# OXFORD

## INTERNATIONAL AQA EXAMINATIONS

# INTERNATIONAL AS

# FURTHER MATHEMATICS

## (9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Wednesday 13 January 2021  07:00 GMT  Time allowed: 1 hour 30 minutes

## Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions
- Use black ink or black ball-point pen.  Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided.  Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book.  Write the question number against your answer(s).
- Do all rough work in this book.  Cross through any work you do not want to be marked.

## Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80.  There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

## Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| **TOTAL** | |

## Section A

## Pure Mathematics

Answer **all** questions in the spaces provided.

**1** The variables $X$ and $Y$ are related by the equation

$$Y = aX + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

It is given that $X = \dfrac{x^2}{y}$ and $Y = y^3$ where $x \geq 0$ and $y > 0$

The following pairs of values of $x$ and $y$ were obtained from an experiment.

| $x$ | 1 | 2 | 3 | 4 |
|-----|------|------|------|------|
| $y$ | 0.64 | 1.62 | 2.23 | 2.65 |

**1 (a)** Complete the table below to show the values of $X$ and $Y$ to one decimal place.

**[2 marks]**
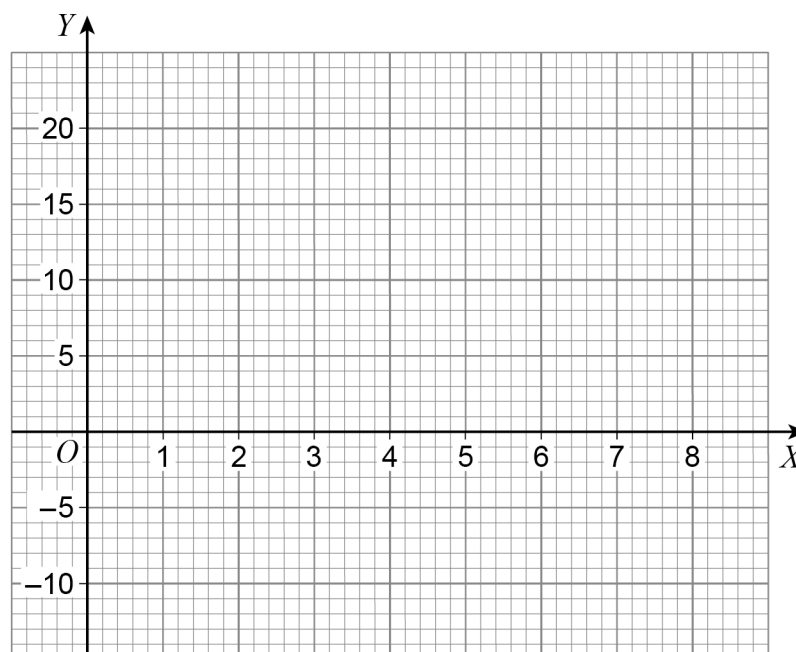
| $X$ | | | | |
|-----|--|--|--|--|
| $Y$ | | | | |

**1 (b)** On the grid below, plot $Y$ against $X$

Draw a line of best fit for the points you have plotted.

**[2 marks]**

**1 (c) (i)** Use your line of best fit to find estimates for $a$ and $b$ to one significant figure.

**[3 marks]**

$a =$ _____     $b =$ _____

**1 (c) (ii)** Using your values of $a$ and $b$ write down an equation relating $x$ and $y$

**[1 mark]**

Answer _____

**1 (d)** A student wants to carry out the experiment to obtain $y = 2$

Estimate the value of $x$ the student should use in the experiment, giving your answer to one decimal place.

**[2 marks]**

Answer _____

**10**

**Turn over ▶**

**2**  The matrix **M** is defined by $\mathbf{M} = \begin{bmatrix} m+1 & m-3 \\ -3 & m+2 \end{bmatrix}$ where $m$ is a constant.

The matrix **N** is defined by $\mathbf{N} = \begin{bmatrix} 2 & p \\ 3 & p \end{bmatrix}$ where $p$ is a constant.

