
INTERNATIONAL AS MATHEMATICS MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$\frac{5}{2}$	B1	
1(a)(ii)	3	B1	
1(b)	$\frac{dy}{dx} = \frac{5}{2} \times 8x^{\frac{3}{2}} - 3 \times 2x^2$ $\frac{dy}{dx} = 20x^{\frac{3}{2}} - 6x^2$	<p>M1</p> <p>A1</p>	<p>oe ft p and q from part (a). Must use coefficients 8 and 2 at this stage. One correct term, simplified or unsimplified.</p> <p>CAO</p>
	Total	4	

Q	Answer	Marks	Comments
2(a)	$(x-5)^2$ or $-\frac{b}{2a} = 5$ or $-\frac{b}{2} = 5$ or $x^2 - 10x + 25$ or Indicates there is a stationary point at $(5, 0)$ $b = -10$ $c = 25$	<p>M1</p> <p>A1</p>	<p>CSO. Both values. Correct b and c but NMS scores 2/2.</p>
2(b)	Correctly orientated symmetrical quadratic parabola tangential to x -axis or symmetrical about the line $x = 5$ $(0, 25)$ labelled on y -axis. Correctly positioned vertex labelled as $(5, 0)$	<p>B1</p> <p>B1ft</p> <p>B1</p>	<p>ft their c Condone label given as the y-value only.</p> <p>Accept correctly positioned vertex with 5 indicated or implied on the x-axis.</p>
	Total	5	

Q	Answer	Marks	Comments
3(a)(i)	$y = \frac{3}{2}x + 3$ (Gradient =) $\frac{3}{2}$	M1 A1	Attempt at $y = mx + c$ or Attempt to find gradient using two correct points on the line. oe
3(a)(ii)	(y -coordinate =) 3	B1	Condone given as coordinates.
3(b)(i)	(m =) $-\frac{2}{3}$	B1ft	ft their gradient for L_1 .
3(b)(ii)	$0 = -\frac{2}{3}x + \frac{22}{3}$ or $\frac{2}{3}x = \frac{22}{3}$ and (x =) 11	B1	AG. Zero substituted for y in the equation of L_2 and correct answer.
3(c)	$(D =) \left(\frac{11}{2}, \frac{3}{2}\right)$ $y - \frac{3}{2} = \text{their gradient} \times \left(x - \frac{11}{2}\right)$ $y - \frac{3}{2} = \frac{3}{2}\left(x - \frac{11}{2}\right)$	B1ft M1 A1	ft their y -coordinate of B. PI in later working. ft their gradient of L_1 . May see $2y - 3x = k$ and substitution of coordinates of D to find k . But must be full method. CAO. oe
	Total	8	

Q	Answer	Marks	Comments
4(a)	$(2-x)^4 =$ $\left[(2)^4\right] + \left[4(2)^3(-x)\right] + 6(2)^2(-x)^2 + 4(2)(-x)^3 + \left[(-x)^4\right]$ $a = 24$ $b = 8$	M1	For either (1), [4], 6, 4, (1) oe unsimplified. or $\binom{4}{2}(2)^2(-x)^2$ or $\binom{4}{3}(2)(-x)^3$ oe, x not needed, Pl.
		A1	If correct working for a seen condone 24 seen in expansion but not explicitly stated.
		A1	a not shown but correct b seen but NMS merits M1A0A1. Condone $24x^2$ or $8x^3$.
4(b)	$\left\{\left(2-\frac{1}{\sqrt{2}}\right)^4 =\right\}$ $16-32\left(\frac{1}{\sqrt{2}}\right)+24\left(\frac{1}{\sqrt{2}}\right)^2-8\left(\frac{1}{\sqrt{2}}\right)^3+\left(\frac{1}{\sqrt{2}}\right)^4$ $16-\frac{32}{\sqrt{2}}+12-\frac{4}{\sqrt{2}}+\frac{1}{4}$ $16-16\sqrt{2}+12-2\sqrt{2}+\frac{1}{4}$ $\frac{113-72\sqrt{2}}{4}$	M1	Substitution of $x = \frac{1}{\sqrt{2}}$ into their expansion.
		M1	Powers correctly evaluated in an expression.
		M1	Denominators rationalised in an unsimplified expression.
		A1	Accept other correct forms, e.g. $\frac{226-144\sqrt{2}}{8}$ NMS= 0
	Total	7	

4(b) ALT	$\left((2-x)^4 =\right)$ $16 - 32\left(\frac{\sqrt{2}}{2}\right) + 24\left(\frac{\sqrt{2}}{2}\right)^2 - 8\left(\frac{\sqrt{2}}{2}\right)^3 + \left(\frac{\sqrt{2}}{2}\right)^4$ $16 - 16\sqrt{2} + 12 - 2\sqrt{2} + \frac{1}{4}$ $\frac{113 - 72\sqrt{2}}{4}$	M2	Substitution of $x = \frac{\sqrt{2}}{2}$ into their expansion.
		M1	Powers correctly evaluated in an expression.
		A1	Accept other correct forms, e.g. $\frac{226 - 144\sqrt{2}}{8}$ NMS= 0

