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FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2021

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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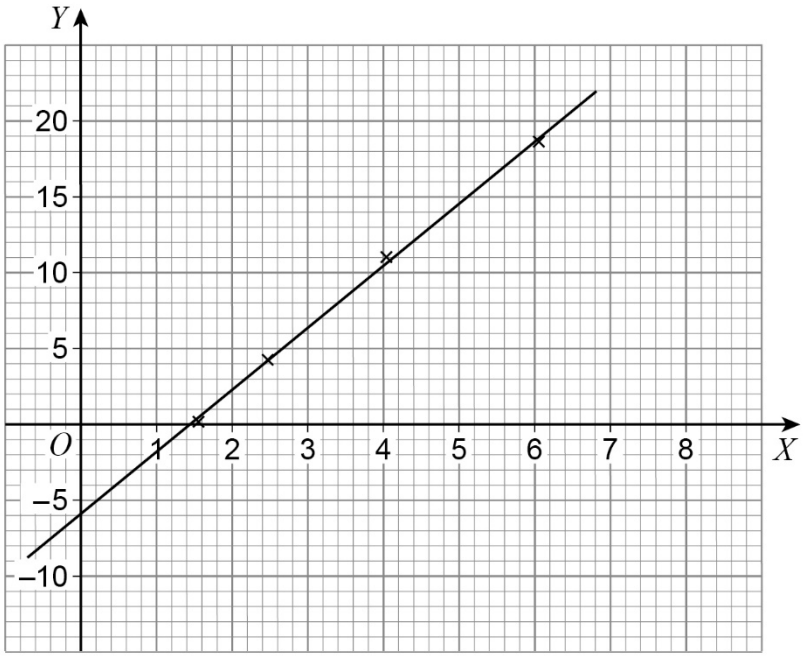
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	<table><tr><td>X</td><td>1.6</td><td>2.5</td><td>4.0</td><td>6.0</td></tr><tr><td>Y</td><td>0.3</td><td>4.3</td><td>11.1</td><td>18.6</td></tr></table>	**M1** **A1**	At least 4 values correct to at least 1 **dp** All 8 values correct to exactly 1 **dp**
		2	

Q	Answer	Marks	Comments
1(b)	See image below	**B1** **B1ft**	All points plotted ± 2 squares 'their' line of best fit drawn Must be a single ruled line
			
		2	

Q	Answer	Marks	Comments
1(c)(i)	"*Their*" intercept $= b$ $b = -6$	B1ft	Note: numerically calculated value gives –6.1, graphical values likely to be between –5 and –7
	"Their" gradient $= a$	M1	Showing method for calculating the gradient
	$a = 4$	A1ft	Values between 3 and 5 accepted
		3	

Q	Answer	Marks	Comments
1(c)(ii)	$y^3 = 4\dfrac{x^2}{y} - 6$	B1ft	$y^3 = \text{'their}\,a\text{'}\dfrac{x^2}{y} + \text{'their}\,b\text{'}$ **ACF**, **eg** $y^4 = 4.2x^2 - 6.1y$
		1	

Q	Answer	Marks	Comments
1(d)	$2^4 + \text{'their}\,a\text{'} \times 2 - \text{'their}\,b\text{'}\,x^2 = 0$	M1	Substituting y=2 and attempts to make x^2 the subject. Answers in the range $2.4 < x < 2.9$ acceptable.
	$x = 2.6$	A1ft	1 **dp** answer
		2	

	Question 1 Total	**10**	

Q	Answer	Marks	Comments
2(a)	$(m+1)(m+2)-(-3)(m-3)=0$ $m^2+6m-7=0$ $m=-7$ and $m=1$	**M1** **A1** **A1**	Setting determinant $=0$ **PI** Obtaining correct quadratic.
		3	

