OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Thursday 16 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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10	
11	
12	
TOTAL	

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point (7.0, 4.3) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{x} + y}{y\sqrt{y}}$$

Use Euler's step-by-step method with a step length of 0.25 to estimate the value of y when $x = 7.5$

Give your answer to three decimal places.

[5 marks]

Answer _____

5

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2 The matrix $\mathbf{B}$ is defined by $\mathbf{B} = \begin{bmatrix} 1 & k \\ 3 & 2 \end{bmatrix}$

2 (a) Find the value of k for which $\mathbf{B}$ is singular.

[2 marks]

Answer _____

2 (b) Given that $\mathbf{B}$ is **not** singular, and that

$$\mathbf{BM} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix}$$

find the matrix $\mathbf{M}$, giving your answer in terms of k.

[5 marks]

Answer _____

7

Turn over ►

3 The variables x and y are related by an equation of the form

$$y^3 = ax^2 + b$$

where a and b are constants.

The following experimental values of x and y are recorded.

x	2	4	6	8	10
y	8.6	8.1	7.4	5.8	–4.2

3 (a) Complete the table below for $X = x^2$ and $Y = y^3$

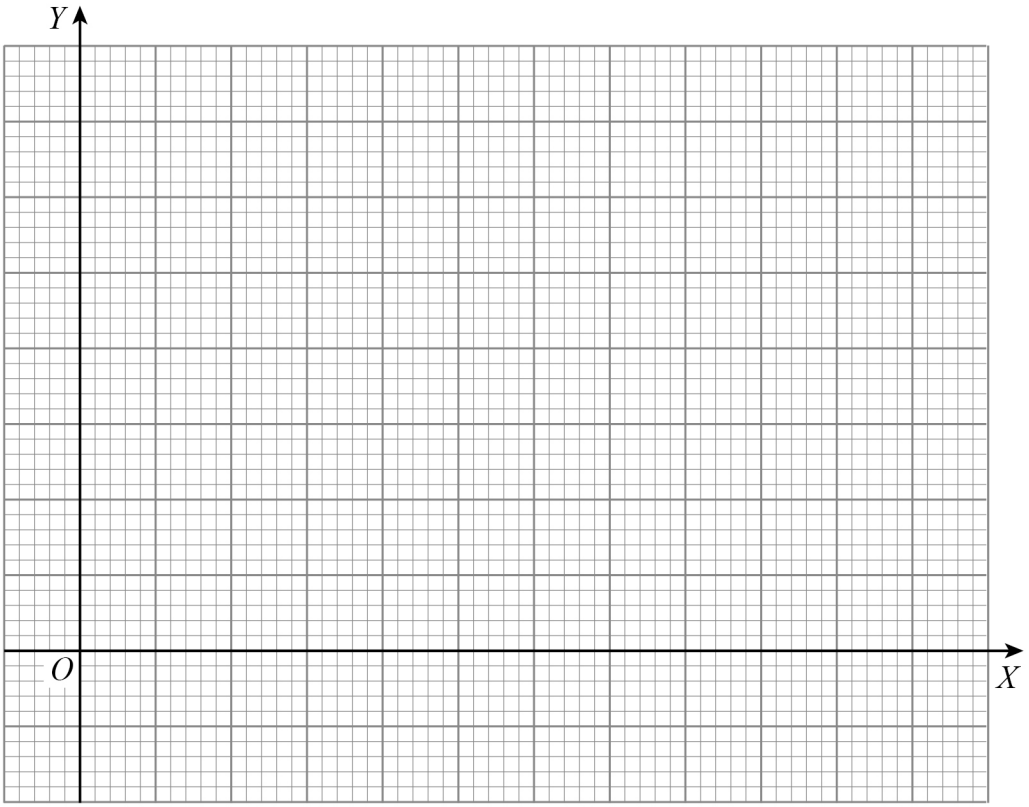
X	4	16	36	64	100
Y	636				

[1 mark]

3 (b) Plot the values of Y against X on the grid below.

