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Centre number	Candidate number
Surname	
Forename(s)	
Candidate signature	I declare this is my own work.

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 18 January 2021 07:00 GMT

Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.



For Exam	iner's Use
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



Answer **all** questions in the spaces provided.

1 A company believes that use of its revision app changes the chance of a candidate passing a particular mathematics examination.

The results of 200 candidates taking the mathematics examination were collected.

Some of these candidates used the app and others did not.

The table below shows the observed frequencies O_i from the collected data and the expected frequencies E_i necessary for a χ^2 test.

	Ра	SS	Fa	ail
	O _i	Ei	O _i	E _i
App used	72	66	28	34
App not used	60	66	40	34

The company conducts a χ^2 test using a 10% level of significance.

1 (a) State the hypotheses for this test.

[1 mark]

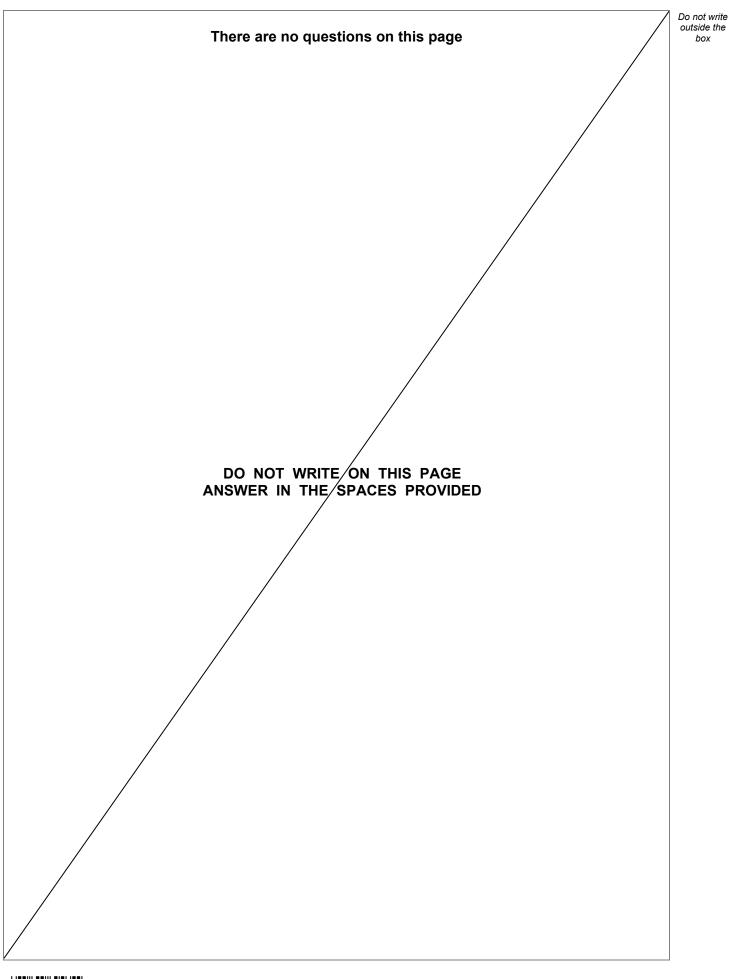
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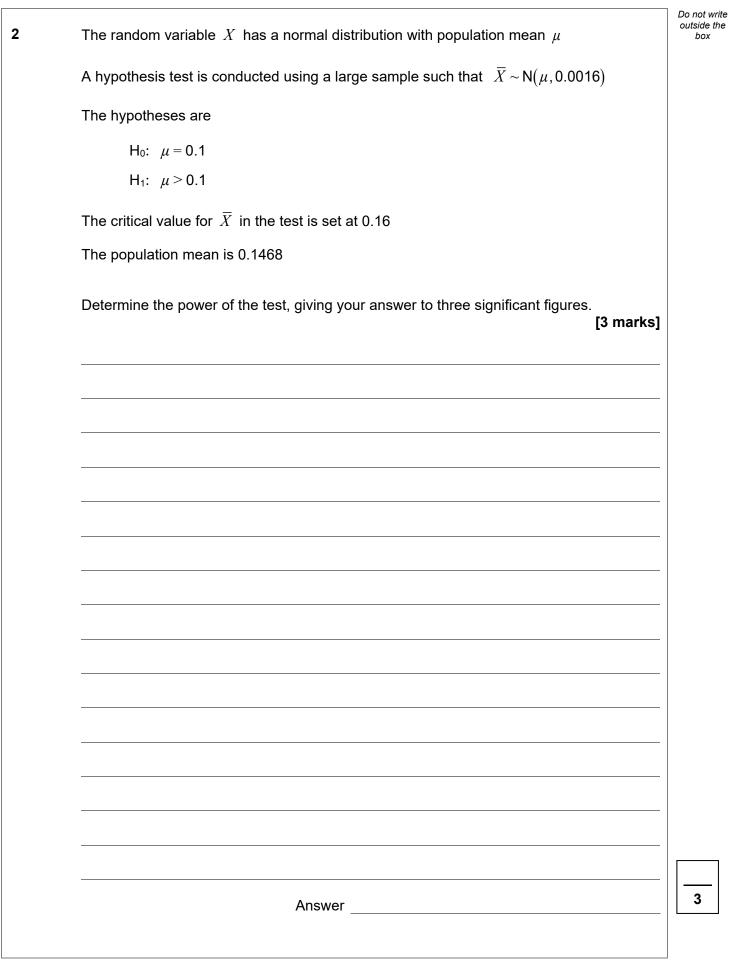
box

