
INTERNATIONAL A-LEVEL MATHEMATICS

MA03

(9660/MA03) Unit P2 – Pure Mathematics

Mark scheme

January 2020

Version: V1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$[5\sin\theta - 12\cos\theta =]$ $R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R = 13$ $\alpha = 67.4$	M1 A1 A1	PI
1(a)(ii)	$\sin(\theta - 67.4) = -\frac{1}{13}$ $\theta = -108.2$ $\theta = 63.0$	M1 A1 A1	Ft their (a) Both correct and no extras in interval (ignore answers outside interval)
1(b)	$2\cot^2 x = 2\operatorname{cosec}^2 x - 2 \quad [= 10 - 5\operatorname{cosec} x]$ $[2\operatorname{cosec}^2 x - 2 = 10 - 5\operatorname{cosec} x]$ $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 12 = 0$ $(2\operatorname{cosec} x - 3)(\operatorname{cosec} x + 4) [= 0]$ $\operatorname{cosec} x = \frac{3}{2}, -4$ $\sin x = \frac{2}{3}, -0.25$ $x = 42, 138,$ $\quad -14, 194$	M1 m1 A1 B1 B1	Correct use of trig identity PI Factorisation or correct use of formula Both correct and no errors seen Either may be seen Sight of any of these values correct All four correct and no extras in interval (ignore answers outside interval)
	Total	11	
Notes: (a) May use cos and sin leading to $\sin p = 2/3, -1/4$ for first M1, m1, A1 (b) Condone more accurate correct answers, but not $-14.5, 194.5$			

Q	Answer	Marks	Comments
2(a)	$0 \leq x \leq 1$	B1	
2(b)		M1 A1	Correct shape and position $(0, -\frac{\pi}{2})$ $(1, \frac{\pi}{2})$ stated or marked on diagram
2(c)	Stretch + either I or II Parallel to x -axis I SF $\frac{1}{2}$ II Followed by Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = \frac{1}{2}$	M1 A1 M1 A1	Alt: Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ M1 $k = 1$ A1 Followed by Stretch in x -direction M1 SF $\frac{1}{2}$ A1
2(d)	$f(x) = \sin^{-1}(2x - 1) + x - 1$ $f(0.6) = -0.198\dots$ $f(0.7) = 0.111\dots$ Change of sign, (the function is continuous), $0.6 < \alpha < 0.7$	M1 A1	Or reverse Both values rounded or truncated to at least 1sf Must have both statements and interval in words or symbols Accept x for α
2(e)	$x_2 = 0.695$ $x_3 = 0.650$	B1 B1	
	Total	11	

Q	Answer	Marks	Comments
3(a)	$u = \ln x, \quad dv = x$ $du = \frac{1}{x}, \quad v = \frac{x^2}{2}$ $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} \, dx$ $= \frac{x^2}{2} \ln x - \frac{x^2}{4} \quad [+c]$	M1 A1 A1	Use of parts formula
3(b)	$u = \ln x, \quad dv = 1$ $du = \frac{1}{x}, \quad v = x$ $\int \ln x \, dx = x \ln x - \int x \times \frac{1}{x} \, dx$ $= x \ln x - x \quad [+c]$	M1 A1 A1	Use of parts formula
	Total	6	

Q	Answer	Marks	Comments
4(a)	$8[(1.5)^3] + b[(1.5)^2] + c[1.5] + 6 = -3.75$ $8[(0.5)^3] + b[(0.5)^2] + c[0.5] + 6 = 5.25$ $2.25b + 1.5c = -36.75$ $0.25b + 0.5c = -1.75$ $b = -21$ $c = 7$	M1 A1 m1 A1	One correct substitution OR for M1 use of long division Attempt to solve Both answers
4(b)	$(4x^2 - 1)(3x - 2)$ $= 12x^3 - 8x^2 - 3x + 2$ $(12x^3 - 8x^2 + x + 7) - (12x^3 - 8x^2 - 3x + 2)$ $= 4x + 5$ OR $12x^3 - 8x^2 + x + 7 = (4x^2 - 1)(3x + d + \frac{ex + f}{4x^2 - 1})$ $4d = -8, d = -2$ $-3 + e = 1, e = 4$ $7 = f - d, f = 5$	B1 B1 M1 A1 (M1) (B1) (B1) (A1)	Accept other correct approaches
	Total	8	

Q	Answer	Marks	Comments
5(a)	$\frac{12}{(3-u)(3+u)} = \frac{A}{3-u} + \frac{B}{3+u} \quad \text{oe}$ $A = 2, \quad B = 2$	M1 A1	
5(b)	$\left[\frac{du}{dx} = \right] \cos x$ $\left[\int \frac{12 \cos x}{8 + \cos^2 x} dx = \right] \int \frac{12 du}{8 + 1 - u^2}$ $= \int \frac{12 du}{(3-u)(3+u)}$ $= \int \frac{A}{3-u} + \frac{B}{3+u} du$ $\left[\int = \right] -2 \ln(3-u) + 2 \ln(3+u)$ $= 2 \ln \frac{3+u}{3-u}$ $\left[x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \left[u \right]_{0.5}^1$ $\left[\int = \right] 2 \ln 2 - 2 \ln 1.4$ $= \ln \frac{100}{49}$	B1 M1 A1 M1 B1 A1	 All in terms of u , condone omission of du Must see du here, or earlier Correct integration Change of limits, maybe seen earlier (may change back to x and not change limits)
	Total	8	

