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MATHEMATICS

MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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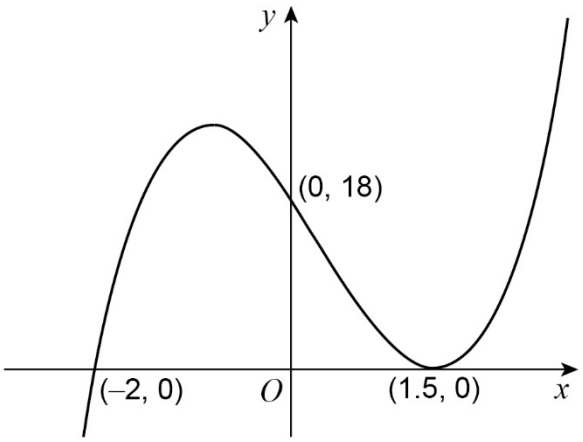
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	18	**B1**	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	-15	**B1**	
		1	

Q	Answer	Marks	Comments
1(a)(iii)	-4	**B1**	
		1	

Q	Answer	Marks	Comments
1(b)	See image below	B1 B1 B1ft	Correct positive cubic graph with maximum in the second quadrant and minimum tangential to the positive x-axis. Correct coordinates of both x-intercepts and no others. Condone given as values rather than coordinates. **ft** their 18 from **part (a)(i)** Correct coordinates of y-intercept. Condone given as value rather than coordinates.
			
		3	

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$\frac{1}{2}\times 90\times(90+1)$	M1	Award **M1** for $\frac{1}{2}\times 90\times(0+89)$
	4095	A1	**CAO NMS** Scores **M1A1**
		2	

Q	Answer	Marks	Comments
2(b)(i)	$a+11d=25$	M1	**oe** **M1** implied by $(d=)\ \frac{57-25}{28-12}$ **oe**
	$a+27d=57$	M1	**oe**
	$a=3 \quad d=2$	A1	
		3	

Q	Answer	Marks	Comments
2(b)(ii)	$\frac{1}{2}\times 65\times(2\times 3+(65-1)\times 2)$ or $3+(65-1)\times 2\ (=131)$ and $\frac{1}{2}\times 65\times(3+131)$ **oe**	M1	Substitutes their values for a and d and $n=65$ into a correct formula for the sum of the first n terms of an arithmetic series.
	4355 dollars	A1	**CAO** condone units omitted
		2	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3(a)	$\frac{15-(-6)}{5-(-1)}=\frac{7}{2}$ $y-15=\frac{7}{2}(x-5)$ **oe** and $7x-2y=5$	**B1** **B1**	Correct gradient Working must be seen. Could come from substituting coordinates of *A* and *B* into $y=mx+c$ Uses gradient and coordinates of *A* or *B* to form equation leading to correct equation in the correct form. May see $y=\frac{7}{2}x+c$ and substitution of coordinates to find c but must be complete method. **AG**
		2	

Q	Answer	Marks	Comments
3(b)	$-\frac{2}{7}$ $\frac{y-4}{x-17}=-\frac{2}{7}$ $2x+7y=62$ $(7x-2y=5$ and $2x+7y=62\Rightarrow)$ $(3, 8)$	**B1** **M1** **M1** **A1** **A1**	Gradient of *CD* **PI** by subsequent working. **oe** Must be in linear form. Correct x-coordinate. Correct y-coordinate. For both **A1** marks condone not given as coordinates, but must be clearly identified.
ALT 3(b)	$-\frac{2}{7}$ $\frac{4-\left(\frac{7}{2}p-\frac{5}{2}\right)}{17-p}$ $\left(=-\frac{2}{7}\right)$ $4-\left(\frac{7}{2}p-\frac{5}{2}\right)=-\frac{2}{7}(17-p)$ $91-49p=4p-68$ $(91-49p=4p-68\Rightarrow p=3\Rightarrow)$ $(3, 8)$	**B1** **M1** **M1** **A1** **A1**	Gradient of *CD* **PI** by subsequent working. **oe** Uses general point on l to form expression for gradient of *CD* **oe** Equates expression for gradient to $-\frac{2}{7}$ and clears the fraction. Brackets must also be expanded. Correct x-coordinate. Correct y-coordinate. For both **A1** marks condone not given as coordinates, but must be clearly identified.
		5	