Q	Answer	Marks	Comments
2(b)	**MN** $=\begin{bmatrix}3 & -1\\ -3 & 4\end{bmatrix}\begin{bmatrix}2 & p\\ 3 & p\end{bmatrix}$ $=\begin{bmatrix}3 & 2p\\ 6 & p\end{bmatrix}$ **NM** $=\begin{bmatrix}2 & p\\ 3 & p\end{bmatrix}\begin{bmatrix}3 & -1\\ -3 & 4\end{bmatrix}$ $=\begin{bmatrix}6-3p & -2+4p\\ 9-3p & -3+4p\end{bmatrix}$ $3=6-3p$ **or** $2p=-2+4p$ **or** $6=9-3p$ **or** $p=-3+4p$ $p=1$	**M1** **M1** **M1** **A1**	Condone one slip in **MN** Condone one slip in **NM** Equating at least one pair of elements **CSO**
		4	

Q	Answer	Marks	Comments
2(c)(i)	$det\begin{bmatrix}2 & 2\\3 & 2\end{bmatrix}=-2$	B1	
		1	

Q	Answer	Marks	Comments
2(c)(ii)	The area has doubled	E1ft	Correctly explains the effect of their $\lvert det\ \mathbf{N}\rvert$ on the area
	But the orientation of the shape is reversed (changed)	E1ft	Correct explains the effect of the $det\ \mathbf{N} < 0$ on the orientation
		2	

	Question 2 Total	10	

Q	Answer	Marks	Comments
3	$\frac{dy}{dx}=\left(\frac{3\times4+2.3}{\sqrt{4+2.3^3}}\right)$ $\left[=3.55649\right]$	M1	**PI** Allow answers correct to 5 decimal places (rounded or truncated)
	$hf\left(x,y\right)=0.1\left(\frac{3\times4+2.3}{\sqrt{4+2.3^3}}\right)$ $=0.3556487759$		2.3 + their value of $hf\left(x,y\right)$
	$y_2=2.3+0.3556487759=2.655648776$	A1ft	
	$y_3=2.655648776+0.1\left(\frac{3\times4.1+2.655648776}{\sqrt{4.1+2.655648776^3}}\right)$	M1	
	$=2.968662163$	A1ft	Values to at least 5 decimal places **(PI)**
	2.9687	A1	**CAO**
		5	

	Question 3 Total	5	

Q	Answer	Marks	Comments
4(a)	$f(3) = -22$ and $f(4) = 59$	M1	
	Change of sign and f is continuous on the interval so α is in the interval $3 < x < 4$	A1	Comment required to show candidate is indicating a change in sign implies a root.
		2	

Q	Answer	Marks	Comments
4(b)	$f(3.5) = -5.625$ $3.5 < x < 4$ $f(3.75) = 17.816$ $3.5 < x < 3.75$	M1	Attempting to calculate f(mid-point) of at least two sets of values Statements can be implied through further calculations
	$f(3.625) = 4.3142$ $3.5 < x < 3.625$	A1	
	$f(3.5625) = -1.054$ $3.5625 < x < 3.625$	A1	
	$[\alpha =] 3.6$ to 2 **sf**	A1	
		4	