**2 (a)**  Find the values of $m$ for which **M** is singular.

[3 marks]

Answer _____

**2 (b)**  In the case when $m = 2$ find the value for $p$ such that

$$\mathbf{MN} = \mathbf{NM}$$

[4 marks]

Answer

**2 (c)** In a different case $p = 2$

**2 (c) (i)** Calculate the determinant of **N**

[1 mark]

Answer

**2 (c) (ii)** Hence explain the effects of the transformation defined by **N** when applied to a unit square.

[2 marks]

10

**Turn over ▶**

**3** A curve passes through the point $(4, 2.3)$ and satisfies the differential equation

$$\frac{dy}{dx} = \frac{3x + y}{\sqrt{x + y^3}} \qquad \text{for} \quad x > 0 \quad \text{and} \quad y > 0$$

Use Euler's step-by-step method with a step length of $0.1$ to calculate an estimated value of $y$ when $x = 4.2$

Give your answer to four decimal places.

**[5 marks]**

Answer _____

5

**4** The function $f$ is defined by $f(x) = 4^x - 3x^3 - 5$

**4 (a)** Show that the equation $f(x) = 0$ has a root $\alpha$ that lies between 3 and 4

**[2 marks]**

**4 (b)** Starting from the interval $3 < x < 4$ use **interval bisection** to find the value of $\alpha$ to two significant figures.

Show clearly each step of your working.

**[4 marks]**

Answer

**Turn over ▶**

**5**      The matrix **T** is defined by $\mathbf{T} = \begin{bmatrix} 9k & 10k \\ -5k & 6 \end{bmatrix}$ where $k$ is a constant.

The point $(1, -1)$ is invariant under the transformation represented by **T**

**5 (a)**      Show that $k = -1$

[2 marks]

**5 (b) (i)**   Show that $y = -x$ is a line of invariant points under the transformation represented by **T**

[2 marks]

**5 (b) (ii)** Find the equations of the invariant lines.

[5 marks]

Answer

9

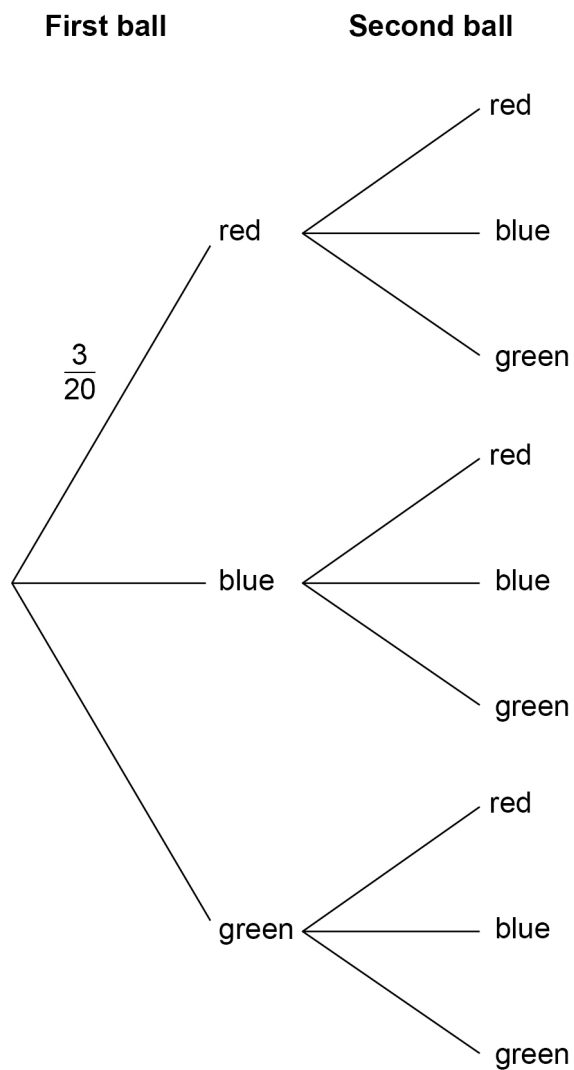**Turn over ▶**

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**Section B**

**Statistics**

Answer **all** questions in the spaces provided.

