OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 17 January 2020 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working: otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) (i) Express $5\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α to the nearest 0.1°

[3 marks]

Answer

1 (a) (ii) Hence solve the equation

$$5\sin\theta - 12\cos\theta = -1$$

in the interval $-180° < \theta < 180°$, giving all solutions to the nearest 0.1°

[3 marks]

Answer

Do not write outside the box

1 (b) Solve the equation

$$2 \cot^2 x = 10 - 5 \operatorname{cosec} x$$

in the interval $-90° < x < 270°$, giving all solutions to the nearest degree.

[5 marks]

Answer

11

Turn over ►

Do not write outside the box

2 The function f is defined by

$$f(x) = \sin^{-1}(2x - 1)$$

2 (a) State the largest possible domain of f.

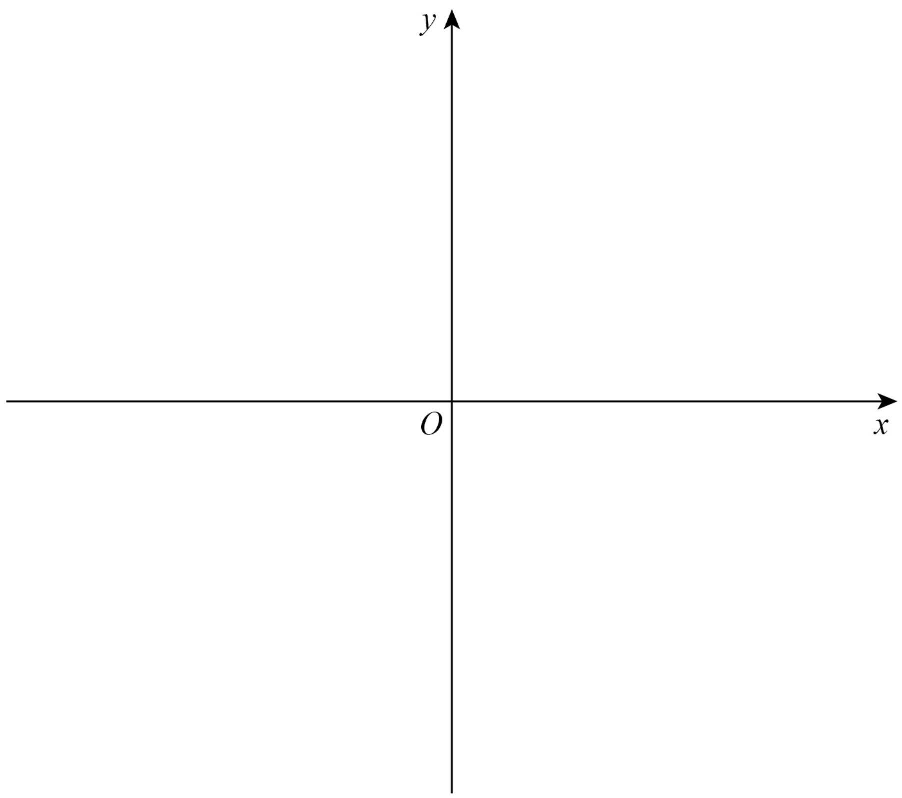
[1 mark]

Answer ______________________

2 (b) On the axes below, sketch the graph of $y = f(x)$.

Show the coordinates of the end-points on the graph.

[2 marks]



Do not write outside the box

2 (c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sin^{-1} x$ onto the graph of $y = \sin^{-1}(2x - 1)$

[4 marks]

2 (d) The root of the equation $\sin^{-1}(2x - 1) + x - 1 = 0$ is α.

Show that α lies between 0.6 and 0.7

[2 marks]

Question 2 continues on the next page

Do not write outside the box

2 (e) Use the iterative formula

$$x_{n+1} = \frac{1 + \sin(1 - x_n)}{2} \quad \text{with } x_1 = 0.6$$

to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

Answer ______________________

11

Do not write outside the box

3 (a) Find $\int x \ln x \, dx$

[3 marks]

Answer ______________________

3 (b) Find $\int \ln x \, dx$

[3 marks]

Answer ______________________

6

Turn over ►

Do not write outside the box

4 (a) The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = 8x^3 + bx^2 + cx + 6$$

where b and c are constants.

When $\mathrm{f}(x)$ is divided by $(2x - 1)$ the remainder is 5.25

When $\mathrm{f}(x)$ is divided by $(2x - 3)$ the remainder is –3.75

Find the value of b and the value of c.

[4 marks]

$b =$ ______________ $c =$ ______________

Do not write outside the box

4 (b) Show that $\dfrac{12x^3 - 8x^2 + x + 7}{4x^2 - 1}$ can be written in the form $3x + d + \dfrac{ex + f}{4x^2 - 1}$

where d, e and f are integers.