Draw a line of best fit.

[2 marks]



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3 (c) Use your graph to estimate the value of x when $y = 0$

[2 marks]

Answer _____

3 (d) Use your graph to estimate the value of a and the value of b.

[3 marks]

$a =$ _____ $b =$ _____

8

Turn over ►

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4 The function f is defined by $f(x) = x^3 - 2x^2 - 3x + 1$

The equation $f(x) = 0$ has one negative root, α.

4 (a) Show that α lies in the interval $-1.5 < \alpha < -1$

[2 marks]

4 (b) Taking $x_1 = -1.5$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α.

Give your answer to three decimal places.

[4 marks]

Answer _____

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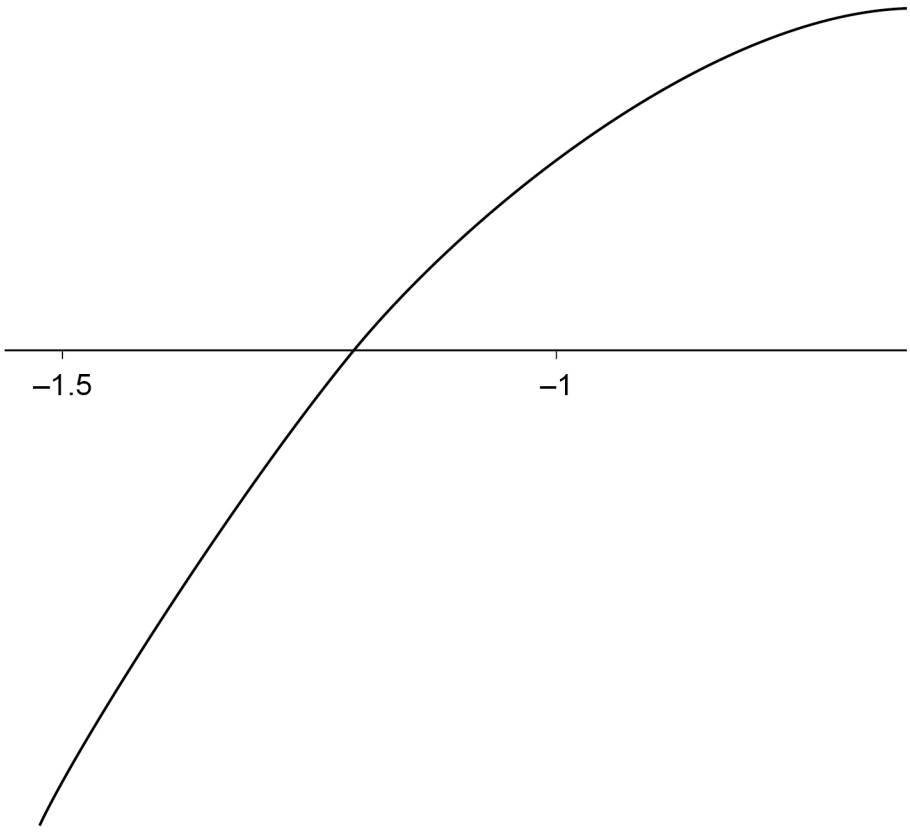
4 (c) The diagram shows part of the graph of $y = \mathrm{f}(x)$



4 (c) (i) Draw a line on the diagram above which illustrates the Newton–Raphson method as used in part **(b)**.

[1 mark]

4 (c) (ii) The point P is $(x_2, 0)$

The point Q is $(\alpha, 0)$

Label the points P and Q on the diagram above.

