

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2021

Version: 1.0 Final



2 1 1 X F M 0 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2021 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$6 \times \left(\frac{2}{3} + h\right)^2 - 8 \times \left(\frac{2}{3} + h\right) + 5$ $= 6 \left(\frac{4}{9} + \frac{4}{3}h + h^2\right) - \frac{16}{3} - 8h + 5$ $= \frac{7}{3} + 6h^2$ <p>Gradient</p> $= \frac{\frac{7}{3} + 6h^2 - \frac{7}{3}}{h}$ $= 6h$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>PI Allow one slip</p> <p>FT their $\frac{7}{3} + 6h^2$ minus $\frac{7}{3}$</p> <p>CAO Must score M1 M1</p>
		3	

Q	Answer	Marks	Comments
1(b)	<p>Gradient of curve</p> $= \lim_{h \rightarrow 0} [6h] [= 0]$ <p>So the curve has a stationary point at</p> $x = \frac{2}{3}$	<p>B1ft</p> <p>E1</p>	<p>FT their '6h' with correct limiting process</p> <p>FT correct conclusion based upon their '6h' and the gradient being zero</p>
		2	

	Question 1 Total	5	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
2	$\text{Let } z = x + iy$ $x + iy - 4 = ai(x + iy + 5)$ $x - 4 + iy = -ay + iax + 5ia$ $x - 4 = -ay$ $y = a(x + 5)$ $x - 4 = -a^2(x + 5)$ $x + a^2x = 4 - 5a^2$ $x = \frac{4 - 5a^2}{1 + a^2}$ $x + 5 = \frac{4 - 5a^2 + 5 + 5a^2}{1 + a^2} = \frac{9}{1 + a^2}$ $y = \frac{9a}{1 + a^2}$ $z = \frac{4 - 5a^2}{1 + a^2} + i\left(\frac{9a}{1 + a^2}\right)$	<p style="text-align: center;">M1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	<p>Equating real and imaginary parts Allow one slip</p> <p>Eliminating x or y from both equations</p>
2 ALT	$z - 4 = aiz + 5ai$ $z(1 - ai) = 4 + 5ai$ $z = \frac{4 + 5ai}{1 - ai} \times \frac{1 + ai}{1 + ai}$ $z = \frac{4 + 5ai + 4ai - 5a^2}{1 + a^2}$ $z = \frac{4 - 5a^2}{1 + a^2} + i\left(\frac{9a}{1 + a^2}\right)$	<p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	
		5	
	Question 2 Total	5	

Q	Answer	Marks	Comments
3	$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{5}{2}}$ $\frac{dy}{dx} = -\frac{1}{162} \quad \text{when } x = 9$ $\delta y \approx \frac{dy}{dx} \times \delta x$ <p>[Estimate =] $0.02 \times \left(-\frac{1}{162}\right)$ or $-\frac{1}{8100}$ oe</p> <p>[Estimate =] $\frac{1}{27} + \text{their } -\frac{1}{8100}$</p> <p>[Estimate =] $\frac{299}{8100}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1F</p> <p>M1</p> <p>A1</p>	<p>PI</p> <p>PI Condone use of = sign</p> <p>FT $0.02 \times \left(\text{their } -\frac{1}{162}\right)$</p> <p>PI</p> <p>CSO Must be $\frac{299}{8100}$</p>
		6	
	Question 3 Total	6	

Q	Answer	Marks	Comments
4(a)	$\frac{x}{2} + \frac{2\pi}{3} = 2n\pi \pm \frac{5\pi}{6}$	B1	oe
	Going from $\left(\frac{x}{2} + \frac{2\pi}{3}\right)$ to x	M1	Including multiplication of all terms by 2
	$x = 4n\pi + \frac{\pi}{3}$	A1	
	$x = 4n\pi + \pi$	A1	A1 A1 for $x = 4n\pi - \frac{4\pi}{3} \pm \frac{5\pi}{3}$ oe
		4	

Q	Answer	Marks	Comments
4(b)	$S_1 = \frac{\pi}{3} + \frac{13\pi}{3} + \dots + \frac{109\pi}{3}$	M1	For forming two series
	and		
	$S_2 = \pi + 5\pi + \dots + 33\pi$	A1	For summing one AP with correct n
	$S_1 = \frac{550\pi}{3}$		
	$S_2 = 153\pi$	A1	For summing a 2nd AP with correct n
Sum = $\frac{550\pi}{3} + 153\pi$	M1		
$\frac{1009\pi}{3}$	A1		
		5	