Q	Answer	Marks	Comments
6(a)(i)	$(1+2x)^{0.5} =$ $1 + 0.5 \times 2x + [0.5 \times -0.5 \times (2x)^2] / 2$ $= 1 + x - \frac{x^2}{2}$	M1 A1	
6(a)(ii)	$(1-4x)^{-0.5} =$ $1 + (-0.5)(-4x) + [(-0.5)(-1.5)(4x)^2] / 2$ $= 1 + 2x + 6x^2$	M1 A1	
6(b)(i)	$\sqrt{f(x)} = (1+x-0.5x^2)(1+2x+6x^2)$ $1+3x+7.5x^2$	M1 A1	
6(b)(ii)	$ x < 0.25$	B1	Accept $-0.25 \leq x < 0.25$
6(c)	$\frac{1+2x}{1-4x} = 2$ $x = 0.1$ $2^{0.5} = 1 + 0.3 + 0.075$ $= 1.375$	M1 A1 A1	
	Total	10	

Q	Answer	Marks	Comments
7(a)	$[\frac{dy}{dx} =] 3e^{3x} - 24$ $[x = 0, \frac{dy}{dx} =] -21$ $y = -21x + 1$	M1 A1 A1	
7(b)	$3e^{3x} = 24$ $3x = \ln 8$ $x = \ln 2$ $y = 8 - 24 \ln 2$ ACF	M1 A1 A1	
7(c)	$\frac{d^2y}{dx^2} = 9e^{3x}$ [= 72] $[x = \ln 2,] \frac{d^2y}{dx^2} > 0$ Hence, min point	B1 B1 E1	Must have scored B1B1 to score this mark
	Total	9	

Q	Answer	Marks	Comments
8(a)	$\left(\frac{dx}{dt}\right)$ rate of change of x (k) is proportional to $(80 - x)$ amount of substance remaining	E1	Complete explanation
8(b)	$\int \frac{dx}{80 - x} = \int k dt$ $-\ln(80 - x) = kt + c$ $t = 0, x = 0, \quad c = -\ln 80$ $t = 60, x = 30, \quad -\ln 50 = 60k - \ln 80$ $k = \frac{1}{60} \ln 1.6 \quad [= 0.00783\dots]$	M1 m1 A1 M1 A1	Separate variables Attempt to find k
8(c)(i)	$-\ln(80 - x) = 2 \ln 1.6 - \ln 80$ $80 - x = \frac{80}{1.6^2}$ $x = 48.75$	M1 A1	
8(c)(ii)	$-\ln(80 - 70) = \frac{t}{60} \ln 1.6 - \ln 80$ $\frac{t}{60} = \frac{\ln 8}{\ln 1.6}$ $t = 265$	M1 m1 A1	Accept 265 - 266
	Total	11	
Notes: 8(c) correct answers with NMS scores full marks			

Q	Answer	Marks	Comments
9	$2y + xy = 1$ $x = \frac{1-2y}{y}$ $[\text{Vol} =] \pi \int_{0.2}^{0.25} \left(\frac{1-2y}{y}\right)^2 dy$ $\int = \int y^{-2} + 4 - \frac{4}{y} [dy]$ $= -y^{-1} + 4y - 4 \ln y$ $= [-4 + 1 - 4 \ln 0.25] - [-5 + 0.8 - 4 \ln 0.2]$ $= 1.2 + 4 \ln 0.8$ $\text{Vol} = \pi(1.2 + 4 \ln 0.8)$ ACF	B1 B1 M1 A1 m1 A1	 Correct including π , limits, dy Attempt to expand Correct simplified integral Correct substitution of correct limits into expression in correct form (PI by final answer of 0.964 – 0.966)
	Total	6	

