

OXFORD

INTERNATIONAL  
AQA EXAMINATIONS

---

# INTERNATIONAL A-LEVEL FURTHER MATHEMATICS FM05

(9665/FM05) Unit FM2 Mechanics

---

Mark scheme

January 2021

---

Version: 1.0 Final



2 1 1 X F M 0 5 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from [oxfordaqaexams.org.uk](http://oxfordaqaexams.org.uk)

#### **Copyright information**

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2021 Oxford International AQA Examinations and its licensors. All rights reserved.

### Key to mark scheme abbreviations

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
<b>✓ or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>–x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$2\begin{bmatrix} 4 \\ 1 \end{bmatrix} + m\begin{bmatrix} 2 \\ U \end{bmatrix} = (m+2)\begin{bmatrix} 2.8 \\ -1 \end{bmatrix}$ $8 + 2m = 2.8m + 5.6$ $2.4 = 0.8m$ $m = 3$	<b>M1</b>       <b>A1</b>	Forms equation based on conservation of momentum in one or two dimensions    Correct value for $m$
		<b>2</b>	

Q	Answer	Marks	Comments
1(b)	$2 + 3U = -5$       $U = -\frac{7}{3}$	<b>M1</b>     <b>A1ft</b>  <b>A1</b>	Forms equation for second component based on conservation of momentum, with at least one side of the equation correct.  Correct equation, for their $m$  Correct value for $U$ <b>AWRT -2.3</b>
		<b>3</b>	

Q	Answer	Marks	Comments
1(c)	$\mathbf{I} = 2\begin{bmatrix} 2.8 \\ -1 \end{bmatrix} - 2\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} -2.4 \\ -4 \end{bmatrix}$ $I = \sqrt{2.4^2 + 4^2} = 4.7 \text{ [Ns]}$	<b>M1</b>     <b>A1</b>    <b>A1</b>	Uses impulse formula in vector form  Obtains correct impulse expression  Obtains correct magnitude <b>AWRT 4.7, such as 4.66476</b>
		<b>3</b>	

	<b>Question 1 Total</b>	<b>8</b>	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
2(a)	$3\cos 60^\circ = v \cos 30^\circ$ $v = \frac{3\cos 60^\circ}{\cos 30^\circ} [= \sqrt{3}]$ $v \sin 30^\circ = e \times 3 \sin 60^\circ$ $e = \frac{3\cos 60^\circ \times \sin 30^\circ}{\cos 30^\circ \times 3 \sin 60^\circ}$ $= \frac{1}{3}$	<b>M1</b> <b>A1</b>  <b>M1</b> <b>M1</b>  <b>A1</b>	Forms equation for motion parallel to the wall Correct $v$  Forms equation for motion perpendicular to the wall. Eliminates $v$  Correct value for $e$ AWRT 0.33
		<b>5</b>	

Q	Answer	Marks	Comments
2(b)	$I = 0.08 \times \frac{3\cos 60^\circ}{\cos 30^\circ} 3 \sin 30^\circ - 0.08 \times (-3 \sin 60^\circ)$ $= \frac{3\sqrt{3}}{25} + \frac{\sqrt{3}}{25}$ $= \frac{4\sqrt{3}}{25} \text{ [Ns]}$	<b>M1</b> <b>A1</b>  <b>A1</b>	Forms equation to find impulse. Allow sign errors and their $v$ Correct equation  Correct impulse in exact form. Accept $0.16\sqrt{3}$
		<b>3</b>	

	<b>Question 2 Total</b>	<b>8</b>	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
3(a)	$4 = \frac{2\pi}{\omega}$ $\omega = \frac{\pi}{2}$ $6 = a \times \frac{\pi}{2}$ $a = \frac{12}{\pi} \text{ [m]}$	<b>B1</b>  <b>M1</b>  <b>A1</b>	Correct $\omega$  Forms equation to find the amplitude using their $\omega$  Correct amplitude
		<b>3</b>	

Q	Answer	Marks	Comments
3(b)	$5^2 = \left(\frac{\pi}{2}\right)^2 \left(\left(\frac{12}{\pi}\right)^2 - x^2\right)$ $x = \pm \frac{\sqrt{44}}{\pi} = \pm \frac{2\sqrt{11}}{\pi} \text{ [m]}$	<b>M1</b>  <b>A1</b> <b>A1</b>	Forms equation to find displacement  At least one correct displacement Both displacements correct and no others.  Accept $\pm \frac{6.6}{\pi}$ and $\pm \frac{\sqrt{44}}{\pi}$
		<b>3</b>	

