OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Tuesday 5 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

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Answer **all** questions in the spaces provided.

1 A curve has equation $y = 6x^2 - 8x + 5$

1 (a) A line passes through two points on the curve, one where $x = \frac{2}{3}$ and the other where $x = \frac{2}{3} + h$

Find the gradient of this line, giving your answer in its simplest form.

[3 marks]

Answer _____

Do not write outside the box

1 (b) Use your answer to **part (a)** to determine whether or not the point on the curve where $x = \frac{2}{3}$ is a stationary point.

[2 marks]

5

Turn over for the next question

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2 The complex number z satisfies the equation

$$z - 4 = a\mathrm{i}(z + 5)$$

where a is a real number and $a \neq 0$

Find z in terms of a

Give your answer in the form $x + \mathrm{i}y$ where x and y are real.

[5 marks]

Answer _____

5

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3 By considering the derivative of $y = x^{-\frac{3}{2}}$ when $x = 9$ find an estimate for $\dfrac{1}{\left(\sqrt{9.02}\right)^3}$

Give your answer as a fraction in its lowest terms.

[6 marks]

Answer _____

6

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4 (a) Find the general solution of the equation

$$\cos\left(\frac{x}{2}+\frac{2\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

Give your answer in terms of π

[4 marks]

Answer _____

Do not write outside the box

4 (b) Find the **sum** of all the solutions between 0 and $\frac{73\pi}{2}$ of the equation

$$\cos\left(\frac{x}{2}+\frac{2\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

Give your answer in terms of π

[5 marks]

Answer _____

9

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5 The equation $x^2 + 6x + p = 0$ where p is a real number has roots α and β

The equation $25x^2 + 450x + 1829 = 0$ has roots $2\alpha + \beta$ and $\alpha + 2\beta$

Find the value of p

[4 marks]

Answer _____

4

Do not write outside the box

6 Show that

$$\sum_{r=1}^{n} r^2$$

is a factor of

$$\sum_{r=1}^{n} \left(8r^3 + r\right)$$

for all positive integers n

[5 marks]

5

Turn over ►

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7 The integral I_n is defined as

$$I_n = \int_0^9 x^{n+0.5}\,\mathrm{d}x$$

where n is an integer.

The integral I_n

- is an improper integral
- has a finite value

Three students, Ahmed, Brian and Catherine, attempt to find the value of n

Ahmed's answer is $n = 0$
Brian's answer is $n = -1$
Catherine's answer is $n = -2$

Only one of the three students is correct.

7 (a) For each student state, with a reason, whether or not their answer is correct.

[5 marks]

Ahmed

Brian

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Catherine

7 (b) Hence find the finite value of the improper integral I_n

[1 mark]