1 (b) Show that the value of the test statistic χ^2 is 2.696 correct to four significant figures. [2 marks]

1	(c)	Determine the conclusion that the company should reach based on the results of the	Do not write outside the box
		test. [4 marks	5]
			_
			_
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			- <u> </u>
		Turn over for the next question	
		Turn over	 ▶









Turn over ►

Do not write outside the box A die is described as fair if the probability of each score from a roll of the die is the same. 3 A six-sided die is rolled 300 times and the score recorded for each roll. The results are shown in the table below. Score on die 4 1 2 3 5 6 50 43 Frequency 38 63 61 45 Investigate whether or not the die is fair at the 1% level of significance. 3 (a) [8 marks]



Do not write outside the box

3	(b)	Carl used the same data but investigated at the 10% level of significance.
•	(~)	ean abea and eans data bat inteeligated at the rever even of eighnedited.

He concluded correctly that the die might **not** be fair.

Using the critical value for Carl's test, explain how this is **consistent** with the result in **part (a)**.

[2 marks]

10



Turn over ►

4	The random variable X has a distribution with unknown mean μ and unknown variance σ^2 A random sample of size <i>n</i> , denoted by X_1, X_2, \dots, X_n is used to determine two statistics $T = \sum_{k=1}^n X_k$ and $V = \left(\frac{1}{n}\sum_{k=1}^n X_k^2\right) - \frac{T^2}{n^2}$	Do not write outside the box
4 (a)	Explain why T and V are both statistics. [2 marks]	
4 (b)	Show that <i>T</i> is not an unbiased estimator. [2 marks]	
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4 (c) (l) Show that
$$E(x_{k}^{2}) = \sigma^{2} + \mu^{2}$$
[1 mark]

4 (c) (l) Show that $E(T^{2}) - n\sigma^{2} + n^{2}\mu^{2}$
[2 marks]

4 (c) (l) Show that $E(T^{2}) - n\sigma^{2} + n^{2}\mu^{2}$
[2 marks]

4 (d) Use your results in part (b) and part (c) to show that $\frac{nV}{n-1}$ is an unbiased estimator of σ^{2}
[3 marks]

10



E In a study of the amount of ourser in 220 ml ages of a soft drink from a manufacturer outsid			
Assume that the mass of sugar in each can is normally distributed. The summary statistics from the study are $\sum x = 328.2 \text{and} \sum x^2 = 10843.9$ 5 (a) Construct a 98% confidence interval for the mean mass of sugar per can. Give your values to two decimal places. [6 marks]	5	In a study of the amount of sugar in 330 ml cans of a soft drink from a manufacturer, 10 cans are sampled.	Do not wi outside ti box
The summary statistics from the study are $\sum x = 328.2$ and $\sum x^2 = 10843.9$ 5 (a) Construct a 98% confidence interval for the mean mass of sugar per can. Give your values to two decimal places. [6 marks]		The mass of sugar in each can, X grams, is measured.	
$\sum x = 328.2$ and $\sum x^2 = 10843.9$ 5 (a) Construct a 98% confidence interval for the mean mass of sugar per can. Give your values to two decimal places. [6 marks]		Assume that the mass of sugar in each can is normally distributed.	
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Give your values to two decimal places. [6 marks]		$\sum x = 328.2$ and $\sum x^2 = 10843.9$	
[6 marks]	5 (a)	Construct a 98% confidence interval for the mean mass of sugar per can.	
		· · · · · · · · · · · · · · · · · · ·	
Answer			
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			Do not write
5	(b)	The manufacturer must print on the can a value for the population mean amount of sugar per can.	Do not write outside the box
		It is required that this value be found from a sample size greater than 100 cans that ensures the width of the 98% confidence interval is less than 1 gram.	
		The population standard deviation is known to be less than 3 grams.	
		Find the minimum number of cans in the sample that the manufacturer needs to use. [4 marks]	
		Answer	10



