OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 18 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 A company believes that use of its revision app changes the chance of a candidate passing a particular mathematics examination.

The results of 200 candidates taking the mathematics examination were collected.

Some of these candidates used the app and others did not.

The table below shows the observed frequencies O_i from the collected data and the expected frequencies E_i necessary for a χ^2 test.

	Pass		**Fail**	
	O_i	E_i	O_i	E_i
App used	72	66	28	34
App **not** used	60	66	40	34

The company conducts a χ^2 test using a 10% level of significance.

1 (a) State the hypotheses for this test.

[1 mark]

1 (b) Show that the value of the test statistic χ^2 is 2.696 correct to four significant figures.

[2 marks]

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1 (c) Determine the conclusion that the company should reach based on the results of the test.

[4 marks]

7

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2 The random variable X has a normal distribution with population mean μ

A hypothesis test is conducted using a large sample such that $\overline{X} \sim \mathrm{N}(\mu, 0.0016)$

The hypotheses are

H_0: $\mu = 0.1$

H_1: $\mu > 0.1$

The critical value for $\overline{X}$ in the test is set at 0.16

The population mean is 0.1468

Determine the power of the test, giving your answer to three significant figures.

[3 marks]

Answer _____

3

Turn over ►

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3 A die is described as fair if the probability of each score from a roll of the die is the same.

A six-sided die is rolled 300 times and the score recorded for each roll.

The results are shown in the table below.

Score on die	1	2	3	4	5	6
Frequency	50	43	38	63	61	45

3 (a) Investigate whether or not the die is fair at the 1% level of significance.

[8 marks]

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3 (b) Carl used the same data but investigated at the 10% level of significance.

He concluded correctly that the die might **not** be fair.

Using the critical value for Carl's test, explain how this is **consistent** with the result in **part (a)**.

[2 marks]

10

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4 The random variable X has a distribution with unknown mean μ and unknown variance σ^2

A random sample of size n, denoted by $X_1, X_2, \ldots, X_n$ is used to determine two statistics

$$T = \sum_{k=1}^{n} X_k \qquad \text{and} \qquad V = \left(\frac{1}{n} \sum_{k=1}^{n} X_k^2 \right) - \frac{T^2}{n^2}$$

4 (a) Explain why T and V are both statistics.

[2 marks]

4 (b) Show that T is not an unbiased estimator.

[2 marks]

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4 (c) (i) Show that $\mathrm{E}\left(X_k^2\right) = \sigma^2 + \mu^2$

[1 mark]

4 (c) (ii) Show that $\mathrm{E}\left(T^2\right) = n\sigma^2 + n^2\mu^2$

[2 marks]

4 (d) Use your results in **part (b)** and **part (c)** to show that $\frac{nV}{n-1}$ is an unbiased estimator of σ^2

[3 marks]

10

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5 In a study of the amount of sugar in 330 ml cans of a soft drink from a manufacturer, 10 cans are sampled.

The mass of sugar in each can, X grams, is measured.

Assume that the mass of sugar in each can is normally distributed.

The summary statistics from the study are

$$\sum x = 328.2 \qquad \text{and} \qquad \sum x^2 = 10843.9$$

5 (a) Construct a 98% confidence interval for the mean mass of sugar per can.

Give your values to two decimal places.

[6 marks]

Answer _____

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5 (b) The manufacturer must print on the can a value for the population mean amount of sugar per can.

It is required that this value be found from a sample size greater than 100 cans that ensures the width of the 98% confidence interval is less than 1 gram.

The population standard deviation is **known** to be less than 3 grams.

Find the **minimum** number of cans in the sample that the manufacturer needs to use.

[4 marks]

Answer _____

10

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6 A particular model of a mobile phone has a battery lifetime X hours with mean μ and variance σ^2

6 (a) A random sample of size n of the phones is taken and the mean $\bar{X}$ is calculated.

6 (a) (i) Write down expressions for the mean and variance of $\bar{X}$ in terms of μ, σ and n

[2 marks]

Mean _____ Variance _____

6 (a) (ii) Hence show that $\bar{X}$ is an unbiased and consistent estimator of μ

[2 marks]

6 (b) Shop *A* and shop *B* both sell this model of mobile phone.

Shop *A* surveys a random sample of 40 customers and calculates the mean $\bar{X}_A$ as an estimate of μ

Shop *B* surveys a different random sample of 60 customers and calculates the mean $\bar{X}_B$ as an estimate of μ

Show that the relative efficiency of $\bar{X}_B$ with respect to $\bar{X}_A$ is 1.5

[2 marks]

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6 (c) The owners of Shops *A* and *B* combine their results in order to improve their estimate of μ

They use a new unbiased estimator T of μ where

$$T = p\bar{X}_A + (1-p)\bar{X}_B \qquad 0 < p < 1$$

Find the value of p that minimises the variance of T

[4 marks]

Answer _____

10

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7 The number of platelets in the blood of males and females is being investigated.

It is assumed that the number of platelets is normally distributed with the same variance for both male and female populations.

The number of platelets, measured in thousands per microlitre, in the blood of 11 males and 9 females was recorded as

Male	261	277	201	219	349	300	321	219	277	280	310
Female	211	190	273	301	220	295	192	231	183		

7 (a) Using the samples, calculate a pooled estimate of the population variance.

Give your answer to four significant figures.

[3 marks]

Answer _____

7 (b) Investigate the hypothesis that the mean number of platelets is greater for males than for females at the 5% level of significance.

[7 marks]

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7 (c) Show that the assumption of same variances for male and female populations is supported at the 5% level of significance.

[6 marks]

16

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8 The discrete random variable X_k has a geometric distribution with parameter p

8 (a) Show that the moment generating function of X_k is given by

$$M_{X_k}(t) = \frac{p}{e^{-t} - (1-p)}$$

[4 marks]

8 (b) Use differentiation to find the mean of X_k

[2 marks]

Answer _____

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8 (c) A fair six-sided die is rolled until two sixes are obtained and the total number of rolls is recorded.

The total number of rolls is calculated as the number of rolls to obtain the first six **plus** the number of further rolls to obtain the second six.

For example, if the second six appears on the 20th roll, then the total number of rolls is 20

8 (c) (i) Show that the moment generating function for the total number of rolls needed is

$$\frac{1}{\left(6e^{-t}-5\right)^2}$$

[2 marks]

8 (c) (ii) Hence show that the mean total number of rolls needed is 12

[2 marks]

Question 8 continues on the next page

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8 (d) The same die is rolled until n sixes are obtained.

Use a moment generating function to find in terms of n the variance of the number of rolls required.

[4 marks]

14

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