OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS
MATHEMATICS
MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2021

Version: 1.0 Final

211xma02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	360°	B1	Condone lack of units. Condone 2π for 360°
		1	

Q	Answer	Marks	Comments
1(b)	$(a+180°, b-1)$	B1 B1	Correct x-coordinate. Condone lack of units. Condone $a+\pi$ for $a+180°$ Correct y-coordinate. For both marks condone not given as coordinates but must be identified as x-coordinate and y-coordinate.
		2	

Q	Answer	Marks	Comments
1(c)	$a+90°$	B1	Condone lack of units. Condone $a+\frac{\pi}{2}$ for $a+90°$ Can be seen as pair of coordinates but ignore the y-coordinate.
		1	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	$y = 5 \Rightarrow 5 = 23 - 3x$ $(6, 5)$	M1 A1	**PI** by correct x-coordinate of centre.
		2	

Q	Answer	Marks	Comments
2(b)	$r^2 = (9-6)^2 + (11-5)^2$ or $r^2 = (6-0)^2 + (5-8)^2$ or $r^2 = (6-0)^2 + (5-2)^2$ or $r^2 = 36 + 9$	M1	**oe PI** Uses coordinates of centre and coordinates of either *P*, *Q* or *R* **ft** their centre.
	$r = \sqrt{45}$ or $r = 3\sqrt{5}$ or $r^2 = 45$	A1	**PI** by correct value of k in the equation of the circle.
	$(x-6)^2 + (y-5)^2 = 45$	B1ft	**ft** their centre. **ft** their r^2 provided **M1** scored.
		3	

Q	Answer	Marks	Comments
2(c)	$\frac{11-5}{9-6}$ $[= 2]$		
	$[\text{Gradient of Tangent} =] -\frac{1}{2}$	M1	**oe ft** their centre.
	$(y-11) = -\frac{1}{2}(x-9)$		
	$y = -\frac{1}{2}x + \frac{31}{2}$	A1ft	**ft** their gradient of tangent provided **M1** scored and any form.
	$x + 2y = 31$	A1	Correct equation in correct form.
		3	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$13^2 = (x-3)^2 + (2x-5)^2 - 2(x-3)(2x-5)\cos\frac{\pi}{3}$ or $13^2 = (x-2)^2 + (x-3)^2 - 2(x-2)(x-3)\cos\frac{2\pi}{3}$	**M1**	Correct substitution into Cosine Rule. Condone 169 for 13^2 **PI**
	$169 = (x^2 - 6x + 9) + (4x^2 - 20x + 25) - (2x^2 - 11x + 15)$ or $169 = (x^2 - 4x + 4) + (x^2 - 6x + 9) + (x^2 - 5x + 6)$	**M1**	**oe** Expansion of brackets. Condone one error.
	$169 = 3x^2 - 15x + 19$ or $3x^2 - 15x - 150 = 0$ and $x^2 - 5x - 50 = 0$	**A1**	**AG** Must be convinced. Must have further simplification before required result stated.
		3	

Q	Answer	Marks	Comments
3(b)	$\left[x^2-5x-50=0\Rightarrow\right]\quad x=10$ **or** $x=-5$	**B1**	Condone omission of $x=-5$ **PI** by correct further working.
	$\left[\text{Arc Length}=\right]\ (10-3)\times\frac{\pi}{3}$	**M1**	**oe ft** their 10 provided their $(10-3)$ is positive.
	$\left[\text{Arc Length}=\right]\ \frac{7\pi}{3}$ or $\left[\text{Arc Length}=\right]\ 7.3[303...]$	**A1**	**CAO**
	$\left[\text{Perimeter}=\right]\ 13+\left((2\times10-5)-(10-3)\right)+\frac{7\pi}{3}$ or $\left[\text{Perimeter}=\right]\ 13+8+\frac{7\pi}{3}$	**m1**	**oe ft** their 10 provided their length of *CD* is positive. **ft** their arc length.
	$\left[\text{Perimeter}=\right]\ 28.3\left[\text{cm}\right]$	**A1**	**CAO AWRT**
		5	

