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MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	Anticlockwise moments about front wheel $0.80\times170\times9.8=1332.8$ Nm Clockwise moments about front wheel $1.4R_A$ Equilibrium – no resultant moment $1.4R_A=1332.8$ $[R_A=]950$ N	**M1** **M1** **A1**	**PI.** Or taking moments about the centre of mass. $0.6R_A=0.8R_B$ Answer is 952 N to 3 **sf**
		3	

Q	Answer	Marks	Comments
1(b)	Equilibrium – no resultant force $952+R_B=170\times9.8$ $[R_B=]710$ N	**M1** **A1ft**	**PI. ft** their **(a)** if less than $170g$ Correct answer is 714 N to 3 **sf**
		2	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)(i)	$\mathbf{a} = 4\cos 2t\,\mathbf{i} - \mathrm{e}^{-t}\,\mathbf{j} + (6t + \sin t)\,\mathbf{k}$ $\mathbf{a} = -2\mathbf{i} - \mathrm{e}^{-\frac{\pi}{3}}\,\mathbf{j} + \left(2\pi + \frac{\sqrt{3}}{2}\right)\mathbf{k}$	**M1** **A1** **A1**	**M1**: At least two components correct. **A1**: All components correct. Accept $\mathbf{j}$ and $\mathbf{k}$ components given to 2 **sf**, eg $\mathbf{a} = -2.0\mathbf{i} - 0.35\mathbf{j} + 7.1\mathbf{k}$
		3	

Q	Answer	Marks	Comments
2(a)(ii)	$[\lvert\mathbf{a}\rvert =] 7.43...\ \mathrm{m\,s^{-2}}$ **or** $[\mathbf{F} =] -20\mathbf{i} - 10\mathrm{e}^{-\frac{\pi}{3}}\,\mathbf{j} + \left(20\pi + 5\sqrt{3}\right)\mathbf{k}$ $[\lvert\mathbf{F}\rvert =] 74\ \mathrm{N}$	 **M1** **A1**	 **PI** by correct answer. **ft** their acceleration from **(a)(i)** **CAO** Correct answer is 74.3 N to 3 **sf**
		2	

Q	Answer	Marks	Comments
2(b)	$\mathbf{r} = \left(-\cos 2t + a\right)\mathbf{i} + \left(-\mathrm{e}^{-t} + b\right)\mathbf{j} + \left(t^3 - \sin t + c\right)\mathbf{k}$	**M1A1**	**M1:** At least two components correct (condone no constants of integration) **A1:** All components correct with (condone no constants of integration)
	$-\cos 0 + a = 1$	**M1**	Substituting $t = 0$ into their position vector [to find their constant(s) of integration]
	$-\mathrm{e}^0 + b = 2$		
	$0 - \sin 0 + c = 3$ $a = 2$ $b = 3$ $c = 3$ $\mathbf{r} = \left(2 - \cos 2t\right)\mathbf{i} + \left(3 - \mathrm{e}^{-t}\right)\mathbf{j} + \left(3 + t^3 - \sin t\right)\mathbf{k}$	**A1**	**CAO, oe**
		4	

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3(a)	Loss in GPE $= 30 \times 9.8 \times 3.0$ $= 882 \text{ J}$	**B1**	Must see full calculation leading to given result.
		1	

Q	Answer	Marks	Comments
3(b)	KE at A $= 0.5 \times 30 \times (1.2)^2$ $= 21.6 \text{ J}$	**B1**	Use of kinetic energy equation.
	KE at B $= 0.5 \times 30 \times (4.0)^2$ $= 240 \text{ J}$	**B1**	**B1 B1** implied by sight of 218.4 [J]
	Conservation of Energy $21.6 + 882 = 240 + W$ $W = 663.6 \text{ J}$	**M1**	Use of the conservation of energy their 21.6 + 882 = their 240 + W
	Resistive Force $R = \frac{663.6}{12} = 55$	**A1**	**CAO**. Correct answer is 55.3 to 3 **sf** Condone inclusion of units in answer.
		4	