	<b>Question 3 Total</b>	<b>6</b>	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
4	$2 \frac{dv}{dt} = -0.1 \times 2 \times 9.8 - 0.49v^2$ $\frac{dv}{dt} = -0.245(4 + v^2)$ $\int \frac{1}{4 + v^2} dv = \int -0.245 dt$ $\frac{1}{2} \tan^{-1}\left(\frac{v}{2}\right) = -0.245t + c$ $t = 0, v = 20 \Rightarrow c = \frac{1}{2} \tan^{-1}(10)$ $v = 0$ $0 = -0.245t + \frac{1}{2} \tan^{-1}(10)$ $t = \frac{1}{0.49} \tan^{-1}(10) = 3.0 \text{ [s]}$	<p><b>M1</b> <b>A1</b></p> <p><b>M1</b> <b>M1</b> <b>A1</b></p> <p><b>M1</b> <b>A1</b></p> <p><b>M1</b> <b>A1</b></p>	<p>Forms a three term differential equation using <math>F = ma</math> Correct differential equation</p> <p>Separates the variables Integrates to obtain a <math>\tan^{-1}</math> term Correct integration. Condone missing constant of integration.</p> <p>Finds <math>c</math> Correct <math>c</math></p> <p>Substitutes <math>v = 0</math> Correct time. <b>AWRT</b> 3.0, such as 3.002</p>
		<b>9</b>	
	<b>Question 4 Total</b>	<b>9</b>	

Q	Answer	Marks	Comments
5(a)	$2.5 \times 9.8 = k \times 0.05$ $k = 490$	<b>M1</b>  <b>A1</b>	Equation to find stiffness Allow $5k$ instead of $0.05k$  Correct stiffness Allow 490.5 from $g = 9.81$
		<b>2</b>	

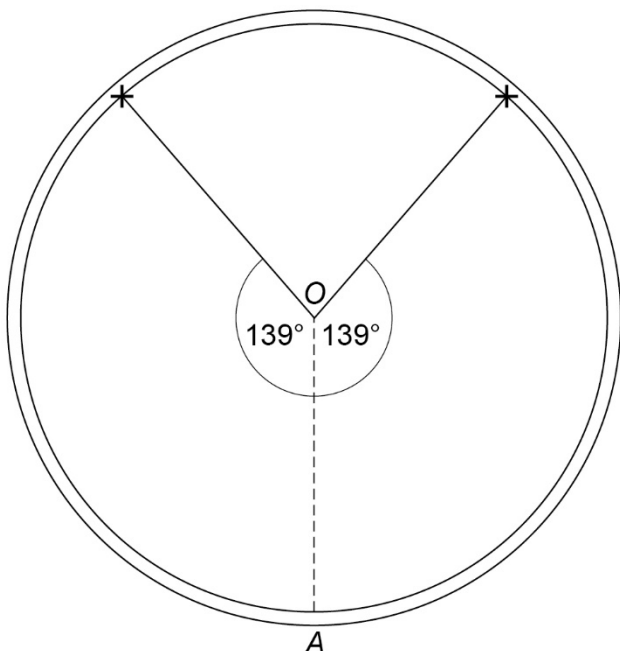
Q	Answer	Marks	Comments
5(b)(i)	$x =$ Displacement below equilibrium position $2.5 \frac{d^2x}{dt^2} = 2.5 \times 9.8 - 490(0.05 + x)$ $2.5 \frac{d^2x}{dt^2} = 24.5 - 24.5 - 490x$ $\frac{d^2x}{dt^2} = -196x$ $\frac{d^2x}{dt^2} \propto -x$ $\therefore$ Simple Harmonic Motion	<b>M1</b> <b>A1</b> <b>A1</b>  <b>A1</b>  <b>A1</b>	Forms three term differential equation At least two correct terms Correct differential equation   Correct simplified differential equation   Correct conclusion Allow 196.2 from $g = 9.81$
		<b>5</b>	

Q	Answer	Marks	Comments
5(b)(ii)	$2 = a \times 14$ $a = \frac{2}{14} = \frac{1}{7} \text{ [m]}$	<b>M1</b>  <b>A1</b>	Equation to find amplitude based on $v_{\max} = a \times \omega$ with their $\omega$  Correct amplitude Allow $\frac{2}{\sqrt{196.2}}$ from $g = 9.81$
		<b>2</b>	

Q	Answer	Marks	Comments
5(b)(iii)	$x = \frac{1}{7} \sin(14t)$ $0.1 = \frac{1}{7} \sin(14t)$ $t = 0.0554$ $\text{Period} = \frac{2\pi}{14} = \frac{\pi}{7} = 0.4488$ $\frac{4 \times 0.0554}{0.4488} \times 100 = 49\%$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Expression for the displacement Accept <math>x = A \sin(\omega t)</math> or <math>x = A \cos\left(\omega t - \frac{\pi}{2}\right)</math> with their <math>A</math> and <math>\omega</math></p> <p>Equation to find time for displacement of 10 cm with their <math>A</math> and <math>\omega</math> Allow 10 instead of 0.1</p> <p>Correct time Note <math>g = 9.81</math> gives 0.0554 to 3 sf</p> <p>Correct period, such as 0.4488 s, for their <math>\omega</math> Note 0.4486 from <math>g = 9.81</math></p> <p>Calculation to find percentage using their time</p> <p>Correct percentage Note <math>g = 9.81</math> gives 49% to 2 sf</p>
		<b>6</b>	
	<b>Question 5 Total</b>	<b>15</b>	