Answer

6

Turn over for the next question

Do not write outside the box

8 The line L is the locus of points on an Argand diagram such that

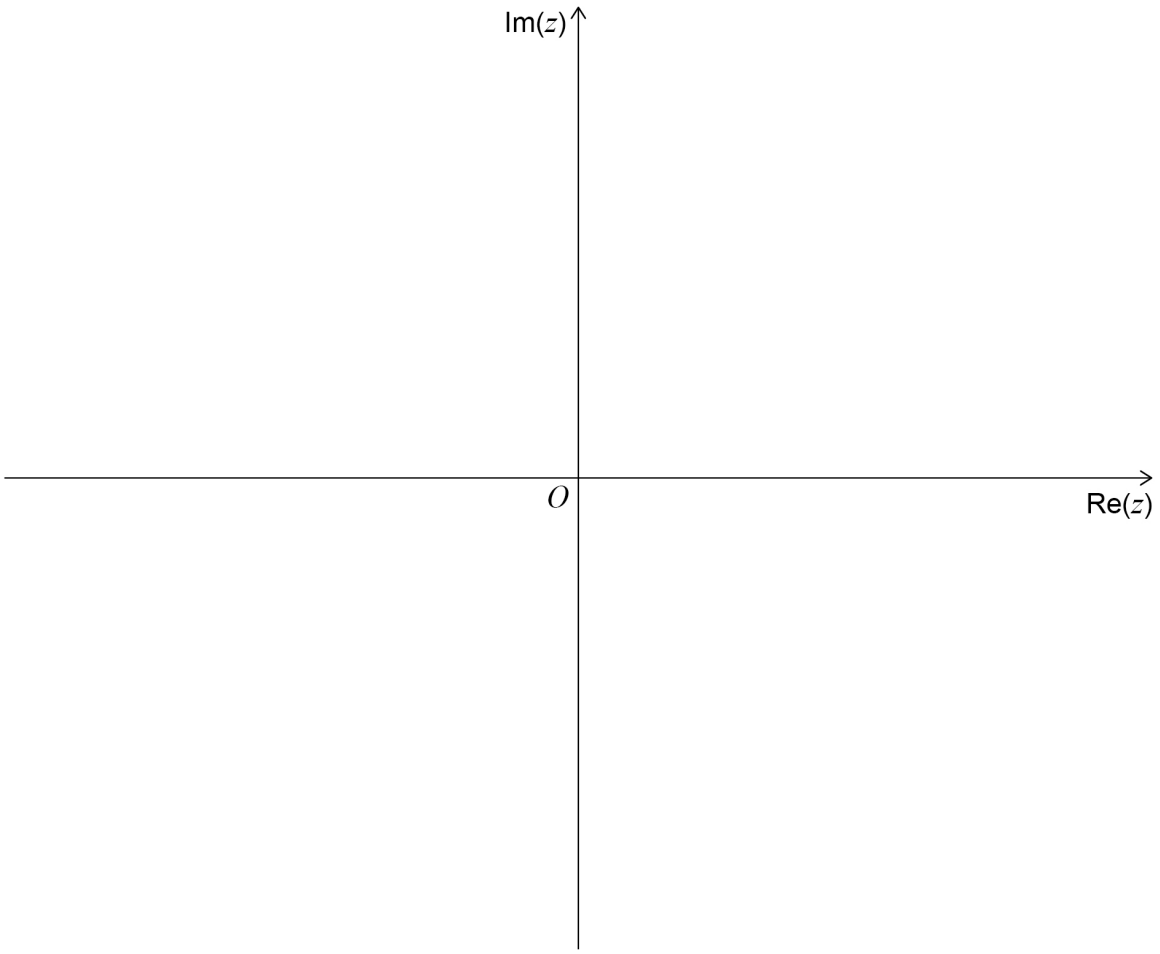
$$|z + 3 - 6\mathrm{i}| = |z + 5 - 2\mathrm{i}|$$

The circle C is the locus of points on an Argand diagram such that

$$|z - 3 - 3\mathrm{i}| = 5$$

8 (a) Draw L and C on the Argand diagram.

[4 marks]



8 (b) L and C intersect at the points P and Q

P and Q represent the complex numbers z_1 and z_2

Find z_1 and z_2

[5 marks]

Do not write outside the box

Answer _____

8 (c) A is the point on C which is the furthest away from L

Find the exact value of the shortest distance from A to L

[5 marks]

Answer _____

14

Turn over ►

Do not write outside the box

9 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2 - 8x}{x^2 - 3x - 18}$$

9 (a) Find the equations of the asymptotes of the graph of $y = \mathrm{f}(x)$

[3 marks]

Answer _____

9 (b) Prove that the line $y = k$ intersects the graph of $y = \mathrm{f}(x)$ for all real values of k

[5 marks]

