OXFORD

INTERNATIONAL AQA EXAMINATIONS

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INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 − Pure Maths, Statistics and Mechanics

Friday 31 May 2019 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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10	
11	
12	
TOTAL	

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Section A

Pure Maths

Answer **all** questions in the spaces provided.

1 A curve passes through the point (5, 7.2) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x + 2y}$$

Use Euler's step-by-step method with a step length of 0.05 to estimate the value of y when $x = 5.1$

Give your answer to four decimal places.

[5 marks]

Answer _____

5

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2 The variables x and y are related by an equation of the form

$$y = a\,b^x$$

where a and b are constants.

Let $Y = \log_{10} y$

2 (a) Show that there is a linear relationship between x and Y.

[3 marks]

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2 (b) It is given that $y = 27$ when $x = 3$ and that $y = 12$ when $x = 8$

Use the linear relationship from part **(a)** to find the value of a and the value of b.

Give your answers to 3 significant figures.

[5 marks]

$a =$ _____

$b =$ _____

8

Turn over for the next question

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3 The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

3 (a) (i) $\mathbf{A}$

[2 marks]

Answer _____

3 (a) (ii) $\mathbf{B}$

[2 marks]

Answer _____

3 (a) (iii) $\mathbf{C}$

[2 marks]

Answer _____

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3 (b) The matrix $\mathbf{M}$ represents a stretch followed by an enlargement, and one of the following statements is true.

$\mathbf{M = AB}$ $\mathbf{M = BA}$ $\mathbf{M = BC}$ $\mathbf{M = CB}$ $\mathbf{M = CA}$ $\mathbf{M = AC}$

Calculate the matrix $\mathbf{M}$.

[2 marks]

Answer _____

3 (c) Triangle T_1 has area 1.5 square units.

The transformation represented by $\mathbf{M}$ maps triangle T_1 to a triangle T_2

Find the area of T_2

[3 marks]

Answer _____

11

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4 The function f is defined by $f(x) = x^4 - 17x^2 - 5x - 12$

The equation $f(x) = 0$ has one positive root, α.

4 (a) Show that α lies in the interval $4.3 < x < 4.4$

[2 marks]

4 (b) Starting from the interval $4.3 < \alpha < 4.4$, use interval bisection **twice** to find an interval of width 0.025 within which α must lie.

[3 marks]

Answer _____

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4 (c) Taking $x_1 = 4.3$ as a first approximation to α, use the Newton-Raphson method to find a third approximation, x_3, to α.

Give your answer to 3 decimal places.

[5 marks]

Answer _____

10

Turn over for the next question

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5 The matrices $\mathbf{C}$ and $\mathbf{D}$ are defined by

$$\mathbf{C} = \begin{bmatrix} 5 & -1 & 0 \\ 0 & 1 & 3 \\ 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} -4 & 3 & -1 \\ 2 & 10 & -11 \\ -3 & -15 & 5 \end{bmatrix}$$

5 (a) Show that

$$\mathbf{CD}^{\mathrm{T}} = k\mathbf{I}$$

where $\mathbf{D}^{\mathrm{T}}$ is the transpose of $\mathbf{D}$, $\mathbf{I}$ is the 3×3 identity matrix and k is an integer.

[3 marks]

5 (b) Hence, without doing any further numerical calculations, prove that

$$\mathbf{CD}^{\mathrm{T}} = \mathbf{DC}^{\mathrm{T}}$$

[3 marks]

6

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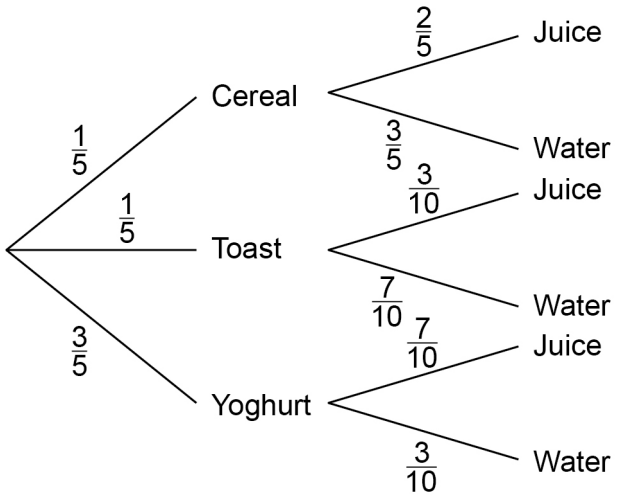
Section B

Statistics

Answer **all** questions in the spaces provided.

6 For breakfast, Barbara chooses one food item and one drink.

The probability of her making each choice is shown on the tree diagram.



Find the exact probability that Barbara chooses cereal given that she chooses juice.

[3 marks]

Answer _____

3

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7 Let X and Y be random variables such that Var(X) = 2, Var(Y) = 8 and ρ = 0.78, where ρ is the product moment correlation coefficient of X and Y.

Find Var($X + Y$).

[4 marks]

Answer _____

4

Turn over for the next question

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8 The random variable X has a geometric distribution with parameter p.

The variance of X is 6

8 (a) Find p.

[3 marks]

Answer _____

8 (b) Find $\mathrm{E}(X)$.

[1 mark]

Answer _____

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8 (c) Find $P(X \leq 2)$

[2 marks]

Answer _____

6

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9 The random variable Y has generating function

$$G_Y(t) = \frac{1}{8} + \frac{1}{16}t + \frac{5}{8}t^2 + \frac{3}{16}t^3$$

9 (a) Find $P(Y \geq 2)$

[2 marks]

Answer _____

9 (b) Using differentiation

9 (b) (i) find the exact value of the mean of Y

[2 marks]

Answer _____

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9 (b) (ii) find the exact value of the variance of Y.

[3 marks]

Answer

7

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Section C

Mechanics

Answer **all** questions in the spaces provided.

10 Two cyclists, A and B, are travelling on horizontal ground.

Initially A is z metres due south of B.

The velocity of A is $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1} and the velocity of B is $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

The distance between the two cyclists is a minimum after 28 seconds.

Find z.

[5 marks]

Answer _____

5

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11 The escape speed, v m s^{-1}, for an object is the minimum speed needed for the object to escape the gravitational field of a planet.

A model for the escape speed is

$$v = kM^a r^b G^c$$

where: k is a dimensionless constant,
M is the mass of the planet in kilograms,
r is the radius of the planet in metres,
and G is the universal gravitational constant, which has units m^3 kg^{-1} s^{-2}

Find the value of a, the value of b and the value of c.

[5 marks]

$a =$ _____

$b =$ _____

$c =$ _____

5

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12 A disc, of mass 0.25 kg, is sliding on a smooth horizontal surface at a speed of 4.8 m s^{-1}

The disc hits a vertical wall that is perpendicular to its path.

The disc rebounds and travels in the opposite direction along its original path.

At time t seconds after the disc first makes contact with the wall, the magnitude of the force, $F(t)$ newtons, acting on the disc is modelled as

$$F(t) = kt^2 (t - 0.1)^2$$

where k is a constant and $0 \leq t \leq 0.1$

The model assumes that the disc is in contact with the wall for 0.1 seconds.

12 (a) Calculate, in terms of k, the magnitude of the maximum force on the disc.

[2 marks]

Answer _____

12 (b) Find, in terms of k, the magnitude of the impulse on the wall.

[4 marks]

Answer _____

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12 (c) The coefficient of restitution between the disc and the wall is 0.6

Find the value of k.

[4 marks]

Answer _____

END OF QUESTIONS

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