Q	Answer	Marks	Comments
5(a)	$x - 15 + \frac{14}{x}$ or $x - 15 + 14x^{-1}$ $(y =) x - 15 + 14x^{-1}$ $\frac{dy}{dx} = 1 - 14x^{-2}$ $x = 2 \Rightarrow \frac{dy}{dx} = 1 - \frac{14}{4} = -\frac{5}{2}$ $x = 2 \Rightarrow y = -6$ $(y - (-6)) = -\frac{5}{2}(x - 2)$ $(y - (-6)) = -\frac{5}{2}(x - 2)$	M1 A1 A1ft B1ft B1 M1ft A1	Expansion of numerator and division by x . Allow error in one term if two other terms correct. oe. Correct simplification oe. ft their rearrangement of y provided their rearrangement contains a term in x^{-1} ft their $\frac{dy}{dx}$ ft their $-\frac{5}{2}$ provided it came from a correct method. ft their -6 May see substitution of $x = 2$, $y = -6$ and $m = -\frac{5}{2}$ into $y = mx + c$ but must be a complete method to find c . ACF. Correct equation simplified or unsimplified.
5(b)	$\frac{dy}{dx} = 1 - 14 \times (-4)^{-2} [= 0.125 > 0]$ Increasing	B1ft E1ft	ft their $\frac{dy}{dx}$ Substituting $x = -4$ into their $\frac{dy}{dx}$ to show it has a positive value. ft. Allow 'decreasing' if their $\frac{dy}{dx} < 0$ for their $\frac{dy}{dx}$ Dependent on B1 scored.
	Total	9	

Q	Answer	Marks	Comments
6(a)(i)	Arithmetic (sequence)	E1	With an attempt at an explanation.
	The sequence increases by 22 each time.	E1	Explanation implying that the difference between each term is constant.
6(a)(ii)	$36 + (n - 1)d$ or $a + (n - 1)22$	M1	$a + (n - 1)d$ with a or d correct and the other a letter.
	$36 + (n - 1)22$	A1	CAO oe
6(b)	(Total boxes sold =) 7500	B1	PI in later working.
	$\frac{1}{2}N(2 \times 36 + (N - 1) \times 22)$	M1	oe. Expression for total number of boxes sold in N months. Condone other letters used instead of N throughout.
	$11N^2 + 25N - 7500 = 0$	A1	oe. Correct quadratic equation in N . PI.
	$\left(N = \frac{-25 \pm \sqrt{25^2 - 4 \times 11 \times (-7500)}}{2 \times 11}\right)$ or $(11N + 300)(N - 25) (= 0)$	M1	Correct use of quadratic formula to solve their quadratic equation, condoning one error in signs or Correct factorisation ignoring signs.
	25	A1	Both values given as final answer scores A0
Total		9	

Q	Answer	Marks	Comments
7(a)	$\int (3x^2 + ax - 36) dx$ $= x^3 + \frac{a}{2}x^2 - 36x (+c)$ $(y =) x^3 + \frac{a}{2}x^2 - 36x (+c)$ $1 + \frac{a}{2} - 36 + c = -7$ or $27 + \frac{9a}{2} - 108 + c = -5$ $\frac{a}{2} + c = 28$ $\frac{9a}{2} + c = 76$ $a = 12 \text{ and } c = 22$ $(y =) x^3 + 6x^2 - 36x + 22$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>A1ft</p>	<p>Attempt to integrate $\frac{dy}{dx}$ with at least two terms correct.</p> <p>oe.</p> <p>oe. Substituting either $x = 1$ and $y = -7$ or $x = 3$ and $y = -5$ into their equation for y in an attempt to find an equation in a and c.</p> <p>oe. For each correct equation simplified or unsimplified. PI.</p> <p>Both correct. PI</p> <p>ft their a and c provided both method marks scored.</p>
7(b)(i)	$\left(\frac{d^2y}{dx^2} = \right) 6x + 12$	B1ft	ft their $\frac{dy}{dx}$ with their a substituted.
7(b)(ii)	$\left(\frac{dy}{dx} = \right) 3x^2 + 12x - 36 = 0$ and $(x + 6)(x - 2) = 0$ $x = 2 \Rightarrow \frac{d^2y}{dx^2} = 6 \times 2 + 12 [= 24 > 0]$ $x = 2$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>oe. Sets their $\frac{dy}{dx}$ equal to zero with their a and attempts to solve for x. PI by $x = 2$</p> <p>Establishes that $\frac{d^2y}{dx^2}$ is positive at $x = 2$</p> <p>Must be working with correct $\frac{d^2y}{dx^2}$ and $x = 2$ at this stage.</p> <p>CSO.</p> <p>Failure to show that $\frac{d^2y}{dx^2}$ is positive at $x = 2$ scores A0.</p>
	Total	11	