6		A particular model of a mobile phone has a battery lifetime X hours with mean μ and variance σ^2	Do not wri outside th box
6	(a)	A random sample of size n of the phones is taken and the mean \overline{X} is calculated.	
6	(a) (i)	Write down expressions for the mean and variance of \overline{X} in terms of μ , σ and <i>n</i> [2 marks]	
		Mean Variance	
6	(a) (ii)	Hence show that \overline{X} is an unbiased and consistent estimator of μ [2 marks]	
6	(b)	Shop <i>A</i> and shop <i>B</i> both sell this model of mobile phone.	
		Shop A surveys a random sample of 40 customers and calculates the mean ${\bar X}_{\!\scriptscriptstyle A}$ as an estimate of μ	
		Shop <i>B</i> surveys a different random sample of 60 customers and calculates the mean $\bar{X}_{\rm B}$ as an estimate of μ	
		Show that the relative efficiency of \overline{X}_B with respect to \overline{X}_A is 1.5 [2 marks]	



6 (c)	The owners of Shops A and B combine their results in order to improve their estimate of μ	
	They use a new unbiased estimator T of μ where	
	$T = p \overline{X}_A + (1 - p) \overline{X}_B \qquad 0$	
	Find the value of p that minimises the variance of T	[4 marks]
	Answer	



Turn over ►

7	The numb	per of pl	atelets	in the	blood o	of male:	s and f	emales	is beir	ng inve	stigate	:d.	Do not outside box
	It is assur for both m						normal	ly distri	buted	with th	e same	e variance	
	The numb of 11 male						ands pe	er micro	olitre, ir	n the bl	ood		
	Male	261	277	201	219	349	300	321	219	277	280	310	
	Female	211	190	273	301	220	295	192	231	183			
7 (a)	Using the Give your	-			•		ate of t	he pop	ulation	varian	ce.	[3 marks]	
					Ans	wer							
7 (b)	Investigat						nber of	fplatele	ets is g	reater	for mal	les than	
	for female	es at the	e 5% le	vel of s	significa	ance.						[7 marks]	



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7 (c)	Show that the assumption of same variances for male and female populations is supported at the 5% level of significance. [6 marks]



Turn over ►

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8		The discrete random variable X_k has a geometric distribution with parameter p	Do not write outside the box
8	(a)	Show that the moment generating function of X_k is given by $M_{X_k}(t) = \frac{p}{e^{-t} - (1-p)}$	
		[4 marks]	
8	(b)	Use differentiation to find the mean of X_k [2 marks]	
		Answer	



Do not write outside the box A fair six-sided die is rolled until two sixes are obtained and the total number of rolls is 8 (c) recorded. The total number of rolls is calculated as the number of rolls to obtain the first six plus the number of further rolls to obtain the second six. For example, if the second six appears on the 20th roll, then the total number of rolls is 20 8 (c) (i) Show that the moment generating function for the total number of rolls needed is $\frac{1}{\left(6e^{-t}-5\right)^2}$ [2 marks] 8 (c) (ii) Hence show that the mean total number of rolls needed is 12 [2 marks] Question 8 continues on the next page

