

## INTERNATIONAL AS MATHEMATICS MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2020

Version: V1 Final Mark Scheme

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## Key to mark scheme abbreviations

M Mark is for method

m Mark is dependent on one or more M marks and is for method

A Mark is dependent on M or m marks and is for accuracy

B Mark is independent of M or m marks and is for method and accuracy

E Mark is for explanation

√or ft Follow through from previous incorrect result

**CAO** Correct answer only

**CSO** Correct solution only

**AWFW** Anything which falls within

**AWRT** Anything which rounds to

**ACF** Any correct form

AG Answer given

**SC** Special case

**oe** Or equivalent

A2, 1 2 or 1 (or 0) accuracy marks

**−x EE** Deduct *x* marks for each error

NMS No method shown

PI Possibly implied

SCA Substantially correct approach

**sf** Significant figure(s)

**dp** Decimal place(s)

| Q    | Answer   | Marks      | Comments                                    |
|------|--|------------|---|
|      |  |            |   |
| 1(a) | p-q  | B1         |   |
| 1(b) | $\log_3(5^2 \times 2)$   |            |   |
|      | or   | B1         | PI  |
|      | $\log_3(5\times5\times2)$  |            |   |
|      | $\log_3(5\times5\times2)$ $2\log_35 + \frac{1}{2}\log_34$              |            |   |
|      | or   | M1         | Complete, correct application of log rules. |
|      | $\log_{3} 5 + \log_{3} 5 + \frac{1}{2} \log_{3} 4$ $2p + \frac{1}{2}q$ |            |   |
|      | $2p+\frac{1}{2}q$  | <b>A</b> 1 |   |
|      | Total  | 4          |   |

| Q    | Answer  | Marks      | Comments   |
|------|---|------------|--|
| 2(a) | $\frac{\sin\beta}{4} = \frac{\frac{3}{7}}{6} \text{ oe}$          |            |  |
|      | and $\sin \beta = 4 \times \frac{3/7}{6} \text{ oe}$              | М1         | Use of sine rule with values substituted and correct unsimplified rearrangement. |
|      | $\sin\beta = \frac{2}{7}$   | <b>A</b> 1 | AG   |
| 2(b) | $\cos^2\beta + \sin^2\beta = 1$                                   | M1         | Stated or used, PI   |
|      | $\cos^2 \beta = \frac{45}{49}$ $\cos \beta = \frac{3\sqrt{5}}{7}$ | m1         | Correct substitution and rearrangement   |
|      | $\cos \beta = \frac{3\sqrt{5}}{7}$                                | <b>A</b> 1 | CSO  |
|      | Total   | 5          |  |

| Q     | Answer  | Marks      | Comments   |
|-------|---|------------|--|
|       | ( - 2 ( - 2   |            |  |
| 3(a)  | $(\sqrt{17})$ + $(\sqrt{17})$   | М1         | oe   |
|       | $\left(\sqrt{17}\right)^2 + \left(\sqrt{17}\right)^2$ $\left(\left(\sqrt{17}\right)^2 + \left(\sqrt{17}\right)^2 = \right)34$ | <b>A</b> 1 |  |
|       | $(x-4)^2 + (y-1)^2 = 34$  | B1         | For correct LHS  |
|       | (x-4) + (y-1) = 34  | B1ft       | ft their 34 provided M1 scored.<br>Equation must be in the correct form                      |
| 3(b)  | $(x-1)^2 + (y-3)^2 = 34$  | B2ft       | B1 for each correct bracketed term in an equation of the correct form or $(x-7)^2 + (y+1)^2$ |
|       |   |            | ft their 34  |
| 3 (c) | $(x-6)^2-36+(y-2)^2-4+2=0$  | M1         | Completes the square twice   |
|       | $(x-6)^{2}-36+(y-2)^{2}-4+2=0$ $((x-6)^{2}+(y-2)^{2}=)38$   | <b>A</b> 1 | Finding correct radius squared for the circle. PI by correct radius.                         |
|       | The two circles have different radii  | E1         | E1 for comparing radii in the context of the question. E1 for statement on Jane's claim.     |
|       | (Hence) Jane is not correct   | E1         | Stating she is not correct and giving a correct reason. No working seen scores E0E0          |
|       | Total   | 10         |  |

| Q | Answer   | Marks      | Comments                                   |
|---|--|------------|--|
| 4 | $3.6 = \frac{1}{2} \times r^2 \times 0.8$                    | M1         | oe. Use of $A = \frac{1}{2}r^2\theta$      |
|   | r=3  | <b>A</b> 1 |  |
|   | OD = 12  | B1         | PI   |
|   | $(CD^2 =) 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos 0.8$ | M1         | Correct substitution into Cosine Rule      |
|   | $(CD^2 =) 79.674233$   | M1         | Correct order of evaluation to find $CD^2$ |
|   | (CD =) 8.93  | <b>A</b> 1 | AWRT                                       |
|   | Total  | 6          |  |

| Q    | Answer  | Marks      | Comments   |
|------|---|------------|--|
| 5(a) | Exponential curve and increasing function in the first and second quadrants with the correct form, asymptotic to the negative $x$ –axis from above. | В1         |  |
|      | 15 marked on $y$ -intercept of curve on positive $y$ —axis.   | В1         | Condone correct coordinates  |
| 5(b) | $\log_5\left(15\times7^x\right) = \log_5625^{2x}$   | M1         | Forms correct equation in $x$ .<br>Condone missing 5 in $\log_5$   |
|      | $\log_5 15 + \log_5 7^x$  | M1         | Correct use of $log(ab) = log(a) + log(b)$<br>Condone missing 5 in $log_5$   |
|      | $x \log_{5} 7$ or $2x \log_{5} 625$ or $8x \log_{5} 5$ or $4x \log_{5} 25$ or $8x$  | <b>M</b> 1 | Correct use of $\log(a^b) = b \log(a)$ for at least 1 term.  Condone missing 5 in $\log_5$                                   |
|      | $1 + \log_5 3 + x \log_5 7 = 8x$  | <b>A</b> 1 | Completely correct unsimplified linear equation in $x$ with $\log_5 5$ replaced by 1 Condone missing 5 in $\log_5$           |
|      | $x(8-\log_5 7) = 1 + \log_5 3$  | M1         | Correct rearrangement of their equation with $x$ factorised out. Must be seen as an equation.  Condone missing 5 in $\log_5$ |
|      | $x = \frac{1 + \log_5 3}{8 - \log_5 7}$   | <b>A</b> 1 | Must see all 5s in log <sub>5</sub>  |
|      | Total   | 8          |  |

| Q | Answer   | Marks      | Comments   |
|---|--|------------|--|
| 6 | $5\sin x (3\cos x - 2)$ or $4(3\cos x - 2)$ or $4(2-3\cos x)$                            | M1         | Attempt at one factorisation.  |
|   | $5\sin x (3\cos x - 2) = 4(2 - 3\cos x)$ or $5\sin x (3\cos x - 2) + 4(3\cos x - 2) = 0$ | M1         | Both sides of equation factorised correctly. Maybe sum of two brackets set equal to zero   |
|   | $(5\sin x + 4)(3\cos x - 2) = 0$   | <b>A</b> 1 | Factorised form set equal to zero.<br>PI by $5\sin x + 4 = 0$<br>and $3\cos x = 2$   |
|   | $\sin x = -\frac{4}{5}$ $\cos x = \frac{2}{3}$   | m1<br>A1   | m1 for one correct<br>A1 for both correct<br>PI by $x = -0.927$ or $-2.214$ (or more accurate)<br>PI by $x = 0.841$ or $-0.841$ (or more accurate)   |
|   | x = -0.927, -2.214, 0.841, -0.841  | В2         | Condone more accurate answers.  (-0.927295, -2.21429, ±0.841068)  B2 for exactly 4 answers to the correct accuracy  B1 for at least 2 answers to the correct accuracy  Ignore answers outside of the interval. If more than two answers for each inside the interval, -1 for each extra from Bs to a min of 0. |
|   | Total  | 7          |  |

| Q    | Answer   | Marks      | Comments  |
|------|--|------------|---|
|      |  |            |   |
| 7(a) | 0.251  | B1         | AWRT  |
| 7(b) | $P(W < 3) = P(W \le 2)$  | M1         | Attempts to find correct probability Allow for 0.833  |
|      | = 0.167  | <b>A</b> 1 | AWRT  |
| 7(c) | Not a good model,  | E1         | Concludes not a good model and any reason, even incorrect   |
|      | Probability of winning unlikely to be constant from game to game | E1         | Comment about probability not being constant or winning in one match unlikely to be independent of winning in another match |
|      | Total  | 5          |   |

| Q           | Answer   | Marks      | Comments   |
|-------------|--|------------|--|
|             |  |            |  |
| 8(a)        | $E(Y^2) = (3 \times 1^2 + 1)^2 \times 0.4 + (3 \times 2^2 + 2)^2 \times 0.25 + (3 \times 3^2 + 3)^2 \times 0.35$ | М1         | Applies formula for $E(Y^2)$<br>Implied by sight of 370.4 or 1852/5 oe |
|             | $Var(Y) = E(Y^2) - (E(Y))^2$<br>= 370.4 - 15.6 <sup>2</sup>  | M1         | Applies formula for $Var(Y)$<br>Condone applied to $X$ instead         |
|             | = 127.04   | <b>A</b> 1 | Accept 3176/25 oe  |
| 8(b)        | $Var(0.5Y - 6) = 0.5^2 Var(Y)$   | M1         | Applies formula for Var (a $Y$ + b)                                    |
| <b>O(D)</b> | = 31.76  | A1ft       | Accept 794/25 oe Follow through their Var ( <i>Y</i> )                 |
| 8(c)        | $E\left(\sum_{i=1}^{3} Y_{i}\right) = 3E(Y)$   | M1         | Applies formula for $E \Biggl( \sum_{i=1}^{3} Y_{i} \Biggr)$           |
|             | = 46.8   | <b>A</b> 1 |  |
|             | Total  | 7          |  |

| Q    | Answer  | Marks | Comments  |
|------|---|-------|---|
| 9(a) | $P(B \cup S) = 0.7$<br>P(S) = 0.4   | B1    | States $P(B \cup S)$ and $P(S)$<br>Accept $P(B \cup S) = 7/10$ oe and $P(S) = 4/10$ oe<br>Also award for $P(S) = 0.4$ and number of customers going to the bank and supermarket is 35 (35 might be implied by Venn diagram) |
|      | $P(B \cap S) = 0.04$  | B1    | $P(B \cap S) = 2/50$ oe   |
|      | $P(B \cup S) = P(B) + P(S) - P(B \cap S)$ $0.7 = P(B) + 0.4 - 0.04$                 | M1    | Uses Addition formulae to find P(B) Also award for number of customers going to the bank = 35 - 20 + 2 = 17   |
|      | P(B) = 0.34   | A1    | Accept 17/50 oe   |
|      | $P(B) \times P(S) = 0.34 \times 0.4 = 0.136$  | A1ft  | Multiplies P(B) and P(S) Follow through their P(B) and P(S)   |
|      | $P(B \cap S) \neq P(B) \times P(S)$<br>Therefore events B and S are not independent | E1    | Mathematically statement of dependence and conclusion   |
| 9(b) | $P(S B') = \frac{20-2}{20-2+15}$ or $\frac{0.36}{0.66}$                             | M1    | Applies conditional probability formula   |
|      | $=\frac{6}{11}$   | A1    | OE<br>Do not accept rounded decimals  |
|      | Total   | 8     |   |

| Q  | Answer                                   | Marks      | Comments  |
|----|--|------------|---|
| 10 | $10.4 = 0.4 \times v - 0.4 \times (-20)$ | M1         | Allow for $10.4 = 0.4 \times v - 0.4 \times (20)$ |
|    | v = 6                                    | <b>A</b> 1 |   |
|    | Total                                    | 2          |   |

| Q     | Answer                          | Marks | Comments   |
|-------|---------------------------------|-------|--|
|       |                                 |       |  |
| 11(a) | $2 + 36t - 6t^2$                | B2    | Three term expression with one error scores B1.  |
| 11(b) | 36 - 12t = 0                    | M1    | Differentiating their $v$ and setting equal to zero or attempt to complete the square to obtain $a(t-3)^2 + b$ . |
|       | <i>t</i> = 3                    | A1ft  | For value of $t$ for which $v$ is a maximum. ft their $v$ from part (a) provided B1 scored.                      |
|       | $7+2(3)+18(3^2)-2(3^3)$ (= 121) | M1    | Substituting their $t = 3$ into the expression for the displacement  |
|       | 121–7                           | M1    | Subtracting 7 from their displacement at $t = 3$   |
|       | 114                             | A1    | CAO  |
|       | Total                           | 7     |  |

| Q            | Answer   | Marks      | Comments  |
|--------------|--|------------|---|
| 12(a)        | R-2g=10  | M1         | Three term equation of motion ignoring signs with $20g$ or 196 and $20 \times 0.5$ or 10  |
|              | R = 206  | <b>A</b> 1 | For correct R   |
| 12(b)        | 11000 - (900 + 20x)g = 0.5(900 + 20x) or                           | M1         | oe. Equation of motion, ignoring signs, including consideration of total mass of lift and $x$ boxes.  May be seen as an inequality, ignoring signs. Condons                           |
|              | $11000 - (900 + 20x)g \ge 0.5(900 + 20x)$                          | <b>A</b> 1 | ignoring signs. Condone > Correct equation or inequality. Condone >   |
|              | $10x + 20gx = 11000 - 900g - 450$ or $10x + 20gx \le 10550 - 900g$ | M1         | oe. Simplification of their equation or inequality isolating terms in $\boldsymbol{x}$ on one side and constant terms on the other. Condone $<$                                       |
|              | $(x =) 8.39$ or $x \le 8.39$                                       | <b>A</b> 1 | AWRT 8.4 Condone truncated to 8.3 PI by 8 in correct concluding statement If working with inequalities correct inequality must be seen here.  |
|              | 8 boxes  | <b>E</b> 1 | Condone < Correct statement including whole number of boxes dependent upon correct working seen.  |
| 12(b)<br>ALT | 11000 - Mg = 0.5M  | <b>M</b> 1 | oe. Equation of motion, ignoring signs, including consideration of total mass of lift and boxes <i>M</i> . May be seen as an inequality,  |
|              | $11000 - Mg \ge 0.5M$  | <b>A</b> 1 | ignoring signs. Condone > Correct equation or inequality. Condone >   |
|              | (M =) 1067.96<br>or<br>$(M \le) 1067.96$                           | М1         | AWRT 1068 Solving equation or inequality for <i>M</i>   |
|              | $(x =) 8.39$ or $x \le 8.39$                                       | <b>A</b> 1 | AWRT 8.4 For number of boxes unrounded. Condone truncated to 8.3 PI by 8 in correct concluding statement If working with inequalities correct inequality must be seen here. Condone < |
|              | 8 boxes  | <b>E</b> 1 | Correct statement including whole number of boxes dependent upon correct working seen.  |
|              | Total  | 7          |   |

| Q  | Answer  | Marks      | Comments  |
|----|---|------------|---|
| 13 | (Displacement =) -75 or 75  | В1         | Both values. PI by later working. Recognises that displacement could be ±75   |
|    | $3 \times 10 + \frac{1}{2} \times a \times 10^{2} = 75$ or $3 \times 10 + \frac{1}{2} \times a \times 10^{2} = -75$ | М1         | Use of constant acceleration formula to gain at least one correct equation. Values substituted but need not be evaluated. |
|    | a = 0.9   | <b>A</b> 1 | oe  |
|    | a = -2.1  | <b>A</b> 1 | oe  |
|    | Total   | 4          |   |