	Question 4 Total	9	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
5	$(2\alpha + \beta)(\alpha + 2\beta) = 73.16$ $2(\alpha + \beta)^2 + \alpha\beta = 73.16$ or $(-6 + \alpha)(-6 + \beta) =$ $36 - 6(\alpha + \beta) + \alpha\beta = 73.16$ Using $\alpha + \beta = -6$ and $\alpha\beta = p$ $p = 1.16$	M1 M1 M1 A1	or $(5x + 31)(5x + 59) = 0$ or $2\alpha + \beta = -\frac{31}{5}$ and $\alpha + 2\beta = -\frac{59}{5}$ or $\alpha = -\frac{1}{5}$ and $\beta = -\frac{29}{5}$ oe CSO
		4	
	Question 5 Total	4	

Q	Answer	Marks	Comments
6	$\sum_{r=1}^n (8r^3 + r) = 8 \sum_{r=1}^n r^3 + \sum_{r=1}^n r$ $= 8 \left(\frac{1}{4} \right) n^2 (n+1)^2 + \frac{1}{2} n(n+1)$ $= \frac{1}{2} n(n+1)(4n(n+1)+1)$ $= \frac{1}{2} n(n+1)(2n+1)^2$ $\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$ <p>so</p> $\sum_{r=1}^n (8r^3 + r) = 3(2n+1) \left(\sum_{r=1}^n r^2 \right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Must see attempt at factorising</p> <p>Factorising $(4n(n+1)+1) = (2n+1)^2$ PI by seeing $3(2n+1)$ and no errors seen</p> <p>Be convinced</p>
		5	
	Question 6 Total	5	

Q	Answer	Marks	Comments
7(a)	<p>Ahmed</p> $n = 0: I_0 = \int_0^9 x^{0.5} dx$ <p>[This is not an improper integral, as all required values of the integrand are finite]</p> <p>Ahmed is incorrect.</p> <p>Brian</p> $n = -1: I_{-1} = \int_0^9 x^{-0.5} dx$ <p>This is an improper integral, because the integrand is not defined at the lower limit.</p> <p>Brian is correct (with reason given)</p> <p>Catherine</p> $n = -2: I_{-2} = \int_0^9 x^{-1.5} dx$ $= \lim_{h \rightarrow 0} \left(\frac{9^{-0.5}}{-0.5} - \frac{h^{-0.5}}{-0.5} \right)$ <p>This does not have a finite value. Catherine is incorrect (with reason given)</p>	<p>E1</p> <p>E1</p> <p>E1</p> <p>B1</p> <p>E1</p>	<p>or shows that $\int_0^9 x^{-0.5} dx = 6$ using a limiting process</p>
		5	

Q	Answer	Marks	Comments
7(b)	$\left[I_{-1} = \lim_{h \rightarrow 0} \left(\frac{\sqrt{9}}{0.5} - \frac{\sqrt{h}}{0.5} \right) = \right] 6$	B1	
		1	

	Question 7 Total	6	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
8(a)	Circle with centre $3 + 3i$, radius = 5 Line of negative gradient through $-4 + 4i$ Correct line	B1 B1 B1 B1	
<p>The diagram shows an Argand diagram with the horizontal axis labeled $\text{Re}(z)$ and the vertical axis labeled $\text{Im}(z)$. Both axes have tick marks from -12 to 12. A circle is drawn with its center at the point $(3, 3)$ and a radius of 5. A straight line with a negative gradient is drawn, passing through the point $(-4, 4)$. There are four '+' signs on the diagram: one at the center of the circle $(3, 3)$, one at the point $(-4, 4)$ on the line, one at the intersection of the circle and the line in the second quadrant, and one at the intersection of the circle and the line in the first quadrant.</p>			
		4	

Q	Answer	Marks	Comments
8(b)	Cartesian equation of L $y - 4 = -\frac{1}{2}(x + 4)$ $x = 4 - 2y$	B1	
	Cartesian equation of C $(x - 3)^2 + (y - 3)^2 = 25$	B1	
	$(1 - 2y)^2 + (y - 3)^2 = 25$ $y^2 - 2y - 3 = 0$	M1	oe quadratic equation in x , ie $x^2 - 4x - 12 = 0$
	Substituting $y = 3$ or -1 into an equation to find the corresponding value for x	M1	oe for values of x to find y
	$z_1 = -2 + 3i \text{ and } z_2 = 6 - i$	A1	or the other way round
		5	