	Question 3 Total	**8**	

Q	Answer	Marks	Comments
4	$(x+5)^2+\ldots+(y-7)^2+\ldots\ [=0]$	M1	**oe** **PI** by $(x^2+10x+25)+(y^2-14y+49)+\ldots\ [=0]$
	$[(x+5)^2+(y-7)^2=]\ 64-k^2$	A1	Rearranges to find expression for square of the radius. Simplified or unsimplified. **PI** by both correct critical values.
	$64-k^2>0$	m1	Forms inequality. **ft** their expression for the square of the radius. Condone $64-k^2\geq 0$
	$-8<k<8$	A1	**CAO oe** such as $\lvert k\rvert<8$ Condone $-8\leq k\leq 8$ **oe**
		4	

	Question 4 Total	**4**	

Q	Answer	Marks	Comments
5(a)	$\left[\log_{16}x^3 =\right] 3y$	B1	
		1	

Q	Answer	Marks	Comments
5(b)	$\left[y = \log_{16}x \Rightarrow\right] 16^y = x$	M1	**PI**
	$2^{4y} = x$ or $y = \dfrac{\log_2 x}{\log_2 16}$ **oe** or $\log_2 x = \dfrac{\log_{16} x}{\log_{16} 2}$ $\left[= 4\log_{16} x\right]$ **oe**	M1	**PI** by subsequent working.
	$\left[\log_2 x =\right] 4y$	A1	
		3	

Q	Answer	Marks	Comments
5(c)	$\log_3 81 = 4$	M1	**PI**
	$4\times 3y + 5\times 4y - 4 = 60 \Rightarrow y = 2$	M1	**PI** Condone $\log_{16} x$ used instead of y
	$\left[x = 16^2 =\right] 256$	A1	**NMS** scores **M0M0A0**
		3	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6(a)	$6\dfrac{\sin^2 x}{\cos x} = 5\left(1+\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x$	M1	**PI** Uses $\tan x = \dfrac{\sin x}{\cos x}$ to write equation in terms of $\cos x$ and $\sin x$ only.
	$\left[5\left(1+\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x = \right] 5\left(\cos^2 x+\sin^2 x\right) = 5$	M1	**PI** Clear use of $\cos^2 x + \sin^2 x = 1$ to show $\left(1+\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x = 1$
	$\left[6\dfrac{\sin^2 x}{\cos x} = 5 \Rightarrow\right] 6\sin^2 x = 5\cos x$ $\Rightarrow \quad 6\left(1-\cos^2 x\right) = 5\cos x$	M1	Clear use of $\cos^2 x + \sin^2 x = 1$ on LHS to form an equation in $\cos x$ only.
	$6 - 6\cos^2 x = 5\cos x$ $\Rightarrow 6\cos^2 x + 5\cos x - 6 = 0$	A1	**AG** Must come from a convincing argument.
		4	

Q	Answer	Marks	Comments
6(b)	$\cos x = \frac{2}{3}$, $\cos x = -\frac{3}{2}$ Indicates that $\cos x$ cannot take a value less than -1 [Hence the only real solutions satisfy $\cos x = \frac{2}{3}$]	B1 E1	Considers the two possible values for $\cos x$ Eliminates $\cos x = -\frac{3}{2}$ by logical argument.
		2	

Q	Answer	Marks	Comments
6(c)	$\cos\left(x + 35^\circ\right) = \frac{2}{3}$ or $x + 35^\circ = \cos^{-1}\left(\frac{2}{3}\right)$ $x = 13.2^\circ$ $x = -83.2^\circ$	M1 A1 A1	**PI** **CAO** Condone more accurate figures given.
		3	

	Question 6 Total	**9**	

Q	Answer	Marks	Comments
7(a)	$\left[\mathrm{E}(2X+100)=\right]$ $2\times 200+100$ $=500$ [dollars]	M1 A1	Applies expectation formula **PI**
		2	

Q	Answer	Marks	Comments
7(b)(i)	100	B1	
		1	

Q	Answer	Marks	Comments
7(b)(ii)	$\left[\mathrm{Var}(X-Y)=\right]$ $25.2+4.7^2$ **or** $25.2+22.09$ $=47.29$	M1 A1	Applies variance formula **PI** Condone $25.2+4.7$
		2	

	Question 7 Total	**5**	

Q	Answer	Marks	Comments
8(a)(i)	$\left[\mathrm{P}(A \cup B) = 0.45 + 0.32 =\right]\ 0.77$	B1	
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$\left[\mathrm{P}(A \cap B) =\right]$ $0.45 \times 0.32\ [= 0.144]$ $\left[\mathrm{P}(A \cup B) = 0.77 - 0.144 =\right]$ 0.626	M1 A1	PI
		2	