[2 marks]

9

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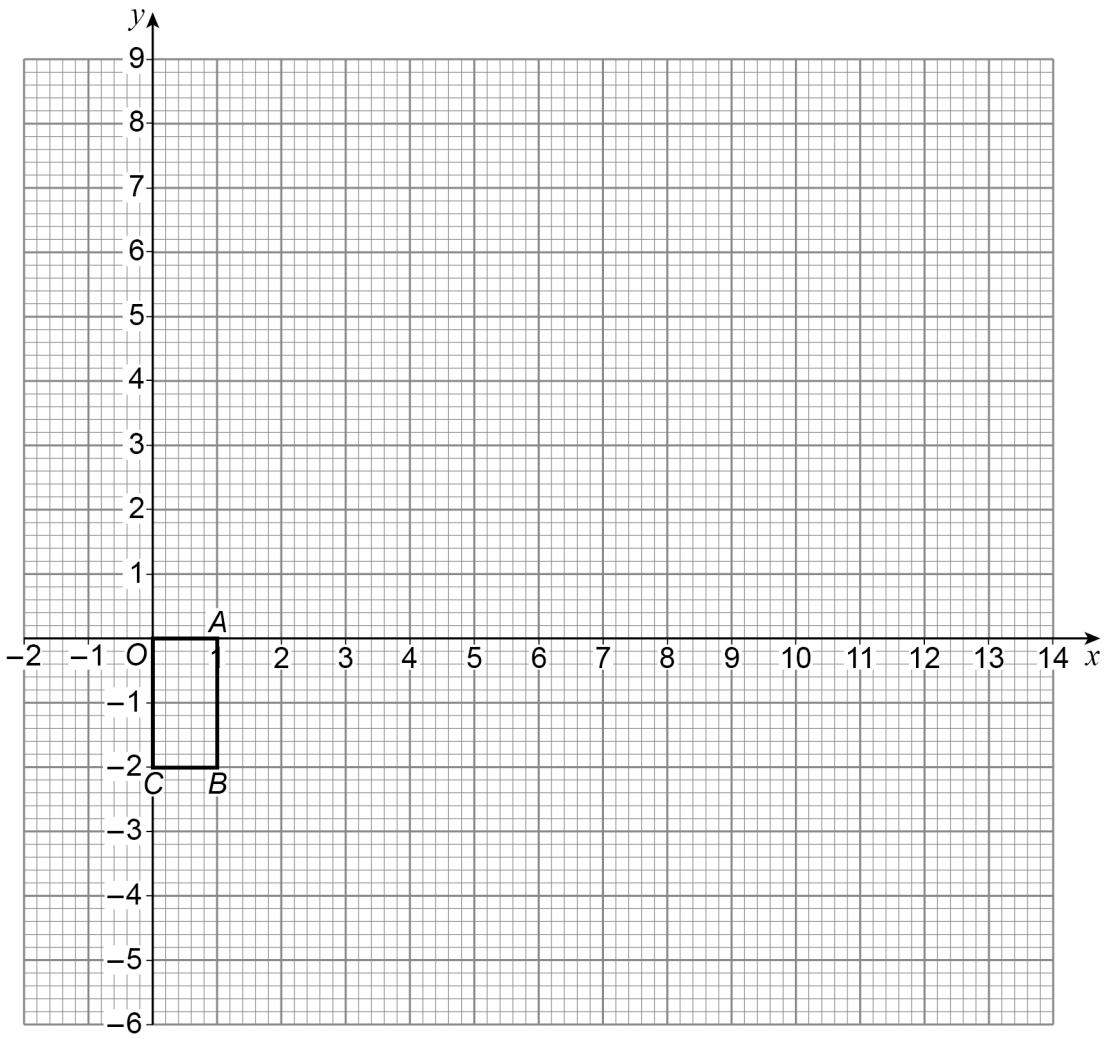
5 The transformation T is a shear which is not parallel to the x-axis or the y-axis.

T is represented by the matrix $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

5 (a) T transforms the rectangle $OABC$ shown on the diagram to $OA'B'C'$

Draw $OA'B'C'$ on the diagram, labelling each of the points A', B' and C' clearly.

[2 marks]



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5 (b) Show that the area of the quadrilateral $OA'B'C'$ is 2 square units.

