OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

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# INTERNATIONAL A-LEVEL
# MATHEMATICS

(9660/MA03) Unit P2 − Pure Mathematics

Wednesday 29 May 2019 07:00 GMT Time allowed: 2 hours 30 minutes

**Materials**
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks for method may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1 (a)** Use Simpson's rule with 7 ordinates (6 strips), to find an estimate for $\int_0^3 3^x \, dx$ ,

giving your answer to 3 decimal places.

**[4 marks]**

Answer _____

**1 (b)** A curve is defined by the equation $y = 3^x$

The curve intersects the line $y = 12 - 4x$ at a single point where $x = \alpha$

**1 (b) (i)** Show that $\alpha$ lies between 1.5 and 1.6

**[2 marks]**

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**1 (b) (ii)** The equation $3^x = 12 - 4x$ can be rearranged into the form $x = \dfrac{\ln(12 - 4x)}{\ln 3}$

Use the iterative formula

$$x_{n+1} = \frac{\ln(12 - 4x_n)}{\ln 3}$$

with $x_1 = 1.5$ to find the values of $x_2$ and $x_3$, giving your answers to 3 decimal places.

**[2 marks]**

$x_2 =$ _____ $x_3 =$ _____

8

**Turn over for the next question**

*Do not write outside the box*

**2 (a)** The number of fish in Lake *P* decreases by 3% each year.

On 1 January 2019 there are 50 000 fish in this lake.

Calculate, to the nearest 100, the number of fish in this lake on

**2 (a) (i)** 1 January 2020,

**[1 mark]**

Answer _____

**2 (a) (ii)** 1 January 2029,

**[2 marks]**

Answer _____

**2 (a) (iii)** 1 January 2009.

**[2 marks]**

Answer _____

*Do not write outside the box*

**2 (b)** The number of fish in Lake *Q* increases by 1.5% each year.

On 1 January 2019 there are 25 000 fish in this lake.

Find the first year in which there are more fish in Lake *Q* than in Lake *P* on 1 January.

**[4 marks]**

Answer _____

**9**

**Turn over ►**

*Do not write outside the box*

**3 (a)** The polynomial $f(x)$ is defined by

$$f(x) = 4x^3 + bx^2 + cx + 6$$

where $b$ and $c$ are constants.

When $f(x)$ is divided by $(2x - 3)$ the remainder is $-6$

When $f(x)$ is divided by $(2x + 1)$ the remainder is 10

Find the value of $b$ and the value of $c$.

**[4 marks]**

$b =$ _____ $c =$ _____

*Do not write outside the box*

**3 (b)** Simplify $\dfrac{4x^2 - 1}{4x^2 + 4x - 3}$, giving your answer in the form $1 + g(x)$.

**[4 marks]**

Answer _____

8

**Turn over ►**

*Do not write outside the box*

**4 (a) (i)** Express $3\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the value of $\alpha$, in radians, to 3 significant figures.

**[3 marks]**

Answer _____

**4 (a) (ii)** Hence solve the equation

$$3\cos(y - 0.1) - 4\sin(y - 0.1) = 2.5$$

giving all values of $y$, to 2 decimal places, in the interval $-\pi < y < \pi$

**[3 marks]**

Answer _____

*Do not write outside the box*

**4 (b)** Solve the equation

$$7 \tan^2 x = 13 - 4 \sec x$$

giving all solutions, to the nearest degree, in the interval $-90° < x < 270°$

**[5 marks]**

Answer _____

**11**

*Do not write outside the box*

**5 (a)** Find $\frac{dy}{dx}$ given that $y = \ln(3x + 2)$

**[2 marks]**

Answer _____

**5 (b)** Find $\frac{dy}{dx}$ given that $y = \frac{e^{3x}}{x^2}$

Express your answer in the form $\frac{dy}{dx} = y\,f(x)$

**[3 marks]**

Answer _____

*Do not write outside the box*

**5 (c)** A curve has equation

$$2xy + y^2 = \frac{1}{x}$$

where $x \neq 0$

Find the coordinates of the stationary point of the curve.

**[6 marks]**

Answer _____

**11**

*Do not write outside the box*

**6 (a)** By writing $\sin 3x$ as $\sin(2x + x)$ show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

**[3 marks]**

**6 (b)** Hence find $\int \sin^3 x \, dx$

**[3 marks]**

Answer _____

6

*Do not write outside the box*

**7 (a)** Given that $x = \dfrac{1}{\cos\theta}$, use the quotient rule to show that $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \sec\theta \tan\theta$

**[2 marks]**

**7 (b)** Use the substitution $x = 2 \sec \theta$, to find the exact value of $\displaystyle\int_{\frac{4\sqrt{3}}{3}}^{4} \frac{1}{x^2\sqrt{(x^2-4)}}\,\mathrm{d}x$

**[7 marks]**

Answer _____

9

**Turn over ►**

*Do not write outside the box*

**8** A curve is defined by the parametric equations

$$x = \frac{1}{t+1} \quad \text{and} \quad y = 3t - t^2$$

**8 (a)** Find the values of $\frac{dy}{dx}$ when $y = 0$

**[5 marks]**

Answer _____

*Do not write outside the box*

**8 (b)** Find a Cartesian equation of the curve, giving your answer in the form $yx^2 = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is expressed as a product of linear factors.