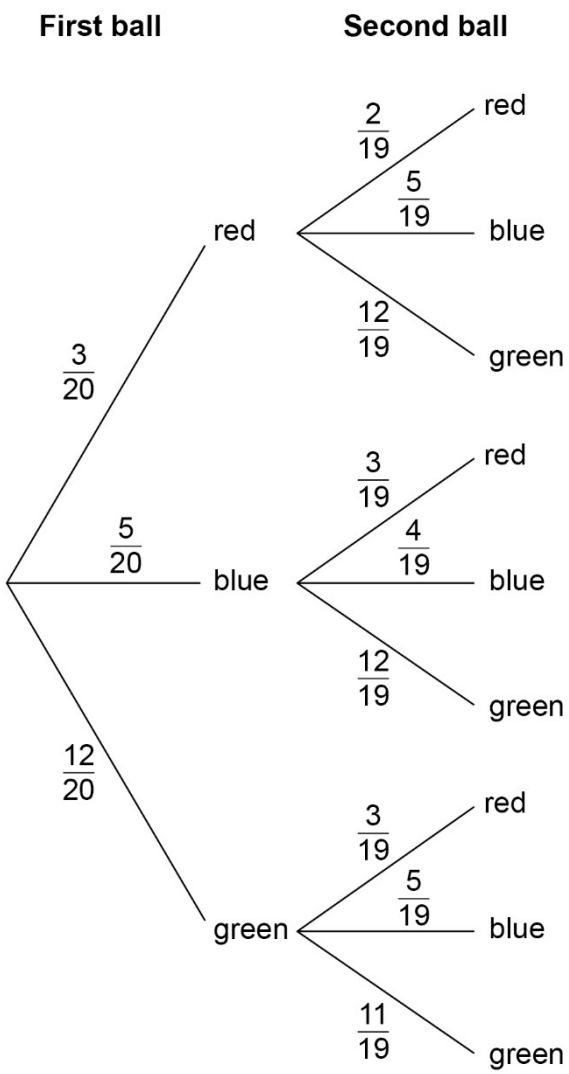
	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	$\begin{bmatrix} 9k & 10k \\ -5k & 6 \end{bmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ Used to obtain $9k - 10k = 1$ **or** $-5k - 6 = -1$	**M1**	Appropriate method of matrix multiplication used
	$k = -1$	**A1ft**	**AG**
		2	

Q	Answer	Marks	Comments
5(b)(i)	$\begin{bmatrix} -9 & -10 \\ 5 & 6 \end{bmatrix}\begin{pmatrix} x \\ -x \end{pmatrix} = \begin{pmatrix} -9x + 10x \\ 5x - 6x \end{pmatrix}$ $= \begin{pmatrix} x \\ -x \end{pmatrix}$	**M1**	Correct matrix multiplication (with concluding statement) **PI**
	$y = -x$ is a line of invariant points	**A1**	
		2	

Q	Answer	Marks	Comments
5(b)(ii)	$\begin{bmatrix} -9 & -10 \\ 5 & 6 \end{bmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$ $\Rightarrow$ $x' = -9x - 10mx - 10c$ $y' = 5x + 6(mx+c)$	**M1**	x' and y', in terms of x, y, m, c
	Invariant lines $\Rightarrow y' = mx' + c$ $5x + 6mx + 6c = m(-9x - 10mx - 10c) + c$	**M1**	Use of $y' = mx' + c$
	$5(2m^2 + 3m + 1)x + 5(2m+1)c = 0$ $2m^2 + 3m + 1 = 0 \qquad (2m+1)c = 0$ $m = -1$ or $m = -\frac{1}{2}$	**M1** **A1**	Attempt at solving equations where coefficients = 0 or compares coefficients Finding the correct values of m Must include $m = -\frac{1}{2}$
	If $m = -1 \Rightarrow c = 0$ $y = -x$		Fully correct line – no restriction on c
	if $m = -\frac{1}{2} \Rightarrow c$ can take any value $y = -\frac{1}{2}x + c$	**A1**	Stating equations for invariant lines as $y = -x$ (given) **and** $y = -\frac{1}{2}x + c$
		5	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)		**M1** **A1**	two branches correct fully correct
		2	

Q	Answer	Marks	Comments
6(b)	$\mathrm{P}(G_1 \mid B_2) = \dfrac{\frac{12}{20}\times\frac{5}{19}}{\frac{3}{20}\times\frac{5}{19}+\frac{5}{20}\times\frac{4}{19}+\frac{12}{20}\times\frac{5}{19}}$	**M1**	Correct numerator: simplifies to $\frac{3}{19}$
		M1	Correct denominator: simplifies to $\frac{1}{4}$
	$=\frac{12}{19}$	**A1**	**AWRT** 0.632
		3	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	$\frac{n^2-1}{12}=65.25$	M1	Forms correct equation
	$n=28$	A1	
		2	