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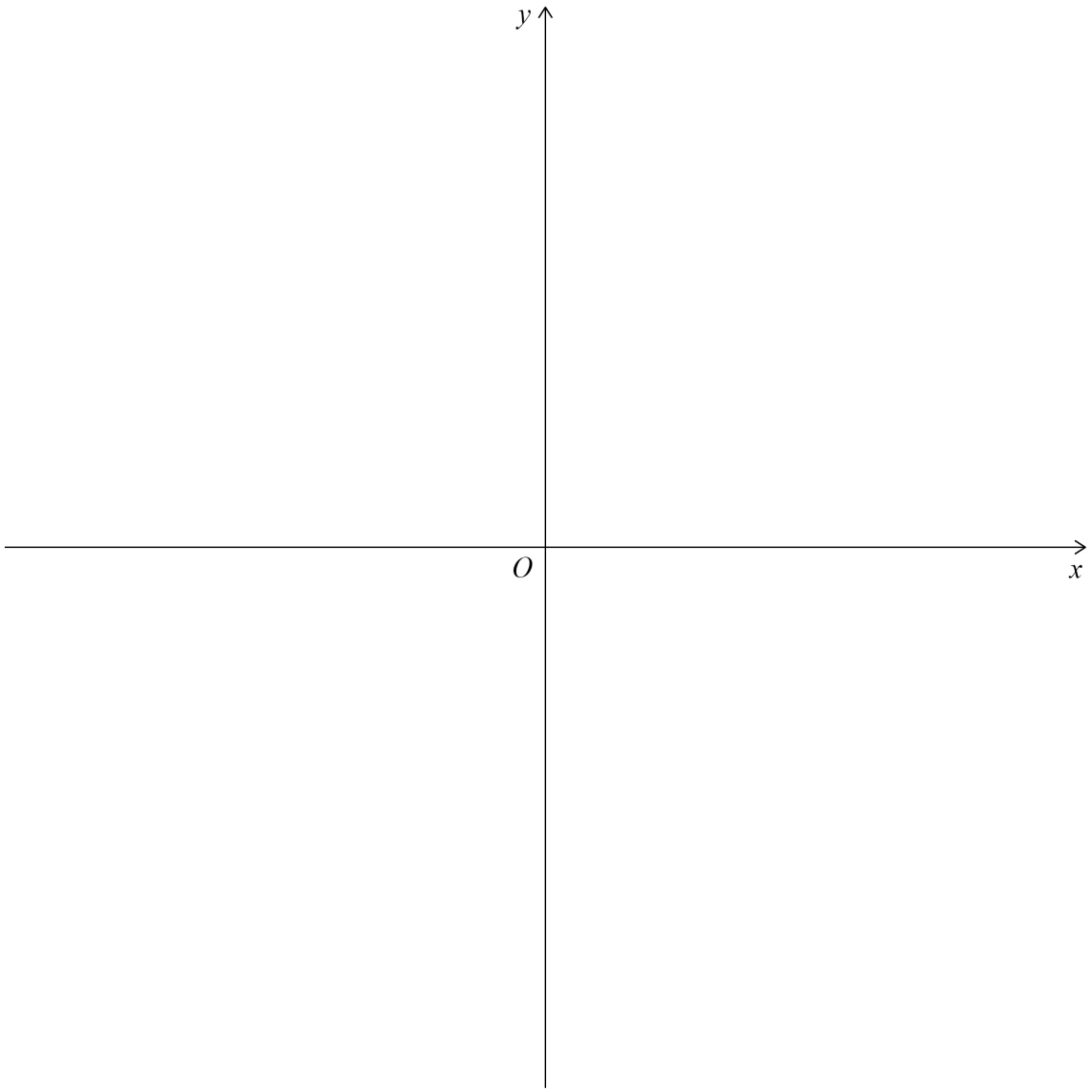
9 (c) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below.

(You are given that the graph of $y = \mathrm{f}(x)$ has no stationary points.)

[3 marks]



11

Turn over for the next question

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10 The hyperbola H_1 has equation $x^2 - 4y^2 = 1$

The hyperbola H_2 has equation $y^2 - 4x^2 = 1$

10 (a) Describe the transformation that maps H_1 onto H_2

[1 mark]

10 (b) Write down the equations of the asymptotes of H_1 and H_2

[2 marks]

Asymptotes of H_1

Asymptotes of H_2

Do not write outside the box

10 (c) Show that if the line $y = mx + c$ is a tangent to H_1 then

$$c^2 = \frac{4m^2 - 1}{4}$$

No credit will be given for solutions using differentiation.

[5 marks]

Question 10 continues on the next page

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10 (d) When the line $y = mx + c$ is a tangent to H_2 then

$$c^2 = \frac{4 - m^2}{4}$$

Find the set of possible values of m in this case.

[2 marks]

Answer _____

10 (e) Find the equations of the four lines which are tangents to both H_1 and H_2

[3 marks]

Answer _____

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10 (f) Find the area of the region enclosed by the lines found in **part (e)**.

[2 marks]

Answer _____

15

END OF QUESTIONS

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