Q	Answer	Marks	Comments
3(c)	$\left(\lvert\mathrm{CD}\rvert =\right) \sqrt{(17-3)^2+(4-8)^2}$	**M1**	**oe ft** their coordinates of *D*
	$\left(\lvert\mathrm{CD}\rvert =\right) 2\sqrt{53}$ or $\sqrt{212}$	**A1**	**CAO**
	$\frac{1}{2}\times 2\sqrt{53}\times 3\sqrt{53} \Rightarrow \text{Area} = 159$	**A1**	Must be seen to use $AB = 3\sqrt{53}$ **PI** by correct $\lvert\mathrm{CD}\rvert$ and area seen with no working showing that they have not used the length of *AB* **CAO**
		3	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$\left(x \pm \frac{9}{2}\right)^2 \ldots$	M1	**PI** by correct x-component in vector.
	$\left(x-\frac{9}{2}\right)^2-\frac{61}{4}$	A1	**PI** by correct vector.
	Translation	E1	Stating translation and no other transformations.
	$\begin{bmatrix} \frac{9}{2} \\ -\frac{69}{4} \end{bmatrix}$	E1ft	Strict **ft** their $-\frac{9}{2}$ and their $-\frac{61}{4}$ minus 2 Must be given in vector form.
		4	

Q	Answer	Marks	Comments
4(b)(i)	$(\mathrm{f}(3k)=)\ 4(3k)^3+5(3k)^2+32k^3-20k^2$	M1	$\mathrm{f}(3k)$ attempted If polynomial division used it must be correct.
	$(\mathrm{f}(3k)=)\ 140k^3+25k^2$	A1	**CAO**
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$(\mathrm{f}(-2k)=)\ 4(-2k)^3+5(-2k)^2+32k^3-20k^2$	M1	$\mathrm{f}(-2k)$ attempted. Must use Factor Theorem. **CSO**
	$(\mathrm{f}(-2k)=)\ -32k^3+20k^2+32k^3-20k^2=0$	A1	Correctly shows $\mathrm{f}(-2k)=0$ All coefficients must be seen simplified.
		2	

Q	Answer	Marks	Comments
4(c)(i)	$-6k$	**B1**	**CAO**
		1	

Q	Answer	Marks	Comments
4(c)(ii)	$\left(\mathrm{f}\left(\frac{1}{3}x\right)=\right) 4\left(\frac{1}{3}x\right)^3+5\left(\frac{1}{3}x\right)^2+32k^3-20k^2$	**M1**	Substitutes $\frac{1}{3}x$ for x in $\mathrm{f}(x)$ **PI** by one correct coefficient of x^3 or x^2 in their final answer.
	$\frac{4}{27}x^3+\frac{5}{9}x^2+32k^3-20k^2$	**A1**	
		2	

	Question 4 Total	11	

Q	Answer	Marks	Comments
5(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 2ax - b$	B1	Correct derivative
	$2a \times 2 - b = 16$ and $4a - b = 16$	B1	**AG** Intention to use $x = 2$ and required result stated.
		2	

Q	Answer	Marks	Comments
5(b)	$\left[\frac{a}{3}x^3 - \frac{b}{2}x^2 + 5x\right]_2^3 = 23$	M1 A1	At least two terms correct. Fully correct integration. Ignore limits and do not need to see $= 23$
	$\left(\frac{a}{3}(3)^3 - \frac{b}{2}(3)^2 + 5(3)\right) - \left(\frac{a}{3}(2)^3 - \frac{b}{2}(2)^2 + 5(2)\right)$	m1	Substitution into $\mathrm{F}(3) - \mathrm{F}(2)$ **ft** their expression. Condone one sign error.
	$\frac{19}{3}a - \frac{5}{2}b + 5 = 23$		Simplifying $\mathrm{F}(3) - \mathrm{F}(2)$ and set equal to 23
	and then $38a - 15b = 108$	A1	**CSO AG**
		4	