**6** In a bag there are 3 red balls, 5 blue balls and 12 green balls.

A ball is selected at random from the bag and its colour is recorded.

The ball is **not** replaced in the bag.

A second ball is then selected at random from the bag and its colour is recorded.

**6 (a)** Complete the tree diagram by writing the probability for each branch.

**[2 marks]**

First ball      Second ball

$\dfrac{3}{20}$ red → red, blue, green

blue → red, blue, green

green → red, blue, green

**6 (b)** Find the probability that the first ball selected is green, given that the second ball selected is blue.

[3 marks]

Answer _____

5

**Turn over for the next question**

**Turn over ▶**

**7** The discrete random variable $X$ has a uniform distribution and takes the values 1, 2, 3, 4, …, $n$

The variance of $X$ is 65.25

**7 (a)** Find the value of $n$

[2 marks]

Answer

**7 (b)** Find P$(X \geq 4)$

[2 marks]

Answer

**7 (c)** The discrete random variable $Y$ has variance 15

The correlation between $X$ and $Y$ is $-0.8$

Find $\text{Var}(2X - Y)$ giving your answer to three significant figures.

**[3 marks]**

Answer _____

7

**Turn over ▶**

**8** The discrete random variable $X$ has the following probability distribution where $a$ is a constant.

| $x$ | 1 | 2 | 4 | 8 |
|---|---|---|---|---|
| $P(X=x)$ | 0.2 | $a$ | 0.1 | $3a$ |

**8 (a)** Find the probability generating function $G_X(t)$

[3 marks]

Answer

**8 (b)** The discrete random variable $Y$ has probability generating function

$$G_Y(t) = \frac{0.35 + 0.65t}{t}$$

The random variables $X$ and $Y$ are independent.

By using the probability generating function for $X + Y$, find $E(X + Y)$

**[5 marks]**

Answer _____

8

**Turn over ▶**

## Section C

### Mechanics

Answer **all** questions in the spaces provided.

**9 (a)** State the dimensions of $g$, the acceleration due to gravity.

**[1 mark]**

Answer _____

**9 (b)** The equation $T = \dfrac{2Mmg}{M + m}$ applies to the motion of two connected particles.

In this equation

$T$ is the force in the string connecting the particles

$M$ and $m$ are the masses of the two particles

$g$ is the acceleration due to gravity

Show that the equation is dimensionally consistent.

**[3 marks]**

**10** A ball of mass $m$ is released from rest at a height $H$ above a horizontal surface.

The ball bounces when it hits the surface and rebounds to a height $h$

It is given that $H : h = 1 : k$

Find, in terms of $k$, the coefficient of restitution between the ball and the surface.

**[4 marks]**

Answer _____

4

**Turn over ►**

**11** A ball of mass 40 grams is moving at a speed of 8 m s⁻¹ on a smooth horizontal surface when it collides with a fixed vertical wall.

The wall is perpendicular to the path of the ball.

The ball is in contact with the wall for 0.1 seconds.

The magnitude of the force, $F$ newtons, exerted by the wall on the ball at time $t$ seconds is modelled by

$$F = ct(0.1 - t) \qquad \text{for} \qquad 0 \le t \le 0.1$$

where $c$ is a constant.

The ball rebounds with a speed of 4 m s⁻¹

Find the value of $c$

**[6 marks]**

Answer _____

6

**12** A ship (S) is initially 2 km north-east of a boat (B).

The ship (S) is travelling due east at a constant speed of 4 m s$^{-1}$

The boat (B) is travelling at a constant speed of 6 m s$^{-1}$

The water is not moving.

Find the shortest time in which the boat (B) can intercept the ship (S).

Give your answer to the nearest second.

**[6 marks]**

Answer

**END OF QUESTIONS**

6

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