Q	Answer	Marks	Comments
3(c)	The student uses 12 metres in their calculation when they should use 3.0 metres	**E1**	Accept any plausible reason.
	The child does not have a uniform acceleration of 9.8 m s^{-2}	**E1**	Accept a second plausible reason, eg the equation $v^2 = u^2 + 2as$ can only be used for uniformly accelerated motion in a straight line.
		2	

	Question 3 Total	**7**	

Q	Answer	Marks	Comments
4(a)(i)	<table><tr><th>Area of Lamina / cm^2</th><th>COM of Lamina</th></tr><tr><td>1600 $(=20\times80)$</td><td>$(40, 10)$</td></tr><tr><td>1000 $(=50\times20)$</td><td>$(90, 25)$</td></tr><tr><td>1000 $(=50\times20)$</td><td>$(75, 60)$</td></tr><tr><td>2000 $(=100\times20)$</td><td>$(10, 70)$</td></tr><tr><td>1600 $(=80\times20)$</td><td>$(60, 110)$</td></tr></table>		
		M1	At least 3 areas and x-coordinates of COMs correct.
		A1	All 5 areas and x-coordinates of COMs correct.
	$1600\sigma\times40+1000\sigma\times90+1000\sigma\times75$ $+2000\sigma\times10+1600\sigma\times60$ $=7200\sigma\bar{X}$	**M1**	Condone not using σ
	$\bar{X}=\frac{575}{12}$ cm	**A1**	**CAO**, such as $\bar{X}=47\frac{11}{12}$ cm **oe** Condone missing units.
		4	

Q	Answer	Marks	Comments
4(a)(ii)	$1600\sigma\times10+1000\sigma\times25+1000\sigma\times60$ $+2000\sigma\times70+1600\sigma\times110$	**M1**	At least 4 y-coordinates of COMs correct [May be seen in **(a)(i)**]
	$=7200\sigma\bar{Y}$	**M1**	Forming COM equation.
	$\bar{Y}=\frac{695}{12}$ cm	**A1**	**CAO**, such as $\bar{Y}=57\frac{11}{12}$ cm **oe** Condone missing units.
		3	

Q	Answer	Marks	Comments
4(a)(iii)	The centre of mass of each lamina is at its centre.	**E1**	
		1	

Q	Answer	Marks	Comments
4(b)	$\tan\theta = \dfrac{\frac{575}{12}}{120 - \frac{695}{12}} \left[= \dfrac{575}{745}\right]$	**M1**	**PI.** $\tan\theta = \dfrac{\text{their } \overline{X}}{120 - \text{their } \overline{Y}}$
	$\theta = \tan^{-1}\left(\dfrac{575}{745}\right) \left[= 37.6...^\circ\right]$	**A1ft**	**ft** their **(a)(i)** & **(a)(ii)**
	$\theta = 38^\circ$	**A1**	**CAO**
		3	

	Question 4 Total	**11**	

Q	Answer	Marks	Comments
5(a)	$\mathbf{F} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$	**B1**	**oe** Ignore units.
		1	

Q	Answer	Marks	Comments
5(b)(i)	$\mathbf{v} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} + t\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	**M1 A1**	**M1**: Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integration **A1**: All correct. **oe** Ignore units.
		2	

Q	Answer	Marks	Comments
5(b)(ii)	$\mathbf{v} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} + 6\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 22 \end{bmatrix}$ $v = \sqrt{12^2 + 22^2} = \sqrt{628}$ $\text{KE} = 0.5 \times 5 \times 628 = 1570 \text{ J}$	**M1** **m1** **A1**	**ft** their final answer to **(b)(i)** with use of $t = 6$ **PI** by correct answer. Condone one slip. Answer is 1600 J to 2 **sf**
		3	