		<b>4</b>	
--	--	----------	--

Q	Answer	Marks	Comments
6(b)(ii)	$v^2 = ga \left( \frac{7}{2} - 2(1 - \cos \theta) \right)$ $v^2 = ga \left( \frac{3}{2} + 2 \cos \theta \right)$ $0 = \frac{3}{2} + 2 \cos \theta$ $\cos \theta = -\frac{3}{4}$ $\theta = 139^\circ \text{ or } -139^\circ$ 	<p><b>M1</b></p> <p>Substitutes for <math>U</math> and sets <math>v = 0</math></p> <p><b>A1</b></p> <p>Obtains correct value for <math>\cos \theta</math></p> <p><b>A1</b></p> <p>Obtains at least one correct value for <math>\theta</math></p> <p><b>A1</b></p> <p>Shows both positions correctly on the diagram</p> <p>Accept <math>139^\circ</math> or <math>221^\circ</math></p> <p>Accept answers in radians (<math>\pm 2.42</math> or <math>3.86</math>)</p>	
		4	

	<b>Question 6 Total</b>	<b>13</b>	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
7	<p>Max KE when rod at equilibrium position.</p> $T_1 = \frac{4mg}{d}e$ $T_2 = \frac{3mg}{d}(2d - e)$ $mg + 2 \times \frac{3mg}{d}(2d - e) = \frac{4mg}{d}e$ $d + 12d - 6e = 4e$ $e = \frac{13d}{10}$ <p>Initial EPE = <math>\frac{1}{2} \times \frac{4mg}{d} \times (2d)^2 = 8mgd</math></p> $8mgd = mg \times \frac{7d}{10} + \frac{1}{2} \times \frac{4mg}{d} \times \left(\frac{13d}{10}\right)^2 + 2 \times \frac{1}{2} \times \frac{3mg}{d} \times \left(\frac{7d}{10}\right)^2 + \text{KE}$ $8mgd = \frac{7mgd}{10} + \frac{169mgd}{50} + \frac{147mgd}{100} + \text{KE}$ $\text{KE} = \frac{49mgd}{20} = 2.45mgd$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Finds tensions in both strings at equilibrium</p> <p>Correct tensions</p> <p>Equation to find extension at equilibrium with three terms</p> <p>Correct equation</p> <p>Correct extension</p> <p>Correct initial EPE</p> <p>Five term energy equation</p> <p>At least three terms correct</p> <p>All terms correct</p> <p>Correct KE</p>
		<b>10</b>	
	<b>Question 7 Total</b>	<b>10</b>	

Q	Answer	Marks	Comments
8(a)	$25 \sin 30^\circ t - \frac{1}{2} g \cos 20^\circ t^2 = U \sin 60^\circ t - \frac{1}{2} g \cos 20^\circ t^2$ $25 \sin 30^\circ t = U \sin 60^\circ t$ $U = \frac{25 \sin 30^\circ}{\sin 60^\circ} = \frac{25\sqrt{3}}{3}$ $25 \cos 30^\circ t + \frac{1}{2} g \sin 20^\circ t^2 = 10 + U \cos 60^\circ t + \frac{1}{2} g \sin 20^\circ t^2$ $25 \cos 30^\circ t = 10 + \frac{25\sqrt{3}}{3} \cos 60^\circ t$ $\frac{25\sqrt{3}}{2} t = 10 + \frac{25\sqrt{3}}{6} t$ $t = \frac{2\sqrt{3}}{5} \text{ s}$	<p><b>M1</b> Equation for motion perpendicular to the plane</p> <p><b>A1</b> Correct equation</p> <p><b>M1</b> Solves for <math>U</math></p> <p><b>A1</b> Correct <math>U</math></p> <p><b>M1</b> Equation for motion parallel to the plane</p> <p><b>A1</b> Correct equation</p> <p><b>M1</b> Solves for <math>t</math></p> <p><b>A1</b> Any correct version of <math>t</math> AWRT 0.69</p> <p><b>A1</b> Correct <math>t</math> in exact form</p>	
		<b>9</b>	

Q	Answer	Marks	Comments
8(b)	$y_{\max} = 25 \sin 30^\circ \times \frac{2\sqrt{3}}{5} - \frac{1}{2} g \cos 20^\circ \left( \frac{2\sqrt{3}}{5} \right)^2$ $= 6.5 \text{ m}$	<p><b>M1</b> Substitutes their time into correct equation</p> <p><b>A1</b> Obtains correct height. <b>AWFW</b> 6.4 to 6.5, such as 6.450097</p>	
		<b>2</b>	

	<b>Question 8 Total</b>	<b>11</b>	
--	-------------------------	-----------	--