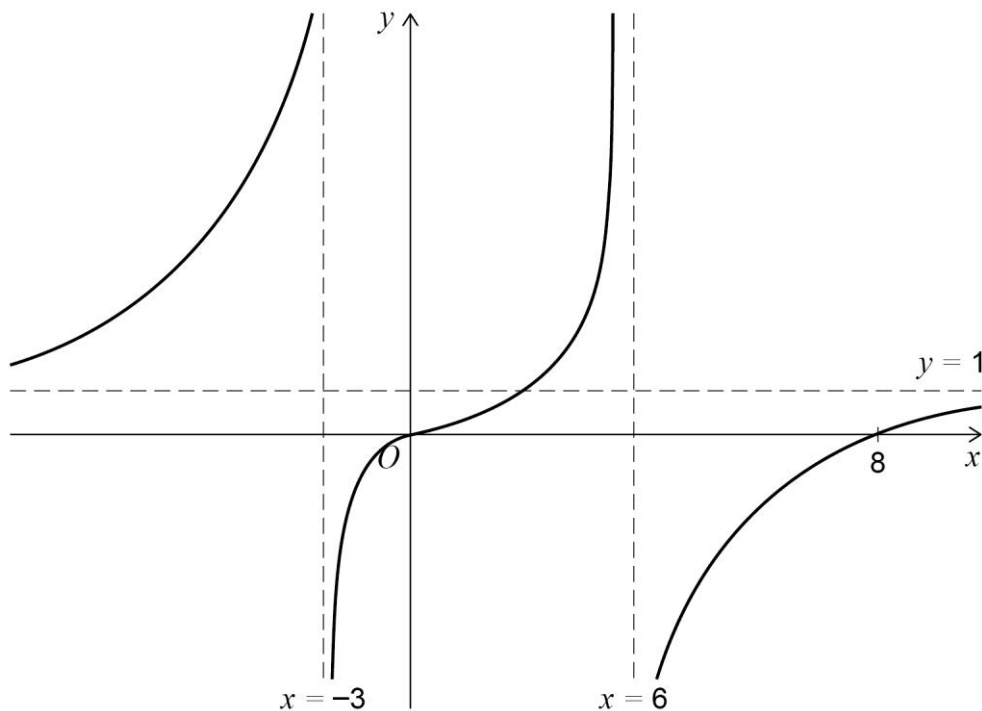
Q	Answer	Marks	Comments
8(c)	$ z_2 - z_1 = \sqrt{8^2 + 4^2} = 4\sqrt{5}$	M1	or $\frac{1}{2}(z_1 + z_2) = 2 + i$
	Let h = distance from $(3, 3)$ to L Then $h^2 = 5^2 - (2\sqrt{5})^2$	M1	$ (2 + i) - (3 + 3i) $
	$h = \sqrt{5}$	A1	$= \sqrt{5}$
	Required distance = h + radius	M1	
	$5 + \sqrt{5}$	A1	CAO
		5	

	Question 8 Total	14	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
9(a)	$x = -3$	B1	
	$x = 6$	B1	
	$y = 1$	B1	
		3	

Q	Answer	Marks	Comments
9(b)	$k(x^2 - 3x - 18) = x^2 - 8x$	M1	
	$(k-1)x^2 + (8-3k)x - 18k = 0$	A1	
	$(8-3k)^2 - 4(k-1)(-18k)$	M1	Discriminant in terms of k
	for real roots $(8-3k)^2 - 4(k-1)(-18k) \geq 0$	m1	Discriminant conditions for real roots being applied
	$81k^2 - 120k + 64 \geq 0$ $\left(9k - \frac{20}{3}\right)^2 + \frac{176}{9} \geq 0$ (or > 0) Always true so there are real roots for all real k	A1	Shows as sum of squares or Shows discriminant of $81k^2 - 120k + 64$ is negative and states k^2 coefficient is positive.
		5	

9(c)	Graph of $y = f(x)$ correct shape	B1
	Asymptotes shown	B1
	Values at axis intercepts shown	B1