Q	Answer	Marks	Comments
8(b)	$0.74 = 0.65 + 0.18 - \mathrm{P}(C \cap D)$	M1	Uses addition formula to form equation to find $\mathrm{P}(C \cap D)$
	$\left[\mathrm{P}(C \cap D) =\right] 0.09$	A1	PI
	$\left[\mathrm{P}(D \mid C) =\right] \frac{0.09}{0.65}$	M1	Applies conditional probability formula with their $\mathrm{P}(C \cap D)$
	$\frac{9}{65}$	A1ft	Anything which rounds or truncates to 0.138 **ft** their 0.09 provided full method seen.
		4	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$np = 22.5$, $np(1-p) = 12.375$ $22.5(1-p) = 12.375$ $1-p = 0.55$ $p = 0.45$ $n = 50$	B1 M1 A1 B1	Forms two equations using mean and variance. Obtains equation in one variable. **AG** Be convinced.
		4	

Q	Answer	Marks	Comments
9(b)	$[P(X = 24) =] \begin{pmatrix} 50 \\ 24 \end{pmatrix} 0.45^{24}(1-0.45)^{50-24}$ $= 0.1026$	M1 A1	Applies probability formula or uses cumulative probability tables with their n **or** $0.7160 - 0.6134$ **PI** **AWRT**
		2	

Q	Answer	Marks	Comments
9(c)	$[P(X > 19) =] 1 - 0.1974$ $= 0.8026$	M1 A1	**PI** Follow through their n **AWRT**
		2	

	Question 9 Total	**8**	

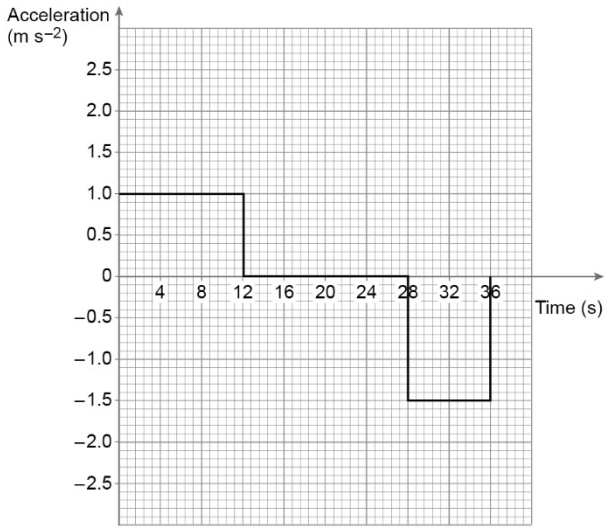
Q	Answer	Marks	Comments
10(a)	$0^2 = 32^2 + 2\times(-9.8)\times s$	M1	Substitutes correct values into $v^2 = u^2 + 2as$ or a correct pair of suvat equations. Condone sign error with -9.8
	$[s =]\ 52.2\ [\text{metres}]$	A1	Anything that rounds or truncates to 52 $(52.24489...)$
		2	

Q	Answer	Marks	Comments
10(b)(i)	$[52.2\times 2 - 2 =]\ 102.4\ [\text{metres}]$	B1ft	**ft** their value in **Part (a)** as long as it is larger than 2 Allow for answers between 102.4 and 102.5 (inclusive).
		1	

Q	Answer	Marks	Comments
10(b)(ii)	$2 = 32\times t + 0.5\times(-9.8)\times t^2$ or $0 = 32 - 9.8t_{up}$ and $50.2 = \frac{1}{2}\times 9.8t_{down}^2$	M1	**PI** Substitutes correct values into $s = ut + \frac{1}{2}at^2$ or a correct pair of suvat equations. Condone sign error with -9.8
	$4.9t^2 - 32t + 2 = 0$ or $0 = 32 - 9.8t_{up}$ and $50.2 = \frac{1}{2}\times 9.8t_{down}^2$ and $t = 3.2[6530...] + 3.2[02196...]$	M1	Forms a completely correct quadratic equation or pair of suvat equations and attempts to find a value of t **PI** By correct final answer.
	$t = 6.47\ [\text{seconds}]$	A1	**AWRT** 6.5 [May see t = 0.063 s rejected **PI**]
		3	