[4 marks]

8

Turn over ►

Do not write outside the box

5 (a) Express $\dfrac{12}{9-u^2}$ in the form $\dfrac{A}{3-u}+\dfrac{B}{3+u}$

[2 marks]

Answer

Do not write outside the box

5 (b) Use the substitution $u = \sin x$ to find the exact value of $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{12\cos x}{8+\cos^2 x}\,dx$

Give your answer in the form $\ln q$, where q is a rational number.

[6 marks]

Answer

8

6 (a) (i) Find the binomial expansion of $(1 + 2x)^{0.5}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer ______________________________

6 (a) (ii) Find the binomial expansion of $(1 - 4x)^{-0.5}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer ______________________________

6 (b) (i) Hence find the binomial expansion of $\sqrt{\dfrac{1+2x}{1-4x}}$ in ascending powers of x up to and including the term in x^2

[2 marks]

Answer ______________________________

Do not write outside the box

6 (b) (ii) State the values of x for which the expansion of $\sqrt{\dfrac{1+2x}{1-4x}}$ is valid.

[1 mark]

Answer ____________________

6 (c) Use your expansion of $\sqrt{\dfrac{1+2x}{1-4x}}$ to find an estimate for $\sqrt{2}$, giving your answer to three decimal places.

[3 marks]

Answer ____________________

10

Turn over ►

Do not write outside the box

7 A curve has equation $y = e^{3x} - 24x$

7 (a) Find an equation of the tangent to the curve at (0, 1)

[3 marks]

Answer ______________________

7 (b) Find the coordinates of the stationary point of the curve, giving your answer in an exact form.

[3 marks]

Answer ______________________

Do not write outside the box

7 (c) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of this stationary point.

[3 marks]

Answer ______________________

9

Turn over for the next question

Do not write outside the box

8 During a scientific experiment a liquid evaporates.

The rate of change of the evaporated mass, x grams, is directly proportional to the mass of the liquid which remains at time t minutes after the start of the experiment.

Initially there were 80 grams of the liquid.

8 (a) Explain briefly why this information can be represented by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(80 - x)$$

[1 mark]

8 (b) After 60 minutes, 30 grams of the liquid has evaporated.

Solve the differential equation in part **(a)** to find the value of k.

[5 marks]

Answer ______________________

Do not write outside the box

8 (c) (i) Find the number of grams of the liquid that has evaporated after 120 minutes.

[2 marks]

Answer ______________________

8 (c) (ii) Find how many minutes it takes for there to be only 10 grams of the liquid remaining.

[3 marks]

Answer ______________________

11

Do not write outside the box

9 The region bounded by the curve $y = \dfrac{1}{2+x}$, the lines $y = 0.2$, $y = 0.25$ and the y-axis is rotated through 2π radians about the ***y*-axis** to form a solid.

Find the exact value of the volume of the solid generated.

[6 marks]

Answer ______________________

6

Do not write outside the box

10 (a) Use the mid-ordinate rule with six strips, to find an estimate for $\int_{1.5}^{3} x^{-x}\,\mathrm{d}x$, giving your answer to three decimal places.

[4 marks]

Answer ______________________

10 (b) By taking logarithms of both sides of $y = x^{-x}$ and then using implicit differentiation, find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in terms of x only.

[4 marks]

Answer ______________________

8

Do not write outside the box

11 The point A has coordinates (10, 2, –3).

The point B has coordinates (2, –2, 5).

11 (a) Find the vector equation of the line AB and hence show that it can be written as

$$\mathbf{r} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

[4 marks]

Answer

11 (b) The point D has coordinates (–2, 1, 7).

The point C lies on the line AB.

The line CD is perpendicular to the line AB.

11 (b) (i) Find the coordinates of C.

[5 marks]

Do not write outside the box

Answer

11 (b) (ii) Show that the distance $CD = \sqrt{q}$ where q is a constant.

[2 marks]

11 (c) The point $P\,(4 + 2p,\ -1 + p,\ 3 - 2p)$ lies on the line AB such that triangle DCP is isosceles.

Find the possible exact values of p, giving your answers in the form $s + t\sqrt{q}$, where s and t are constants.

[5 marks]

Answer

16

Do not write outside the box

12 A curve has equation $6y^2 + 2e^{4x} = y^3 e^x$

12 (a) Given that the curve has a stationary point (p, q), show that $2e^p = q$

[5 marks]

12 (b) Find the exact value of p and the exact value of q.

[4 marks]

Answer ______________________

9

Do not write outside the box

13 A curve C is defined by the parametric equations $x = at^2$, $y = 2at$, where a is a constant.

The tangent to the curve at the point $P\ (ap^2, 2ap)$ meets the tangent to the curve at the point $Q\ (aq^2, 2aq)$ at the point R.

Given that p and q vary so that $p^2 + q^2 = 1$, find the Cartesian equation of the curve on which R lies, giving your answer in the form $y^2 = \mathrm{f}(x)$

[7 marks]

Answer ________________________

7

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	Additional page, if required. Write the question numbers in the left-hand margin.

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED