OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Friday 24 January 2020 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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ANSWER IN THE SPACES PROVIDED

Answer **all** questions in the spaces provided.

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1 A spring has stiffness 14.7 N m^{-1} and natural length 20 cm

One end of the spring is attached to a fixed point *O*.

A particle of mass 0.6 kg is attached to the other end of the spring.

Find the length of the spring when the particle is in equilibrium directly below *O*.

[3 marks]

Answer _____

3

Turn over for the next question

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2 A particle moves with simple harmonic motion between two end points, *A* and *B*, that are 3 metres apart.

The particle takes 2 seconds to move directly from *A* to *B*.

2 (a) Find the maximum speed of the particle.

[3 marks]

Answer _____

2 (b) Find the speed of the particle when it is at the point *C,* which is 1 metre from *A*.

[3 marks]

Answer _____

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2 (c) Find the time taken for the particle to move directly from A to C.

[3 marks]

Answer _____

9

Turn over for the next question

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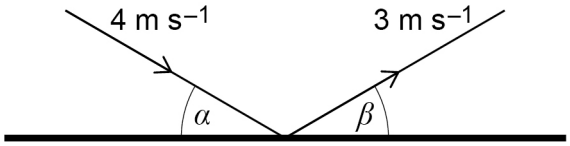
3 A disc, of mass 0.2 kg, moving on a smooth horizontal surface hits a smooth vertical wall.

When it hits the wall, the disc is moving at 4 m s^{-1} at an angle α to the wall.

The disc rebounds with a speed of 3 m s^{-1} at an angle β to the wall.



The coefficient of restitution between the disc and the wall is e.

3 (a) Show that

$$\tan\alpha = \frac{\tan\beta}{e}$$

[4 marks]

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3 (b) It is given that $\alpha = 2\beta$, where $0° < \beta < 45°$

3 (b) (i) Find $\tan \beta$ in terms of e.

[3 marks]

Answer _____

3 (b) (ii) Find the set of possible values of e.

[1 mark]

Answer _____

8

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4 A particle, of mass 2 kg, slides in a straight line on a horizontal surface.

Initially, at point *A*, the particle has speed 12 m s^{-1}

At time t seconds after leaving *A*, the speed of the particle is v m s^{-1}

The coefficient of friction between the particle and the surface is 0.2

Air resistance also acts on the particle with a magnitude of $4v$ newtons.

The particle comes to rest at point *B*.

4 (a) Show that, as the particle moves between *A* and *B*, the speed of the particle is given by

$$v = 12.98e^{-2t} - 0.98$$

[6 marks]

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4 (b) Show that the distance *AB* is 4.73 metres, correct to three significant figures.

[6 marks]

12

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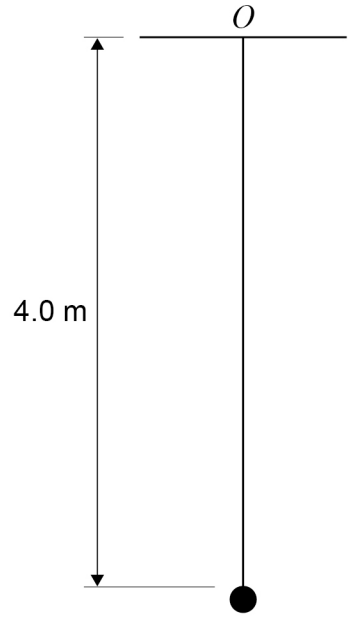
5 A light elastic string has natural length 2.5 metres and modulus of elasticity 10 newtons.

One end of the string is attached to a fixed point, *O*. The other end of the string is attached to a small sphere of mass 0.4 kg

The sphere is held at a point 4 metres below *O* and then released from rest.

The string remains taut in the subsequent motion.



5 (a) Find the minimum distance between *O* and the sphere, giving your answer to three significant figures.

[5 marks]

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Answer _____

5 (b) Find the extension of the string when the speed of the sphere is a maximum.

[3 marks]

Answer _____

Question 5 continues on the next page

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5 (c) Show that the motion of the sphere is simple harmonic motion.

[5 marks]

5 (d) Find the period of the motion.

[2 marks]

Answer _____

15

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Turn over for the next question

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6 A light inextensible string has length 80 cm

One end of the string is attached to a fixed point, *O*.

A particle, of mass 0.2 kg, is attached to the other end of the string.

Initially the particle is at rest directly below *O*.

The particle is then subject to a horizontal impulse so that it starts to move with speed u m s^{-1}

6 (a) In one case the particle completes vertical circles with centre *O*.

Show that the minimum value of u is 6.3 , correct to two significant figures.

[4 marks]

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6 (b) In a **different** case the string becomes slack when the string makes an angle of 30° with the upward vertical.

Find u.

[6 marks]

10

Turn over for the next question

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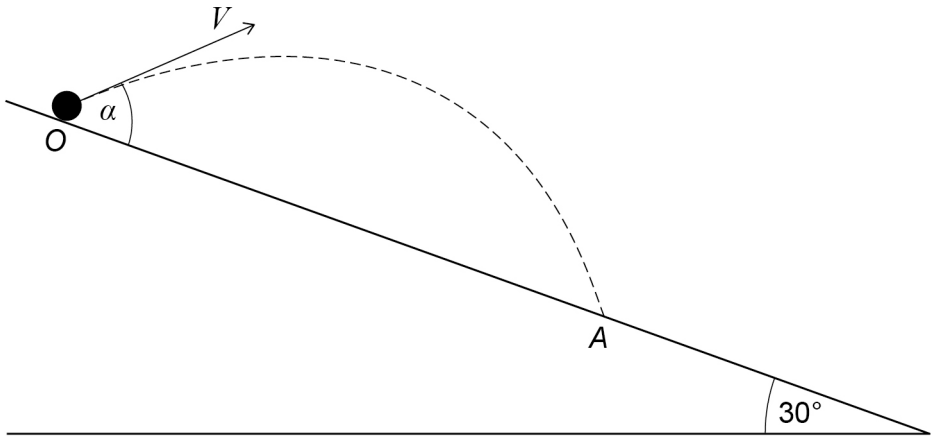
7 A plane is inclined at an angle of 30° to the horizontal.

A ball is projected from the point *O* on the plane and hits the plane again at the point *A*, which is further down the plane than *O*. *OA* is a line of greatest slope of the plane.

The initial velocity of the ball is V m s^{-1} at an angle α above the plane.



7 (a) Show that the distance *OA* is given by

$$\frac{2V^2}{g\cos^2 30°}\sin\alpha\cos(\alpha - 30°)$$

[7 marks]

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7 (b) Find the value of α for which the distance OA is a maximum.

[3 marks]

Answer _____

10

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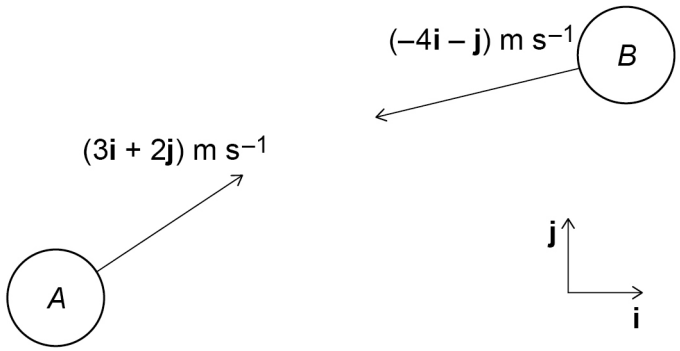
8 Two smooth spheres, *A* and *B*, are moving on a smooth horizontal surface when they collide.

The two spheres have the same radius.

The mass of *A* is 2 kg and the mass of *B* is 4 kg

Before the collision the velocity of *A* is $\left(3\mathbf{i}+2\mathbf{j}\right)\ \text{m s}^{-1}$

Before the collision the velocity of *B* is $\left(-4\mathbf{i}-\mathbf{j}\right)\ \text{m s}^{-1}$



After the collision the velocity of *A* is $\left(-1.5\mathbf{i}-\mathbf{j}\right)\ \text{m s}^{-1}$

8 (a) Find the velocity of *B* after the collision.

[3 marks]

Answer _____

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8 (b) Find the magnitude of the impulse on A during the collision.

[3 marks]

Answer _____

8 (c) Find the coefficient of restitution between the spheres, giving your answer as a fraction.

[7 marks]

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Answer _____

13

END OF QUESTIONS

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