Q	Answer	Marks	Comments
7(b)	$P(X\geq 4)=(28-3)\times\frac{1}{28}$	M1	Uses correct formula for their n Alternative methods accepted **eg** $P(X\geq 4)=1-P(X<4)$ $=1-P(X\leq 3)$
	$=\frac{25}{28}$	A1	**AWRT** 0.893
		2	

Q	Answer	Marks	Comments
7(c)	$\text{Cov}(X, Y)=-0.8\sqrt{65.25\times 15}$ -25.028	M1	Uses covariance formula **PI**
	$\text{Var}(2X-Y)=$ $2^2\text{ Var }(X)+\text{Var }(Y)-2\times 2\text{ Cov}(X, Y)$	M1	Applies variance formula
	$=376$	A1	**AWRT**
		3	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)	$a = 0.175$ $G_X(t) =$ $0.2t + at^2 + 0.1\,t^4 + 3\,a\,t^8$ $G_X(t) =$ $0.2t + 0.175t^2 + 0.1t^4 + 0.525t^8$	**B1** **M1** **A1**	7/40 **oe** Finds $G_X(t)$ in terms of ***a*** or using their value of a **PI** **oe**
		3	

Q	Answer	Marks	Comments
8(b)	$G_{X+Y}(t) =$ $(0.2t + 0.175t^2 + 0.1t^4 + 0.525t^8)$ $\times \frac{0.35 + 0.65t}{t}$	**M1**	Multiplies probability generating functions together
	$= 0.07 + 0.19125t + 0.11375t^2 + 0.035t^3 + 0.065t^4 + 0.18375t^7 + 0.34125t^8$	**A1ft**	Multiplies out their expression Implied by correct use of product rule
	$G'_{X+Y}(t) = 0.19125 + 0.2275t + 0.105t^2 + 0.26t^3 + 1.28625t^6 + 2.73t^7$	**M1** **A1**	Attempts to differentiate '*their function'* by reducing each power by 1 or attempts product rule Differentiates term by term or uses product rule correctly
	$E(X + Y) = G_{X+Y}(1) = 4.8$	**A1**	**AWRT** 4.8 **SC2** for using E(X+Y)=E(X)+E(Y)
		5	

	Question 8 Total	**8**	

Q	Answer	Marks	Comments
9(a)	LT^{-2}	B1	Correct dimensions
		1	

Q	Answer	Marks	Comments
9(b)	$[T] = MLT^{-2}$	B1	Correct dimensions of T
	$\left[\frac{2Mmg}{M+m}\right] = \frac{M^2LT^{-2}}{M}$ $= MLT^{-2}$	M1	Finds dimensions of RHS Condone correct use of units
	$\therefore$ Dimensionally consistent	A1	Correct dimensions of RHS and conclusion Condone correct use of units
		3	