Q	Answer	Marks	Comments														
10(a)	<table><thead><tr><th>x</th><th>y</th></tr></thead><tbody><tr><td>1.625</td><td>$1.625^{-1.625} = 0.45432$</td></tr><tr><td>1.875</td><td>$1.875^{-1.875} = 0.30770$</td></tr><tr><td>2.125</td><td>$2.125^{-2.125} = 0.20154$</td></tr><tr><td>2.375</td><td>$2.375^{-2.375} = 0.12817$</td></tr><tr><td>2.625</td><td>$2.625^{-2.625} = 0.07939$</td></tr><tr><td>2.875</td><td>$2.875^{-2.875} = 0.04802$</td></tr></tbody></table> $0.25[0.45432 + 0.30770 + 0.20154 + 0.12817 + 0.07939 + 0.04802]$ $= 0.305$	x	y	1.625	$1.625^{-1.625} = 0.45432$	1.875	$1.875^{-1.875} = 0.30770$	2.125	$2.125^{-2.125} = 0.20154$	2.375	$2.375^{-2.375} = 0.12817$	2.625	$2.625^{-2.625} = 0.07939$	2.875	$2.875^{-2.875} = 0.04802$	B1 M1 m1 A1	All six correct x values (and no extra used) PI by five correct y values At least five correct y values in exact form or decimals, rounded or truncated to three dp or better (in table or formula) (PI by AWRT correct answer) Correct sub into formula with $h = 0.25$ OE and at least five correct y values either listed, with + signs, or totalled. (PI by AWRT correct answer) CAO , must see this value exactly and no error seen
x	y																
1.625	$1.625^{-1.625} = 0.45432$																
1.875	$1.875^{-1.875} = 0.30770$																
2.125	$2.125^{-2.125} = 0.20154$																
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2.625	$2.625^{-2.625} = 0.07939$																
2.875	$2.875^{-2.875} = 0.04802$																
10(b)	$y = x^{-x}$ $\ln y = -x \ln x$ $\frac{1}{y} \frac{dy}{dx} = -x \times \frac{1}{x} - \ln x$ $= -1 - \ln x$ $\frac{dy}{dx} = (-1 - \ln x)y$ $= (-1 - \ln x)x^{-x}$	B1 M1 A1 A1	ACF														
	Total	8															

Notes:
10(a) 0.305 with NMS scores 4/4
(b) Correct answer without using implicit differentiation scores SC2

Q	Answer	Marks	Comments
11(a)	$[\text{Equation } \vec{AB}] \mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ -3 \end{pmatrix} + f \begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$ $= \begin{pmatrix} 10 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ $\lambda = -4f$ $\lambda = -3$ $(4, -1, 3) \text{ is on line QED}$	M1 A1 m1 A1	
11(b)(i)	Coords of C $(4 + 2c, -1 + c, 3 - 2c)$ $\vec{DC} = \begin{pmatrix} 6 + 2c \\ -2 + c \\ -4 - 2c \end{pmatrix} \quad \text{oe}$ $\begin{pmatrix} 6 + 2c \\ -2 + c \\ -4 - 2c \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0$ $12 + 4c - 2 + c + 8 + 4c = 0$ $c = -2$ $C = (0, -3, 7)$	M1 m1 A1 m1 A1	
11(b)(ii)	$CD^2 = (0 - -2)^2 + (-3 - 1)^2 + (7 - 7)^2$ $= 4 + 16$ $CD = \sqrt{20}$	M1 A1	
11(c)	$CP^2 = (4 + 2p)^2 + (2 + p)^2 + (-4 - 2p)^2$ $= 9p^2 + 36p + 36$ $9p^2 + 36p + 16 = 0$ $p = \frac{-36 \pm \sqrt{36^2 - 4 \times 9 \times 16}}{2 \times 9}$ $p = -2 + \frac{1}{3}\sqrt{q}, \quad p = -2 - \frac{1}{3}\sqrt{q}$	M1 A1 M1 M1 A1	oe
	Total	16	

Q	Answer	Marks	Comments
12(a)	$12y \frac{dy}{dx} + 8e^{4x} = y^3 e^x + e^x 3y^2 \frac{dy}{dx}$ $(12y - e^x 3y^2) \frac{dy}{dx} = y^3 e^x - 8e^{4x}$ $\frac{dy}{dx} = \frac{y^3 e^x - 8e^{4x}}{(12y - e^x 3y^2)}$ $\frac{dy}{dx} = 0, y^3 e^x = 8e^{4x}$ $q^3 = 8e^{3p}$ $q = 2e^p$	M1 A1 M1 A1 A1	Either implicit differential correct Or using $\frac{dy}{dx} = 0$ AG
12(b)	$6 \times 4e^{2p} + 2e^{4p} = 8e^{3p} e^p$ $24e^{2p} = 6e^{4p}$ $e^{2p} = 4$ $p = \ln 2$ $q = 4$	M1 A1 m1 A1	Equation all in terms of one variable ACF
	Total	9	

Q	Answer	Marks	Comments
13	$\frac{dx}{dt} = 2at$ $\frac{dy}{dt} = 2a$ $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$ $\text{At } P, y - 2ap = \frac{1}{p}(x - ap^2)$ $yp = x + ap^2$ $yq = x + aq^2$ $\text{At } R, y(p - q) = a(p^2 - q^2)$ $y = a(p + q)$ $x = apq$ $y^2 = a^2(p^2 + q^2 + \frac{2x}{a})$ $= a(ap^2 + aq^2 + 2x)$ $p^2 + q^2 = 1,$ $y^2 = a(a + 2x)$	<p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>Either differential correct</p> <p>Equation of a tangent with t replaced</p> <p>Both correct (unsimplified)</p> <p>Attempt to solve</p> <p>Both correct</p>
	Total	7	