	Question 10 Total	**6**	

Q	Answer	Marks	Comments
11(a)	$[\text{Total Distance} =] \ \frac{12\times(16+36)}{2}$	**M1**	Attempt to find total distance travelled. Minimum of either one fully correct suvat formula, or intention to add at least two correct, relevant areas if area not treated as a trapezium. **PI**
	$[\text{Total Distance} =] \ \frac{312}{36}$	**M1**	**oe** Divides their 'total distance' by 36
	$\frac{26}{3} \ \left[\text{m s}^{-1}\right]$	**A1**	**CAO** Correct answer implies **M1M1A1** Fraction must be simplified. Could be a mixed number. **AWRT** 8.7 if given as a decimal.
		3	

Q	Answer	Marks	Comments
11(b)		**B1** **B1**	3 horizontal line segments between correct time intervals (give bod for unclear, or missing, segment in the interval $12 < t < 28$). Ignore graph for values of t greater than 36. or $a = 1$ and $a = -1.5$ and no other values seen (could include $a = 0$). A fully correct graph (give bod for unclear, or missing, segment in the interval $12 < t < 28$) Condone omission of vertical lines.
		2	

	Question 11 Total	**5**	

Q	Answer	Marks	Comments
12(a)	Statement that acceleration is zero. or Statement that resultant force is zero. $5000 - 1500 - 3500 = 0$	**B1** **B1**	Condone $a = 0$ **oe** Shows that the resultant force is zero by giving numerical values in a suitable equation. May form an equation for R and solve.
		2	

Q	Answer	Marks	Comments
12(b)	Thrust**:** $-3500-500=10\,000a$ $a_1=-0.4$	**M1**	Considers correct forces on the trailer with 500 [N] as thrust. [a may be taken in either direction] Allow one error but must lead to a value for a
	Tension: $-3500+500=10\,000a$ $a_2=-0.3$	**M1**	Considers correct forces on the trailer with 500 [N] acting as tension. [a may be taken in either direction] Allow one error but must lead to a value for a
	Thrust**:** $D-1500+500=2000\times(-0.4)$ Tension: $D-1500-500=2000\times(-0.3)$	**M1**	Considers forces on the tractor with their value for a from considering thrust and/or tension. Condone a written instead of its value. Must be a correct equation.
	Thrust: [$D=$] 200 [N]	**A1**	**CAO**
	Tension: [$D=$] 1400 [N]	**A1**	**CAO**
			Candidates do not need to identify the 'type' of force in the rod at any point in this question. Whether they are considering thrust or tension should be ascertained by their shown working.

12(b) ALT	$D-1500-3500=12000a$ or $D-5000=12000a$	**M1**	**oe** Considers tractor and trailer as a single particle and forms equation of motion simplified or unsimplified.
	Thrust: $D+500-1500=2000a$ or $D-1000=2000a$	**M1**	**oe** Forms equation of motion for tractor with 500 [N] as thrust simplified or unsimplified.
	Thrust: $\left[\begin{array}{l} D+500-1500=2000a \\ \text{and } D-1000=2000a \Rightarrow \end{array}\right]$ $D=200\ [\text{N}]$	**A1**	**CAO**
	Tension: $D-500-1500=2000a$ or $D-2000=2000a$	**M1**	**oe** Forms equation of motion for tractor with 500 [N] as tension simplified or unsimplified.
	Tension: $\left[\begin{array}{l} D-500-1500=2000a \\ \text{and } D-2000=2000a \Rightarrow \end{array}\right]$ $D=1400\ [\text{N}]$	**A1**	**CAO**
			Candidates do not need to identify the 'type' of force in the rod at any point in this question. Whether they are considering thrust or tension should be ascertained by their shown working.
		5	

Q	Answer	Marks	Comments
12(c)	$-3500 = 10\,000a$ $a = -0.35$ $0 = 2 + (-0.35)t$ $t = 5.7$	**M1** **A1**	Considers horizontal forces to find the correct value of acceleration (allow positive or negative) and substitutes their value for the acceleration into a correct suvat formula involving t **PI** by $\frac{20\,000}{3500}$ or better (eg 5.714) **oe**
		2	

	Question 12 Total	**9**	