**[4 marks]**

Answer _____

9

**Turn over ►**

*Do not write outside the box*

**9** The function f is defined by

$$f(x) = |x^2 - 5| - 3 \quad \text{for } -5 \leq x \leq 5$$

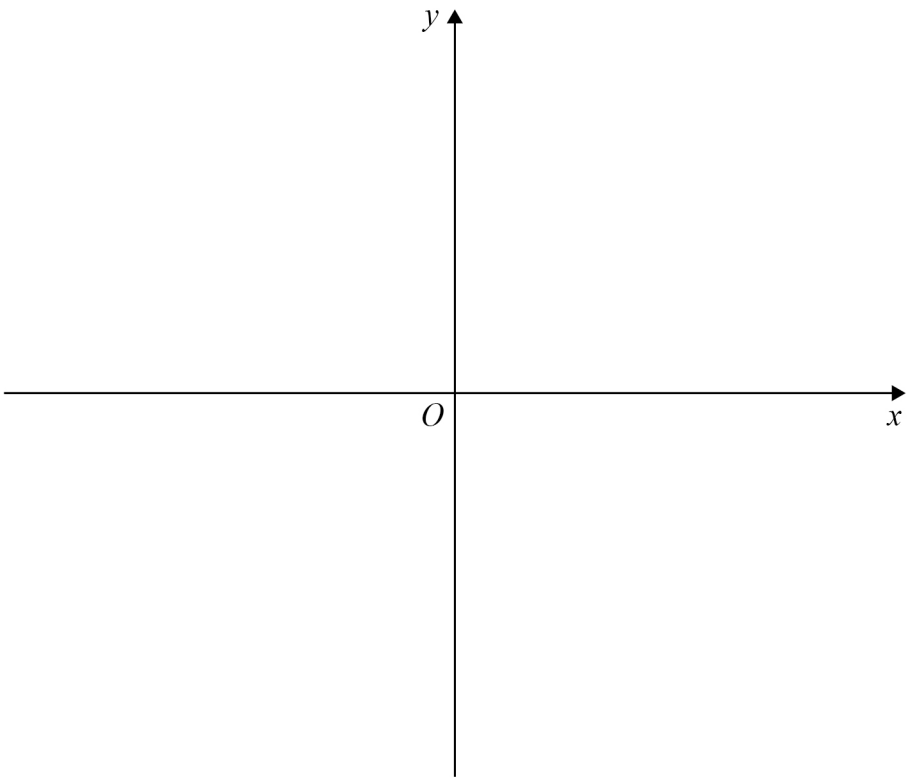
**9 (a) (i)** Write down the range of f.

**[1 mark]**

Answer _____

**9 (a) (ii)** Sketch the graph of $y = f(x)$, indicating the value where the curve crosses the $y$-axis.

**[3 marks]**



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**9 (a) (iii)** Solve $f(x) = 1$

**[3 marks]**

Answer _____

**9 (b)** The function g is defined by

$$g(x) = \frac{1}{x} \qquad \text{where } x \neq 0$$

**9 (b) (i)** Find an expression for $fg(x)$.

**[1 mark]**

Answer _____

**9 (b) (ii)** Solve $fg(x) < 0$

**[3 marks]**

Answer _____

11

**Turn over ►**

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**10** It is given that

$$\frac{dy}{dx} = \frac{y}{\sqrt{2x-1}} \quad \text{where } x > 0.5$$

**10 (a)** Solve the differential equation such that $y = 1$ when $x = 5$

**[4 marks]**

Answer _____

**10 (b)** Hence find the value of $x$ when $y = e^4$

**[2 marks]**

Answer _____

6

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**11 (a)** Express $\dfrac{6}{(1-x)^2(1+2x)}$ in the form $\dfrac{A}{1-x}+\dfrac{B}{(1-x)^2}+\dfrac{C}{1+2x}$

**[5 marks]**

Answer _____

**11 (b)** Find the binomial expansion of $(1-x)^{-1}$ up to and including the term in $x^3$

**[1 mark]**

Answer _____

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**11 (c)** Use your answers to parts **(a)** and **(b)** to show that

$$\frac{6}{(1-x)^2(1+2x)} = D + Ex^2 + Fx^3$$

for small values of $x$, stating the values of $D$, $E$ and $F$.

**[6 marks]**

Answer _____

**12**

**Turn over ►**

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**12** The region bounded by the curve $y = xe^{-1.5x}$, the line $x = 1$ and the $x$-axis from $x = 0$ to $x = 1$, is rotated through $2\pi$ radians about the $x$-axis to form a solid.

Use integration by parts twice to find the exact value of the volume of the solid generated, giving your answer in the form $\pi\,(p + q\,e^{-3})$, where $p$ and $q$ are rational.

**[7 marks]**

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Answer _____

7

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**13** The line $l_1$ has equation $\mathbf{r} = \begin{bmatrix} -4 \\ 1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} -3 \\ -4 \\ -5 \end{bmatrix}$

and the line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} 6 \\ 10 \\ c \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$

**13 (a)** Given that the two lines intersect, find the value of $c$ and the coordinates of the point of intersection.

**[4 marks]**

$c =$ _____ and ( _____ , _____ )

**13 (b)** Find the cosine of the acute angle between the two lines.

**[4 marks]**

Answer _____

*Do not write outside the box*

**13 (c)** The point *A* has coordinates (6, 16, 7). The perpendicular from *A* to the line $l_1$ meets $l_1$ at the point *B*.

Find the coordinates of *B*.

**[5 marks]**

Answer _____

13

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