Q	Answer	Marks	Comments
5(c)	$P = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \text{g} \begin{bmatrix} 2t \\ 4 + 3t \end{bmatrix}$ $P = 20t + 60 + 45t$ $P = 65t + 60$ $65t + 60 > 580$ $t > 8$	**M1** **A1** **A1**	**ft** their vectors from **(a)** & **(b)(i)** Correct expression for P Condone $t \geq 8$ Ignore units.
		3	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)(i)	The length of the string does not increase	**E1**	Allow '[length] does not change'
		1	

Q	Answer	Marks	Comments
6(a)(ii)	The particle is accelerating	**E1**	
	as its velocity continuously changes direction	**E1**	Accept any valid alternative such as references to resultant or centripetal force. Must not contradict statement about acceleration.
		2	

Q	Answer	Marks	Comments
6(b)(i)	$\sin(BAC) = \frac{1.2}{1.5} = 0.8$ $[\text{Forces Vertically Upwards} =]\ T_B \sin(BAC)$ $[\text{Forces Vertically Downwards} =]\ mg$ $T_B \sin(BAC) = mg$ $T_B = \frac{2.4 \times 9.8}{0.8} = 29.4\,\text{N}$	**B1** **M1** **A1**	Seen or used, or *BAC* = 53.13…° seen or used or $\cos(ABC) = \frac{1.2}{1.5} = 0.8$ seen or used. $T_B \cos(ABC) = mg$ Condone missing units.
		3	

Q	Answer	Marks	Comments
6(b)(ii)	$\cos(BAC) = \frac{0.9}{1.5} = 0.6$ $[\text{Forces Towards } C =]\ T_B \cos(BAC) + T_C$ or $[\text{Resultant Force} =]\ \frac{2.4 \times 6^2}{0.90}$ $\left[0.6T_B + T_C = \frac{mv^2}{r}\right]$ $T_C = \frac{2.4 \times 6^2}{0.90} - 0.60 \times 29.4$ $T_C = 78.4\,\text{N}$	**B1** **M1** **m1** **A1**	Seen or used or $\sin(ABC) = \frac{0.9}{1.5} = 0.6$ $T_B \sin(ABC) + T_C$ **PI** by 96 N **ft** their **(b)(i)** **CAO** to 3 **sf** Condone missing units.
		4	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	$2000-500=\left(\frac{500}{9.8}\right)a$ $a=\frac{1500}{\left(\frac{500}{9.8}\right)}$ $a=29.4\ \text{m s}^{-2}$	**M1** **A1**	Forming equation of motion with no resistive force. Intermediate step showing calculation leading to given result, be convinced.
		2	

Q	Answer	Marks	Comments
7(b)	$\left[\text{Magnitude of Resistive Force}=5^2k=\right]\ \ 25k$ $2000-500-25k=\left(\frac{500}{9.8}\right)\times 9.8$ $25k=1000$ $k=40$	**M1** **m1** **A1**	Using resistive force as $25k$ Forming equation of motion, condone one sign error. **PI** by correct answer.
		3	

Q	Answer	Marks	Comments
7(c)	$2000-500-40v^2=0$ $40v^2=1500$ $\left[v=\right]\ \ 6.1\ \text{m s}^{-1}$	**M1** **A1ft**	**M1**: Use of resultant force being zero with their k **ft** their k, provided $k\neq 60$ [i.e. $v\neq 5$] **oe** such as $\frac{5\sqrt{6}}{2}\ \text{m s}^{-1}$
		2	

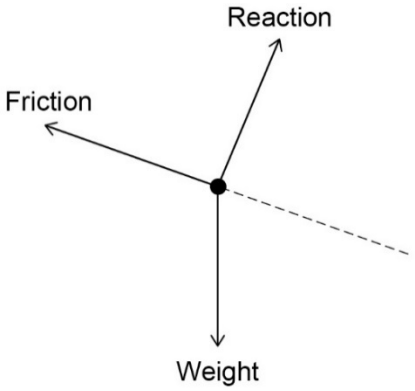
	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)(i)	$v = u + at$ $t = \dfrac{v-u}{g}$ $t = \dfrac{0-(u\sin\theta)}{-g}$ $t = \dfrac{u\sin\theta}{g}$	**M1** **A1**	Use of $v = u + at$ with $v = 0$ and $u\sin\theta$ Condone $a = +g$ for **M1** **AG**, intermediate steps shown and no errors made. Be convinced.
		2	

