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Centre number		Candidate number	
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Forename(s)			
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INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM04) Unit FS2 – Further Statistics

Monday 24 June 2019 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.



For Examiner's Use				
Question	Mark			
1				
2				
3				
4				
5				
6				
7				
8				
TOTAL				



Answer all questions in the spaces provided.	Do not write outside the box
The random variable $X \sim N(\mu, 4^2)$	
A random sample of size 50 is taken.	
A hypothesis test is conducted at the 1% level of significance with the hypotheses	
$H_0: \mu = 75$	
H ₁ : μ > 75	
Find the power of the hypothesis test if the true population mean is equal to 77, giving	
your answer to two significant figures. [5 marks]	
Answer	
	5



A random sample of 20 tins of beans is taken. The variance of the mass of these tins of beans is 4 grams ² Assume that the mass of tins of beans is normally distributed. Investigate the claim at the 5% level of significance.	2	A business claims that the standard deviation of the mass of their tins of beans is 1.6 grams.	outside the box
Assume that the mass of tins of beans is normally distributed. Investigate the claim at the 5% level of significance.		A random sample of 20 tins of beans is taken. The variance of the mass of these tins of beans is 4 grams ²	
Investigate the claim at the 5% level of significance. [6 marks]		Assume that the mass of tins of beans is normally distributed.	
		Investigate the claim at the 5% level of significance. [6 marks]	
6			
			6



IB/M/Jun19/FM04

3	The random variable $X_{\rm i}$ has a Poisson distribution with mean $\lambda_{\rm i}$		outside the box
3 (a)	Show that the moment generating function of X_{i} is given by		
	$M_{X_i}(t) = e^{\lambda_i (e^t - 1)}$	[4 marks]	



Do not write

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(b)	The random variables X_1 and X_2 are independent.		box
	Given that $\lambda_1 = 2$ and $\lambda_2 = 3$, find the moment generating function of $X_1 + X_2$		
	Simplify your answer.		
		[2 marks]	
(c)	Describe the distribution of $X_1 + X_2$		
		[1 mark]	
			/
	Turn over for the next question		



3

The following table s	hows the contents of	a bag o	of coins		
	Coin (cents)	10	20	50	
	Frequency	25	55	20	
Find the mean μ of the mean	ne value of the coins	in the b	ag.		[1 mark]
A random sample of	two coins is selected	from th	ne bag i	n the fc	llowing way.
Step 1: A first coin is coin is put ba	s taken randomly from ack into the bag.	n the ba	ag, its v	alue is	recorded and then the
Step 2: A second co coin is put ba	in is taken randomly ack into the bag.	from th	e bag, i	ts value	e is recorded and then the
For example, a 10 ce	ent coin followed by a	20 cer	it coin is	s record	ded as (10, 20).
Write down all the dis	stinct possible sample	es.			[2 marks]
	The following table s Find the mean μ of the	The following table shows the contents of Coin (cents) Frequency Find the mean μ of the value of the coins A random sample of two coins is selected Step 1: A first coin is taken randomly fron coin is put back into the bag. Step 2: A second coin is taken randomly r coin is put back into the bag. For example, a 10 cent coin followed by a Write down all the distinct possible sample	The following table shows the contents of a bag of Coin (cents) 10 Frequency 25 Find the mean μ of the value of the coins in the b A random sample of two coins is selected from the Step 1: A first coin is taken randomly from the bag. Step 2: A second coin is taken randomly from the coin is put back into the bag. For example, a 10 cent coin followed by a 20 cert Write down all the distinct possible samples.	The following table shows the contents of a bag of coins Coin (cents) 10 20 Frequency 25 55 Find the mean μ of the value of the coins in the bag. A random sample of two coins is selected from the bag, its very coin is put back into the bag. Step 1: A first coin is taken randomly from the bag, its very coin is put back into the bag. For example, a 10 cent coin followed by a 20 cent coin is Write down all the distinct possible samples.	The following table shows the contents of a bag of coins.