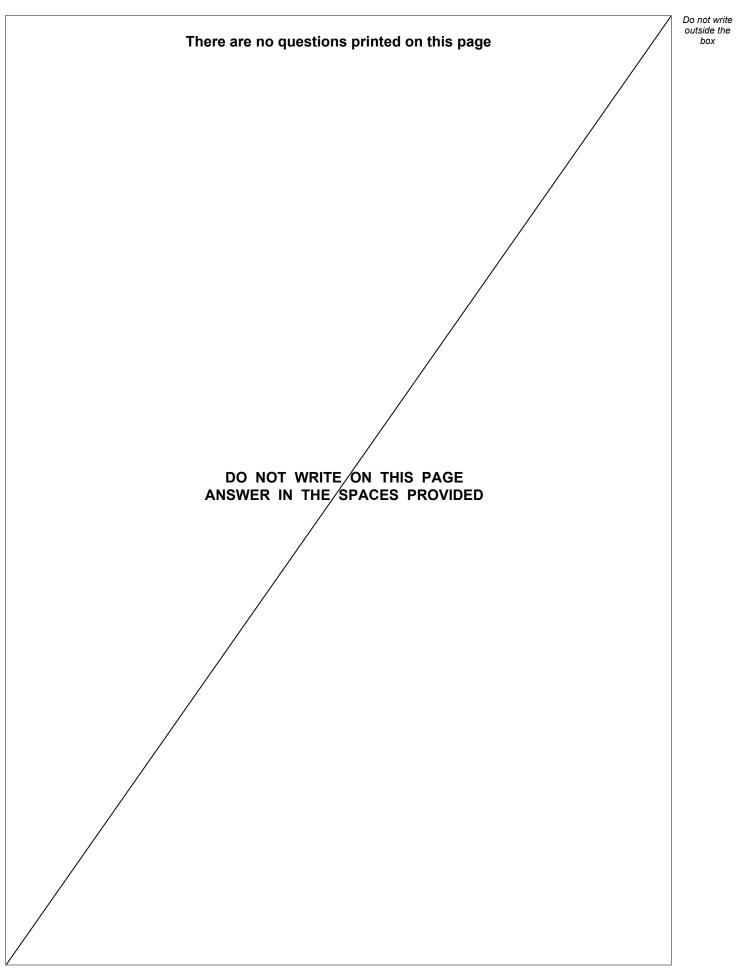




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8	(d)	The same die is rolled until n sixes are obtained.	
		Use a moment generating function to find in terms of n the variance of the	
		number of rolls required.	
			[4 marks]
		END OF QUESTIONS	

14

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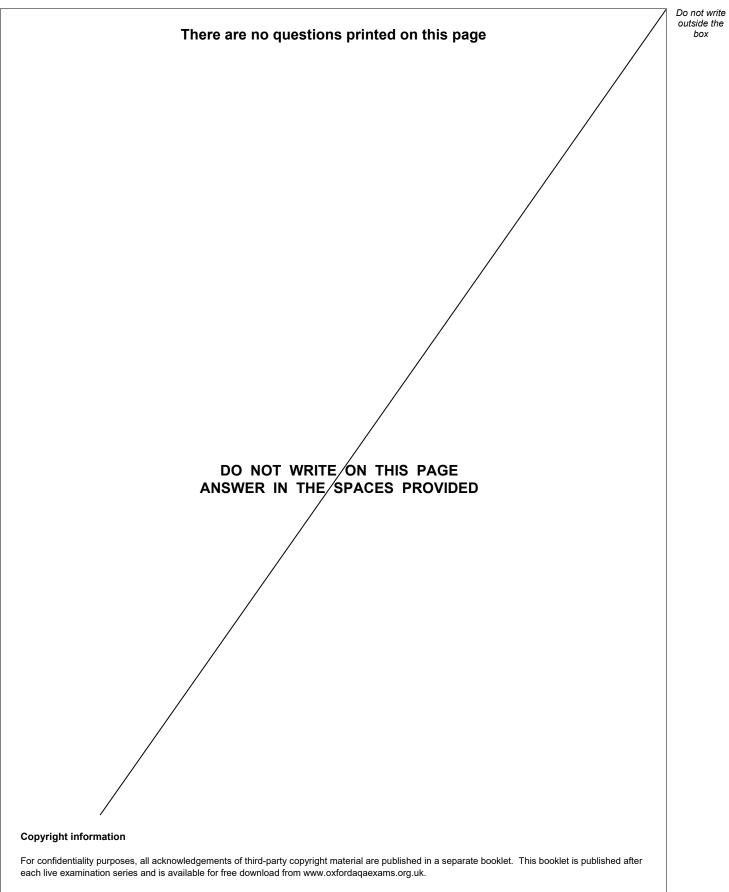


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