Q	Answer	Marks	Comments
5(c)	$a = 6 \quad b = 8$	**B1**	Both correct values.
		1	

Q	Answer	Marks	Comments
5(d)	$\left(6x^2 - 8x + 5 = x + d \Rightarrow\right) 6x^2 - 9x + 5 - d \ (= 0)$	**B1ft**	**ft** their a and b
	$(-9)^2 - 4 \times 6 \times (5 - d)$	**M1**	Correct discriminant for their quadratic containing d. May be partially simplified.
	$d = \frac{13}{8} \ (= 1.625)$	**A1**	**oe** Correct critical value.
	$d < \frac{13}{8}$ or $d < 1.625$	**A1**	**oe** Must be seen as an inequality.
5(d) Alt	$12x - 8 = 1$	**M1**	Correct derivative of equation for *C* set equal to 1 **ft** their a and b
	$\left(\frac{3}{4}, \frac{19}{8}\right)$	**A1**	**oe** Correct coordinates for the point on *C* where the gradient of the tangent is 1 May not be given as coordinates. **PI** in later working.
	$\left(\frac{19}{8} = \frac{3}{4} + d \Rightarrow\right) \ d = \frac{13}{8}$	**A1**	**oe** Correct critical value.
	$d < \frac{13}{8}$ or $d < 1.625$	**A1**	**oe** Must be seen as an inequality.
		4	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	$\left[(2)^8\right]+8(2)^7\left(\frac{1}{4}x\right)+28(2)^6\left(\frac{1}{4}x\right)^2$ $+56(2)^5\left(\frac{1}{4}x\right)^3\left[+70(2)^4\left(\frac{1}{4}x\right)^4\right]$	M1	For either $(1), 8, 28, 56, [70]$ **oe** unsimplified. or $\binom{8}{1}(2)^7\left(\frac{1}{4}x\right)$ or $\binom{8}{2}(2)^6\left(\frac{1}{4}x\right)^2$ or $\binom{8}{3}(2)^5\left(\frac{1}{4}x\right)^3$ **oe** x not needed, **PI**
	$p=256$	A1	Condone $256x$
	$q=112$	A1	Condone $112x^2$
	$r=28$	A1	Condone $28x^3$
		4	

Q	Answer	Marks	Comments
6(b)(i)	$\left[(2)^8\right]+8(2)^7\left(-\frac{1}{4}x\right)+28(2)^6\left(-\frac{1}{4}x\right)^2$ $+56(2)^5\left(-\frac{1}{4}x\right)^3+\left[70(2)^4\left(-\frac{1}{4}x\right)^4+\ldots\right]$	**M1**	Substitution of $-x$ for x in any term in their expansion in **part (a)**. Ignore terms of order 4 and higher.
	$\left(256+256x+112x^2+28x^3\left[+\frac{35}{8}x^4+\ldots\right]\right)$ $-\left(256-256x+112x^2-28x^3\left[+\frac{35}{8}x^4+\ldots\right]\right)$	**M1**	$\left(2+\frac{1}{4}x\right)^8-\left(2-\frac{1}{4}x\right)^8$ with expansions simplified.
	$512x+56x^3$	**A1**	**AG** Be convinced. Must see evidence of second **M1**
		3	

Q	Answer	Marks	Comments
6(b)(ii)	$x=0.4$	**B1**	Must be seen or **PI** by final answer.
	208.38	**B1**	**CAO**
		2	

	Question 6 Total	**9**	

Q	Answer	Marks	Comments
7(a)	$4x^2 + 2xh + 8xh$	**M1**	**oe** Attempt at surface area introducing variable for height of tank. Simplified or unsimplified. Condone one slip.
	$4x^2 + 10xh = 75$	**A1**	**oe** Correct equation linking dimensions of tank to surface area.
	$xh = \frac{1}{10}\left(75 - 4x^2\right)$ or $h = \frac{1}{10x}\left(75 - 4x^2\right)$	**B1ft**	**oe** **ft** their surface area equation.
	$\left(V = 4x^2h =\right)\ 4x \times \frac{1}{10}\left(75 - 4x^2\right)$ and $V = 30x - \frac{8}{5}x^3$	**B1**	**oe** Correct unsimplified expression for volume of tank with height variable eliminated and **AG** Be convinced.
		4	

Q	Answer	Marks	Comments
7(b)(i)	$\left(\frac{\mathrm{d}V}{\mathrm{d}x}=\right)\ 30-\frac{24}{5}x^2$	B1	Correct derivative. **PI** by later working.
	$30-\frac{24}{5}x^2=0$	M1	**PI** By $x=2.5$ Sets their derivative equal to zero.
	$x=2.5$	A1	**oe CAO** Ignore $x=-2.5$ if seen. **PI** by correct final answer.
	$\left(x=2.5\Rightarrow V=30\times2.5-\frac{8}{5}\times2.5^3=\right)\ 50$	A1	**CAO**
		4	