[2 marks]

5 (c) The line $y = mx$ is a line of invariant points of T.

Show that $m = \frac{1}{2}$

[2 marks]

Question 5 continues on the next page

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5 (d) T maps the line $y = kx$ to the line $y = 3x$

Find the value of k.

[5 marks]

Answer _____

11

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The discrete random variable X is binomially distributed such that $X \sim \mathrm{B}(n, p)$

Show that the probability generating function of X is given by

$$G_X(t) = (1 - p + pt)^n$$

[3 marks]

3

Turn over for the next question

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7 The random variable D is a discrete uniform distribution which can take the values 1, 2, 3, …, n

7 (a) Given that $\mathrm{E}(D) = \frac{n+1}{2}$, prove that

$$\mathrm{Var}(D) = \frac{n^2-1}{12}$$

[4 marks]

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7 (b) The random variable D represents the value on the upper face of a dice after it is rolled.

The dice has n faces which are labelled with the integers 1, 2, 3, …, n

The variance of D is 33.25

7 (b) (i) Find the value of n.

[2 marks]

Answer _____

7 (b) (ii) Find $\mathrm{P}(D > 18)$.

[1 mark]

Answer _____

7 (b) (iii) The dice is rolled until a value greater than 18 is obtained.

Find the probability that the dice is rolled exactly 5 times.

[2 marks]

Answer _____

9

Turn over ►

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8 The discrete random variable X has mean 6 and standard deviation 0.5

The discrete random variable Y has mean 8 and standard deviation 0.4

The correlation between X and Y is 0.25

8 (a) Show that

$$\mathrm{Var}\left(aX + (1-a)Y\right) = 0.31a^2 - 0.22a + 0.16$$

[4 marks]

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8 (b) The value of a is chosen so that $\mathrm{Var}\left(aX+(1-a)Y\right)$ is a minimum.

8 (b) (i) Find the exact value of a.

[3 marks]

Answer _____

8 (b) (ii) Find the value of $\mathrm{E}\left(aX+(1-a)Y\right)$.

[1 mark]

Answer _____

8

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 For a body moving around a circle at constant speed, the magnitude of its acceleration, a, is given by

$$a = r\omega^2$$

where r is the radius of the circle.

Find the dimensions of ω.

[3 marks]

3

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10 A circular disc, of mass 0.2 kg, is sliding on a smooth horizontal surface when it hits a vertical wall and rebounds.

The wall is perpendicular to the path of the disc.

The disc hits the wall with speed 8 m s^{-1} and rebounds with speed 5 m s^{-1}

10 (a) Find the coefficient of restitution between the disc and the wall.

[1 mark]

Answer _____

10 (b) Find the magnitude of the impulse on the disc due to the wall.

[2 marks]

Answer _____

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10 (c) The disc is in contact with the wall for 0.25 seconds.

A simple model assumes that the wall exerts a constant force on the disc.

Find the magnitude of this force.

[2 marks]

Answer _____

5

Turn over for the next question

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11 A sailing boat is initially 5 km due north of a patrol boat.

The sailing boat is travelling north-east at a speed of 4 m s^{-1}

The patrol boat travels at 9 m s^{-1} on a bearing of α degrees, so that it intercepts the sailing boat in the shortest possible time.

11 (a) Find α, giving your answer to the nearest integer.

[4 marks]

Answer _____

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11 (b) Find the time that it takes for the patrol boat to reach the sailing boat.

[3 marks]

Answer _____

7

Turn over for the next question

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12 Two smooth spheres, A and B, are moving directly towards each other and collide.

A has mass 2 kg and is moving at 5 m s^{-1} before the collision.

B has mass 3 kg and is moving at 2 m s^{-1} before the collision.

During the collision B exerts an impulse of magnitude 9 N s on A.

Find the coefficient of restitution between A and B, giving your answer as a fraction.

[5 marks]

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Answer _____

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END OF QUESTIONS

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