\circ$, show that

$$6\cos^2 x + 5\cos x - 6 = 0$$

[4 marks]

Do not write outside the box

6 (b) Explain why the only real solutions of the equation

$$6\tan x \sin x = 5\left(1+\tan^2 x\right)\cos^2 x$$

satisfy $\cos x = \frac{2}{3}$

[2 marks]

6 (c) Hence solve the equation

$$6\tan\left(x+35^\circ\right)\sin\left(x+35^\circ\right) = 5\left(1+\tan^2\left(x+35^\circ\right)\right)\cos^2\left(x+35^\circ\right)$$

in the interval $-90^\circ < x < 90^\circ$, giving your answers to the nearest 0.1°

[3 marks]

Answer _____

9

Turn over ►

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Section B

Statistics

Answer **all** questions in the spaces provided.

7 The score in a game in a television show can be represented by the discrete random variable X with mean 200 and variance 25.2

7 (a) The prize money, in dollars, for the game is calculated using the following formula.

$$\text{prize money} = 2 \times \text{score} + 100$$

Sophie plays the game.

Find the expected value of her prize money.

[2 marks]

Answer _____

Do not write outside the box

7 (b) The score in a second game can be represented by the discrete random variable Y with mean 100 and standard deviation 4.7

The random variables X and Y are independent.

7 (b) (i) Find $\mathrm{E}(X-Y)$

[1 mark]

Answer _____

7 (b) (ii) Find $\mathrm{Var}(X-Y)$

[2 marks]

Answer _____

5

Turn over ►

Do not write outside the box

8 (a) The events A and B have probabilities $P(A) = 0.45$ and $P(B) = 0.32$

8 (a) (i) Find $P(A \cup B)$ if A and B are mutually exclusive.

[1 mark]

Answer _____

8 (a) (ii) Find $P(A \cup B)$ if A and B are independent.

[2 marks]

Answer _____

8 (b) Chan and Dalila are taking part in a race. The first 10 runners who finish the race receive a medal.

The event that Chan receives a medal is represented by C

The event that Dalila receives a medal is represented by D

It is given that $P(C) = 0.65$, $P(D) = 0.18$ and $P(C \cup D) = 0.74$

Find the probability that Dalila receives a medal given that Chan receives a medal.

[4 marks]

Do not write outside the box

Answer _____

7

Turn over for the next question

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9 The random variable $X \sim \text{B}(n, p)$ has mean 22.5 and variance 12.375

9 (a) Show that $p = 0.45$ and find the value of n

[4 marks]

Answer _____

Do not write outside the box

9 (b) Find $P(X = 24)$, giving your answer to four decimal places.

[2 marks]

Answer _____

9 (c) Find $P(X > 19)$, giving your answer to four decimal places.

[2 marks]

Answer _____

8

Turn over for the next section

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Section C

Mechanics

Answer **all** questions in the spaces provided.

10 **The acceleration due to gravity, g, should be taken as 9.8 m s^{-2}**

A ball is projected vertically upwards from ground level at a speed of 32 m s^{-1}

The ball moves freely under gravity.

10 (a) Find the greatest height reached by the ball.

[2 marks]

Answer _____

10 (b) As the ball descends it is caught 2 metres above ground level.

10 (b) (i) Find the **total** distance travelled by the ball.

[1 mark]

Answer _____

Do not write outside the box

10 (b) (ii) Find the time taken between the ball being projected and being caught.

[3 marks]

Answer _____

6

Turn over for the next question

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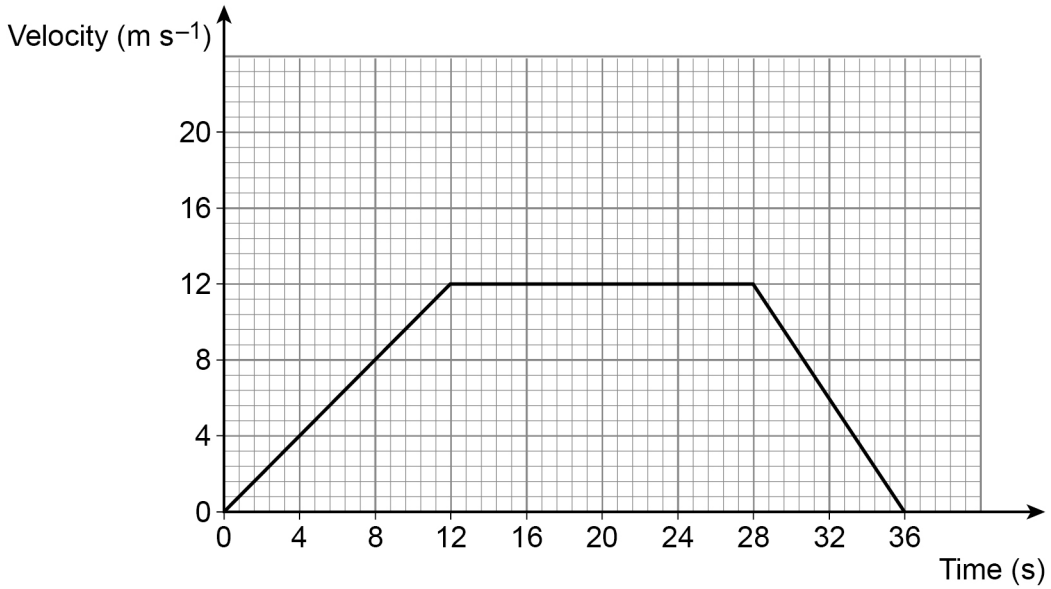
11 A car travels along a straight road for 36 seconds.

The motion of the car is shown in the velocity–time graph below.



11 (a) Find the average speed of the car over this 36-second period.

[3 marks]

Answer _____

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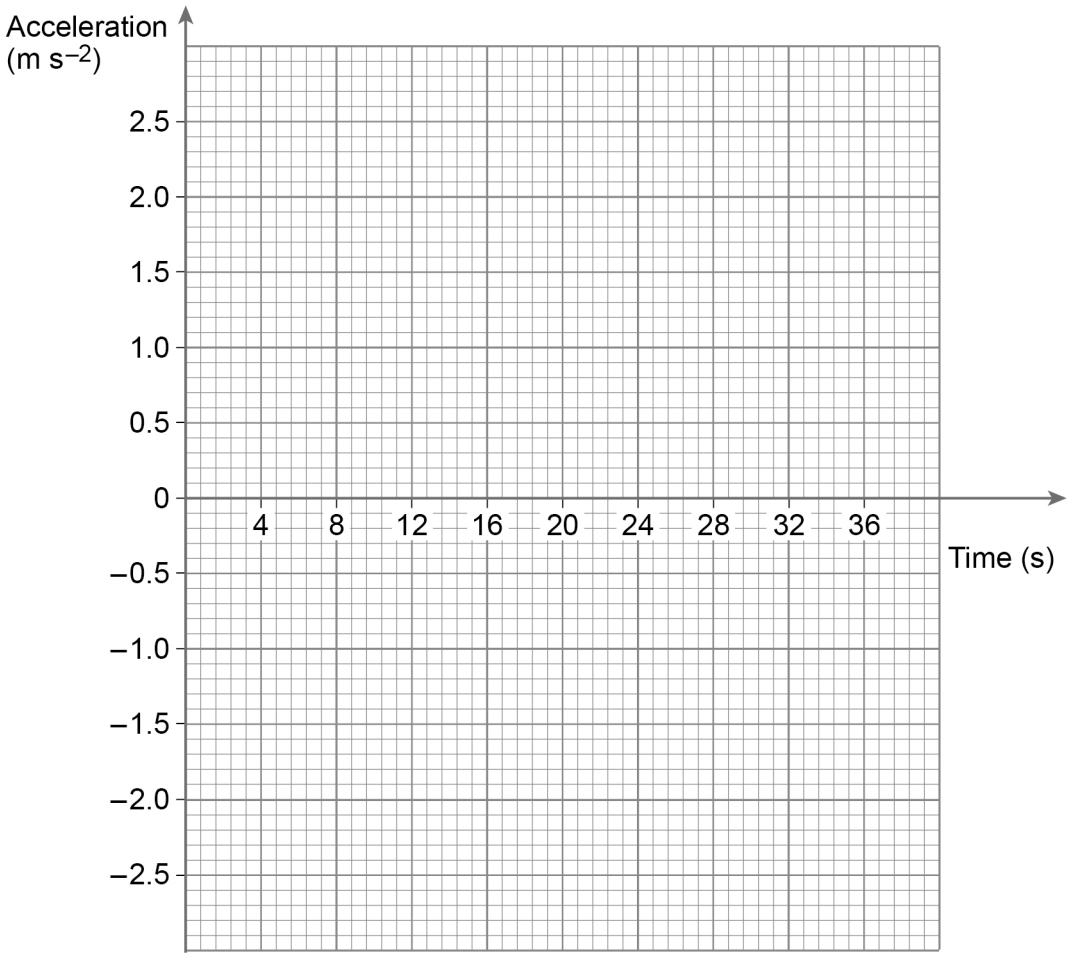
11 (b) On the axes below draw an acceleration–time graph for the motion of the car over this 36-second period.

[2 marks]



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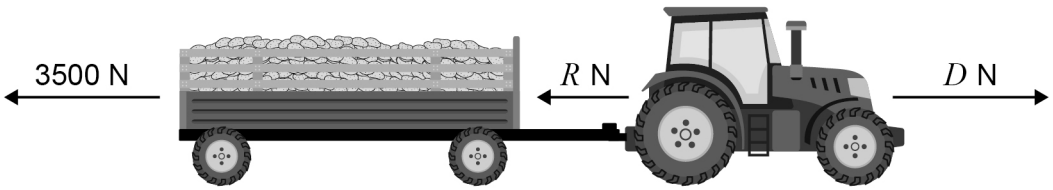
12 The diagram below shows a tractor of mass 2000 kg connected to a trailer of mass 10 000 kg by a light horizontal rod.

The tractor and trailer are moving in a straight line on horizontal ground.

A constant horizontal driving force of magnitude D newtons acts on the tractor.

A constant horizontal resistance force of magnitude R newtons acts on the tractor.

A constant horizontal resistance force of magnitude 3500 N acts on the trailer.



12 (a) Initially $D = 5000$ and the tractor and trailer are moving at a constant speed.

Explain why $R = 1500$

[2 marks]

12 (b) The driving force D newtons is reduced.

The magnitude of the force in the rod is 500 N

The resistance forces acting on the tractor and the trailer remain the same.

Find the **two** possible values of D

[5 marks]

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Answer _____ and _____

12 (c) The trailer is released from the tractor.

At this time the trailer has a momentum of 20 000 N s

The magnitude of the horizontal resistance force acting on the trailer remains at 3500 N

Find the time taken for the trailer to come to rest.

[2 marks]

Answer _____

9

END OF QUESTIONS

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