Q	Answer	Marks	Comments
7(b)(ii)	$\left(\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=\right)\ -\frac{48}{5}x$	B1ft	**ft** their first derivative.
	$-\frac{48}{5}\times2.5\ \left[=-24\right]$ and	E1ft	Substitutes $x=2.5$ into their second derivative. **ft** their $x=2.5$ provided it is positive and the value of their second derivative would be negative.
	Since $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}<0$ then it is a maximum value of V		Indicates that the value of the second derivative is negative and states it is a maximum.
		2	

	Question 7 Total	**10**	

Q	Answer	Marks	Comments
8	$x^{\frac{1}{2}} + \frac{1}{6}x^{-\frac{1}{2}}$	**M1**	Correct expression for integrand in index form.
	$\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{3}x^{\frac{1}{2}}$ or $\frac{2}{3}x\sqrt{x} + \frac{1}{3}\sqrt{x}$	**M1** **A1ft**	**M1** for one correct term in integral simplified or unsimplified. **ft** their integrand.
	$\left(\frac{2}{3}(16k)^{\frac{3}{2}} + \frac{1}{3}(16k)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(64)^{\frac{3}{2}} + \frac{1}{3}(64)^{\frac{1}{2}}\right)$ or $\left(\frac{2}{3}(16k)\sqrt{16k} + \frac{1}{3}\sqrt{16k}\right) - \left(\frac{2}{3}(64)\sqrt{64} + \frac{1}{3}\sqrt{64}\right)$	**M1**	Attempted substitution into $F(16k) - F(64)$ using their integral. Condone incorrect calculation of F(64)
	$(16k)^{\frac{1}{2}} = 4k^{\frac{1}{2}}$ or $\sqrt{16k} = 4\sqrt{k}$	**A1**	**PI** by subsequent working.
	$c = 344$	**A1**	Correct value of c Or equivalent fraction.
	$a = \frac{128}{3}$ or $b = \frac{4}{3}$	**A1**	Either correct value. Ignore powers of k Accept values of a and b rounded or truncated to at least 2**dp**
	$\sqrt{k}\left(\frac{128}{3}k + \frac{4}{3}\right) - 344$	**A1**	In the correct form with $a = \frac{128}{3}$ and $b = \frac{4}{3}$ Accept values of a and b rounded or truncated to at least 2**dp** Ignore incorrect value of c
		8	

	Question 8 Total	**8**	

Q	Answer	Marks	Comments
9(a)	$ar^2 + ar^4 = 20$ and $ar^3 = -6$ or $\frac{ar^2 + ar^4}{ar^3} = -\frac{10}{3}$ $\frac{1+r^2}{r} = -\frac{10}{3}$ or $\left(\frac{-6}{r^3}\right) \times r^2 + \left(\frac{-6}{r^3}\right) \times r^4 = 20$ $3 + 3r^2 = -10r$ or $-6 - 6r^2 = 20r$ and $3r^2 + 10r + 3 = 0$	**M1** **M1** **A1**	$ar^2 + ar^4 = 20$ and $ar^3 = -6$ seen or used. **oe** **PI** by correct further working. **oe** Correct equation with a eliminated, unsimplified. **oe** Must be an additional line of working before required result stated. Need not be linear. Additional line of working not needed if $r^2 + \frac{10}{3}r + 1 = 0$ seen. **AG** Be convinced.
		3	

Q	Answer	Marks	Comments
9(b)	$\left(3r^2+10r+3=0\Rightarrow\right)\ (3r+1)(r+3)(=0)$	M1	Attempt to solve quadratic equation. Could be correct substitution into the quadratic formula. **PI** by $r=-\frac{1}{3}$ and $r=-3$
	$a=162$ and $r=-\frac{1}{3}$ and $r=-3$ rejected	A1	Correct values for a and r Rejecting $r=-3$ **PI** by later working
	$\sum_{n=k}^{\infty}u_n=\frac{162}{1-\left(-\frac{1}{3}\right)}-\frac{162\left(1-\left(-\frac{1}{3}\right)^{k-1}\right)}{1-\left(-\frac{1}{3}\right)}$ or $\frac{162\left(-\frac{1}{3}\right)^{k-1}}{1-\left(-\frac{1}{3}\right)}$	M1	Correct expression with values substituted.
	$\frac{243}{2}(-3)^{1-k}$	M1	Uses $\left(\frac{1}{3}\right)^{k-1}=3^{1-k}$ and the denominator simplified.
	$\sum_{n=k}^{\infty}u_n=\frac{(-1)^{k-1}3^{6-k}}{2}$	A1	Be convinced. **NMS** scores **M1A1M0M0A0**
		5	

	Question 9 Total	**8**	