Q	Answer	Marks	Comments
8(a)(i)	$\frac{a}{1-r} = 425$ $a + ar = 408$ $425(1-r) + 425(1-r)r = 408$ or $\frac{408}{1-r^2} = 425$ $r = \frac{1}{5}$	M1 A1 M1 A1	oe. One correct equation. oe. Both correct equations. Attempts to solve simultaneously by elimination and forms equation in r . Simplified or unsimplified. AG. From correct working.
8(a)(ii)	$\frac{a}{1-\frac{1}{5}} = 425$ or $a + \frac{1}{5}a = 408$ $(a =) 340$	M1 A1	oe. Substitutes $r = \frac{1}{5}$ into either of their equations in part (a).
8(b)	$\left(\sum_{n=k}^{k+1} u_n \right) = \sum_{n=1}^{k+1} u_n - \sum_{n=1}^{k-1} u_n$ or $ar^{k-1} + ar^k$ $\frac{340 \left(1 - \left(\frac{1}{5} \right)^{k+1} \right)}{1 - \frac{1}{5}} - \frac{340 \left(1 - \left(\frac{1}{5} \right)^{k-1} \right)}{1 - \frac{1}{5}}$ or $340 \left(\frac{1}{5} \right)^{k-1} + 340 \left(\frac{1}{5} \right)^k$ $425 \left(\frac{1}{5} \right)^{k-1} \left(1 - \left(\frac{1}{5} \right)^2 \right)$ or $340 \left(\frac{1}{5} \right)^{k-1} \left(1 + \frac{1}{5} \right)$ $\left(\sum_{n=k}^{k+1} u_n \right) = 408 \left(\frac{1}{5} \right)^{k-1}$	B1 M1 M1 A1	PI in later working. oe. Correct substitution of r and a . Simplified or unsimplified. ft their a . Simplification and factorisation, unsimplified. CSO
	Total	10	

Q	Answer	Marks	Comments
9(a)	$(y =) x^{\frac{2}{3}} - 8x^{\frac{1}{3}} + 5$ $\int \left(\left(x^{\frac{1}{3}} - 4 \right)^2 - 11 \right) dx =$ $\frac{3}{5}x^{\frac{5}{3}} - 6x^{\frac{4}{3}} + 5x(+c)$	B1 B2ft	<p>Correct expansion simplified or unsimplified.</p> <p>Simplified or unsimplified. For B2 condone omission of $+c$ Two correct terms only scores B1B0ft their expansion.</p>
9(b)	$\left(\frac{3}{5}(8)^{\frac{5}{3}} - 6(8)^{\frac{4}{3}} + 5(8) \right) - \left(\frac{3}{5}(1)^{\frac{5}{3}} - 6(1)^{\frac{4}{3}} + 5(1) \right)$ $\frac{-182}{5} \quad \text{or} \quad -36.4$ $\frac{182}{5} \quad \text{or} \quad 36.4$	M1ft A1 A1ft	<p>Identifies correct definite integral and attempts $F(8) - F(1)$. ft their indefinite integral from part (a).</p> <p>oe. CSO PI by correct final answer.</p> <p>oe. Correct area ft the value of their definite integral provided it was negative.</p>
9(c)(i)	$\left(\frac{182}{5} \right) + 14$ $\frac{252}{5} \quad \text{or} \quad 50.4$	M1 A1ft	<p>oe. Recognises that they need to increase the area in part (b) by 14.</p> <p>oe. ft their area from part (b) provided it was positive. Re-integrating the equation of C_2 scores M0A0.</p>
9(c)(ii)	$(y =) \left(x^{\frac{1}{3}} - 4 \right)^2 - 13$	B1	ACF
	Total	9	

Q	Answer	Marks	Comments
10(a)	$16(p+3)^2 - 4(2)(12p+q+12) [> 0]$ $16p^2 + 96p + 144 - 96p - 8q - 96 (> 0)$ $16p^2 - 8q + 48 > 0$ $2p^2 - q + 6 > 0$	M1 A1 A1	Use of $b^2 - 4ac$ with a , b and c substituted. oe. Correct expansion. AG. From correct working. Brackets must be seen to be expanded before final answer given. > 0 must appear before final line.
10(b)	$2(0)^2 + 4(p+3)(0) + 12p + q + 12 = 32$ $q = 20 - 12p$ $2p^2 - (20 - 12p) + 6 [> 0]$ $p = -7$ and $p = 1$ $p < -7$ or $p > 1$	M1 A1 m1 A1 A1	Attempts to use $(0,32)$ to find an equation in p and q . oe. oe. Uses their equation in p and q to eliminate q in the inequality. Correct critical values. Condone 'or' omitted but do not condone inclusion of 'and'.
	Total	8	