4 (b) (ii)	The random variable X is the value of a coin taken randomly from the bag		Do not write outside the box
	Find the sampling distribution of the mean, \overline{X} , of the two coins.	[5 marks]	
4 (b) (iii)	Find $E(\overline{X})$.		
		[2 marks]	
			[]
			10



Devon is modelling the number of traffic accidents per day in a town.

Devon records the number of traffic accidents in a town each day for 100 days.

Devon records the results in the following table.

Accidents per day	0	1	2	3	4 or more
Frequency	22	30	33	13	2

There was a day with 4 accidents and a day with 7 accidents.

Devon claims that the number of accidents per day in the town can be modelled by a Poisson distribution.

Investigate Devon's claim using a 5% level of significance.

[10 marks]



Turn over ►	
	10
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6	A random variable <i>X</i> has a normal distribution with mean μ and variance σ^2 A sample is taken from <i>X</i> , taking the values $X_1,, X_n$		Do not write outside the box
6 (a)	Determine whether or not $\frac{\sum_{i=1}^{n} X_{i}}{\sum_{i=1}^{n} + 1}$		
	is a consistent estimator of μ .	[4 marks]	
6 (b) (i)	Show that		
	$E(X_i^2) = \sigma^2 + \mu^2$	[2 marks]	



6 (b) (ii)	Show that	Do not write outside the box
0 (D) (II)	$\sum_{i=1}^{n} X_{i}^{2}$	
	$\frac{\sum_{i=1}^{n}}{n} - \overline{X}^2$	
	is a biased estimator of σ^2	
	[5 marks]	
		11
	Turn over ►	



Sean mod distributio	lels the	e month known :	nly milk pr standard	roductio deviati	on of a co on 8 kilog	ow on grams	a farm s.	, in kilogr	ams, w	ith a normal
A random	samp	le of the	e monthly	milk p	roductior	i, <i>X</i> , c	of five co	ows on th	e farm	was taken.
The result	s can	be sum	marised	as						
		$\mathbf{\nabla}$	4.400				Σ^{2}	404000	0	
		$\sum x =$	= 4499		and		$\sum x =$	= 404899	3	
Using Sea	an's m	odel, co	onstruct a	ı 95% c	onfidenc	e inte	erval for	the mear	n month	ıly milk
production	n of a (cow on	tnis farm.							[4 marks]





		Do not write
7 (c)	Millie assumes the monthly milk production of a cow on the same farm is a normal distribution but with the standard deviation not known.	box
7 (c) (i)	Using Millie's assumption, construct a 95% confidence interval for the mean monthly milk	
	production by a cow on the farm. [5 marks]	



outside the box 7 (c) (ii) Sean and Millie each carry out a hypothesis test on the mean monthly milk production with the following hypotheses $H_0: \mu = 890$ $H_1: \mu \neq 890$ Sean and Millie each use their confidence interval to carry out the test. State whether Sean and Millie reach the same conclusion. Explain your answer. [2 marks] 15 Turn over for the next question



Do not write

8 A chocolate factory has two machines, **A** and **B**.

The chocolate produced in one hour, in tonnes, by the machines can be modelled by normal distributions.

Measurements of the chocolate produced during random 1 hour intervals by Machine **A** and Machine **B** were taken, which are summarised in the table below.

	Number of Measurements	Mean (tonnes)	Standard Deviation (tonnes)
Machine A	5	5.6	0.20
Machine B	7	6.0	0.24

Beth claims that on average, Machine **B** produces more chocolate in 1 hour than Machine **A**.

8 (a) Using a pooled estimate of variance, investigate at the 1% level of significance whether Beth's claim is valid.

[9 marks]



Do not write outside the box

8	(b)	State the assumption required for the test in part (a) to be valid.
-	(~)	

[1 mark]

Turn over for the next question



8	(c)	Investigate at the 10% level of significance whether the assumption stated in part (b)	Do not write outside the box
		is valid. [6 marks]	
			16
		END OF QUESTIONS	







Question number	Additional page, if required. Write the question numbers in the left-hand margin.



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