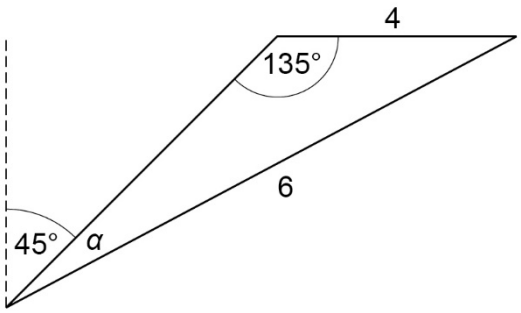
	Question 9 Total	**4**	

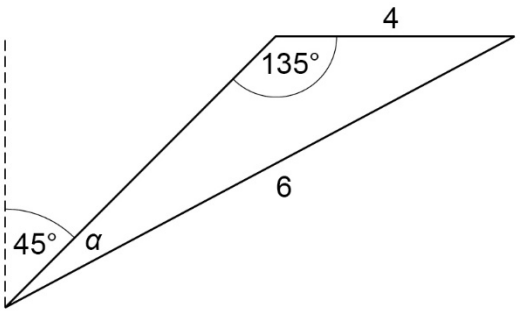
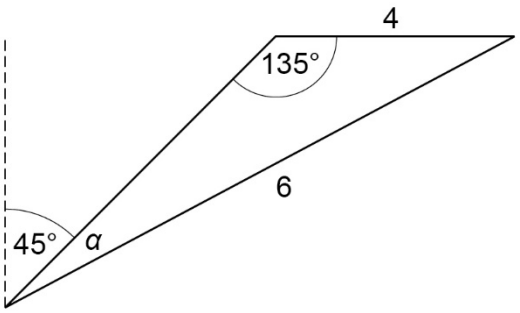
Q	Answer	Marks	Comments
10	$u = \sqrt{2gH}$ $h = kH$ $v = \sqrt{2gkH}$ $e = \frac{\sqrt{2gkH}}{\sqrt{2gH}}$ $= \sqrt{k}$	B1 M1 A1ft A1	Correct expression for u Uses ratio to find rebound height Correct v Alternative $v = \sqrt{2gh}$ Correct e May have $e = \frac{\sqrt{2gh}}{\sqrt{2gH}} = \sqrt{k}$

	Question 10 Total	**4**	

Q	Answer	Marks	Comments
11	$I = \int_0^{0.1} ct(0.1-t)\mathrm{d}t$	**M1**	Forms integral to find the impulse
	$= c\left[\frac{t^2}{20} - \frac{t^3}{3}\right]_0^{0.1}$	**A1**	Correct integration
	$= \frac{c}{6000}$	**A1**	Correct value for integral
	$I = 0.04\times 4 - 0.04\times(-8)$ $\left[= 0.48\right]$	**M1**	Uses $I = mv - mu$ where different directions of u and v are recognised
	$\frac{c}{6000} = 0.48$	**M1**	Equates their result from their integration to their impulse
	$c = 2880$	**A1**	Correct c
		6	

	Question 11 Total	**6**	

Q	Answer	Marks	Comments
12		**M1**	Draws a suitable vector triangle (**PI**)
	$\frac{\sin\alpha}{4}=\frac{\sin 135°}{6}$	**M1**	Applies sine rule to find unknown angle
	$\alpha = 28.13°$	**A1**	Correct angle to at least 2 significant figures
	$v^2 = 4^2 + 6^2 - 2\times 4\times 6\cos(180-135-28.13)$	**M1**	Uses cosine rule or sine rule to find magnitude of relative speed
	$v = 2.4629\ \text{m s}^{-1}$	**A1**	Correct relative speed
	$t = \frac{2000}{2.4629} = 812\ \text{s}$	**A1**	Correct time (812 s to 3 **sf**)

12 ALT	**Alternative method 1:**  		
		M1	Draws a suitable vector triangle (**PI**)
	$(6t)^2 = 2000^2 + (4t)^2 - 2\times 2000\times(4t)\cos 135$ $36t^2 = 2000^2 + 16t^2 - 16000t\cos 135$	**M1** **A1**	Using Cosine rule to obtain quadratic equation in t Correct values for relative speeds used
	$20t^2 - 8000\sqrt{2}t - 2000^2 = 0$ $t = 812$ or $t = -246$	**M1** **A1**	Solving the quadratic equation Correct values for t
	$\Rightarrow t = 812$ seconds	**A1**	States $t = 812$ seconds
	Alternative method 2:  		
		M1	Draws a suitable vector triangle (**PI**)
	$(6)^2 = c^2 + (4)^2 - 2\times 4\times c\times\cos 135$ $36 = c^2 + 16 - 8c\cos 135$ $c^2 - 4\sqrt{2}c - 20 = 0$	**M1** **A1**	Using Cosine rule to obtain quadratic equation in c Correct values for relative speeds per second used
	$c = 2.46$ or $c = -8.11$	**A1**	Solving the quadratic equation
	$t = \dfrac{2000}{2.46}$	**M1**	Correct values for c Correct method for calculating time.
	$\Rightarrow t = 812$ seconds	**A1**	States $t = 812$ seconds
		6	

	Question 12 Total	**6**	