	3
--	----------

Question 9 Total	11
-------------------------	-----------

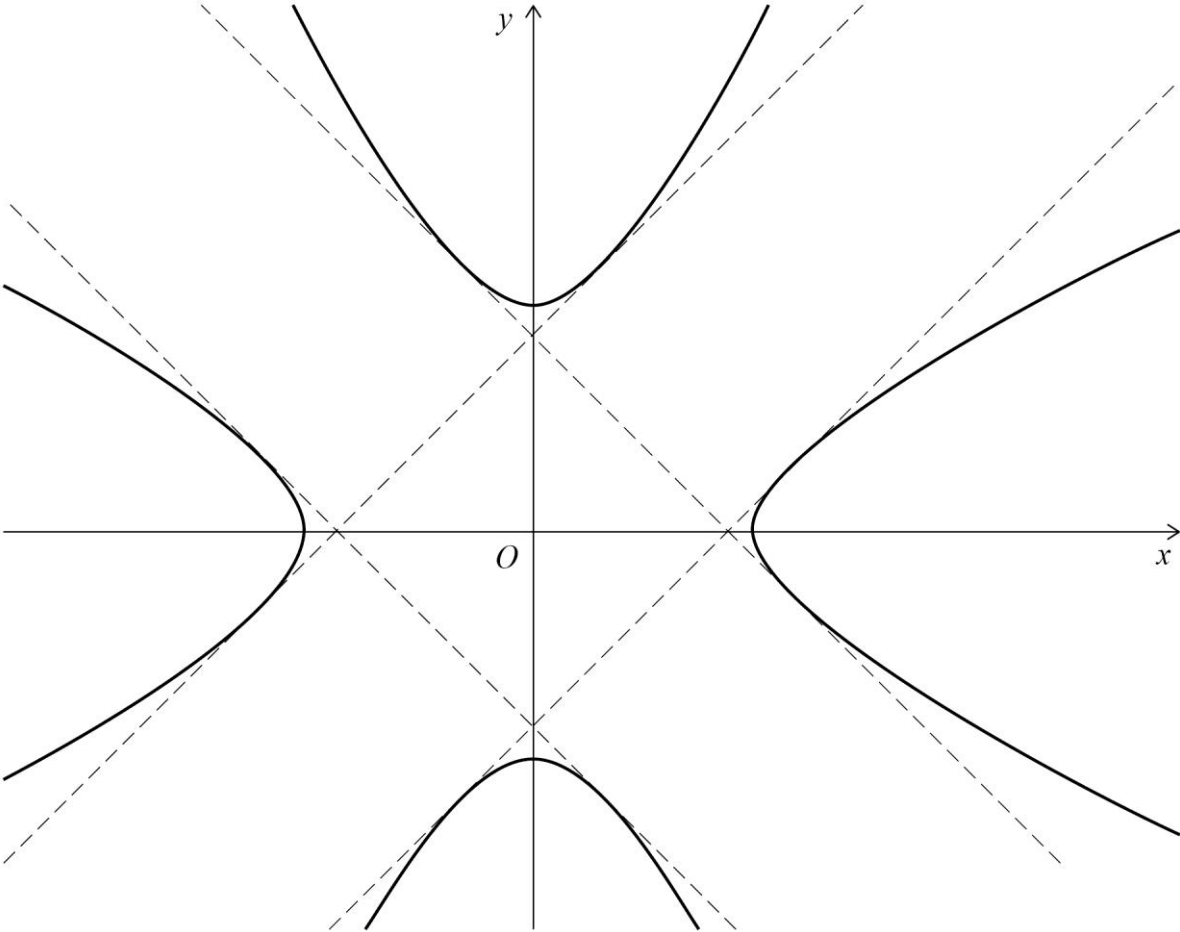
Q	Answer	Marks	Comments
10(a)	Reflection in the line $y = x$	B1	or reflection in the line $y = -x$
		1	

Q	Answer	Marks	Comments
10(b)	$H_1: y = \frac{1}{2}x, \quad y = -\frac{1}{2}x$	B1	oe
	$H_2: y = 2x, \quad y = -2x$	B1	oe
		2	

Q	Answer	Marks	Comments
10(c)	$x^2 - 4(mx + c)^2 = 1$	M1	Their discriminant set equal to zero Correct expansion of their discriminant
	$(1 - 4m^2)x^2 - 8mcx - (4c^2 + 1) = 0$	A1	
	$[\Delta = 0]$ $(-8mc)^2 + 4(1 - 4m^2)(1 + 4c^2) = 0$	M1	
	$64m^2c^2 + 4(1 - 4m^2 + 4c^2 - 16m^2c^2) = 0$	m1	
	$4 - 16m^2 + 16c^2 = 0$ $c^2 = \frac{4m^2 - 1}{4}$ as required	A1	
		5	

Q	Answer	Marks	Comments
10(d)	$c^2 \geq 0 \Rightarrow 4 - m^2 \geq 0$ $-2 \leq m \leq 2$ $m = 2$ or $m = -2 \Rightarrow c = 0$ and $y = \pm 2x$ These lines are asymptotes, not tangents. So $-2 < m < 2$	M1	Allow $-2 \leq m \leq 2$
	A1	2	

Q	Answer	Marks	Comments
10(e)	$4 - m^2 = 4m^2 - 1$ $5m^2 = 5$ $m^2 = 1$	M1	oe
	$c^2 = \frac{4-1}{4} = \frac{3}{4} \quad \left[\Rightarrow c = \pm \frac{\sqrt{3}}{2} \right]$	M1	
	$y = x + \frac{\sqrt{3}}{2}, y = x - \frac{\sqrt{3}}{2}, y = -x + \frac{\sqrt{3}}{2},$ $y = -x - \frac{\sqrt{3}}{2}$	A1	
		3	

Q	Answer	Marks	Comments
10(f)	<p>Area in 1st quadrant = $\frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{8}$</p> <p>Total area = $\frac{3}{2}$</p>	<p>M1</p> <p>A1</p>	<p>or for distance between adjacent vertices followed by squaring</p> <p>or other valid method</p> <p>or for finding axis intercepts and using them to calculate an area</p>
			
		2	
	Total	15	