Q	Answer	Marks	Comments
8(a)(ii)	The object does not experience any air resistance	**E1**	
		1	

Q	Answer	Marks	Comments
8(b)	Time of flight $t = \dfrac{2u\sin\theta}{g}$	**M1**	
	Distance *OA* $\left[\left(u\cos\theta\right)t = \right]\dfrac{2u^2\sin\theta\cos\theta}{g}$	**A1**	**oe**, such as $\dfrac{u^2\sin 2\theta}{g}$
	Maximum height $\left[\dfrac{u\sin\theta}{2}\times\dfrac{u\sin\theta}{g} = \right]\dfrac{u^2\sin^2\theta}{2g}$	**B1**	$u\sin\theta\times\dfrac{u\sin\theta}{g} - \dfrac{1}{2}g\times\left(\dfrac{u\sin\theta}{g}\right)^2$
	Maximum height = *OA* $\dfrac{u^2\sin^2\theta}{2g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$	**M1**	**ft** their max height and their *OA*
	$\sin^2\theta - 4\sin\theta\cos\theta = 0$		
	$\tan\theta = 4$	**A1ft**	$\tan\theta = k$
	$\theta = \tan^{-1}(4) = 76^\circ$	**A1ft**	$\theta = 75.963...^\circ$ **ft** their $\theta = \tan^{-1}\left(k\right)$
		6	

	Question 8 Total	**9**	

Q	Answer	Marks	Comments
9(a)		B1	Must have clear labels on arrows. Do not accept symbols (N, W, mg etc) as labels unless they are defined. Do not condone 'gravity' in place of weight.
		1	

Q	Answer	Marks	Comments
9(b)	Force down the slope $mg\sin\theta$	M1	**PI,** $117.6\sin\theta$
	Force up the slope $\mu mg\cos\theta$	M1	**PI,** $47.04\cos\theta$
	Resultant force on the block $mg\sin\theta-\mu mg\cos\theta=ma$	A1	$117.6\sin\theta-47.04\cos\theta=38.4$
	$g\left(\sin\theta-0.4\cos\theta\right)=a$		$9.8\sin\theta-3.92\cos\theta=3.2$ **oe**
	$\sin\theta-0.4\cos\theta$ $=R\sin\theta\cos\alpha-R\cos\theta\sin\alpha$	M1	**M1**: For use of compound angle formulae. **PI** by correct R or correct α
	$R=\dfrac{\sqrt{29}}{5}$, $\alpha=21.801...^\circ$	A1 A1	**A1**: Correct R (allow 1.08 **oe** correct multiples such as 126.659…) **A1**: Correct α $[\alpha=\tan^{-1}(0.4)]$
	$g\times\dfrac{\sqrt{29}}{5}\sin\left(\theta-21.801...^\circ\right)=a$		
	$\sin\left(\theta-21.801...^\circ\right)=\dfrac{3.2\times5}{9.8\times\sqrt{29}}$ $\theta-21.801...^\circ=17.648...^\circ$	m1	
	$\theta=39[.449...]^\circ$	A1	**CAO**
		8	

Q	Answer	Marks	Comments
9(c)(i)	Angle would not change	**E1**	
	as the situation is independent of the mass of the block	**E1**	**oe** (both component of weight down the slope and friction increase by the same factor)
		2	

Q	Answer	Marks	Comments
9(c)(ii)	Angle would be larger	**E1**	
9(c)(ii)	[as the component of the weight down the slope would need to be greater to overcome the] increased friction force	**E1**	
		2	

	Question 9 Total	**13**	