OXFORD
INTERNATIONAL AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA03

Pure Mathematics Unit P2

Mark scheme

June 2019

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I96yMA03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

<table>
<tr><th>Q1</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>$3^0 = 1$</td></tr>
<tr><td>0.5</td><td>$3^{0.5} = 1.73205$</td></tr>
<tr><td>1</td><td>$3^1 = 3$</td></tr>
<tr><td>1.5</td><td>$3^{1.5} = 5.19615$</td></tr>
<tr><td>2</td><td>$3^2 = 9$</td></tr>
<tr><td>2.5</td><td>$3^{2.5} = 15.58846$</td></tr>
<tr><td>3</td><td>$3^3 = 27$</td></tr>
</table>

$\frac{1}{3} \times 0.5 \times [1 + 27 + 4(1.73205 + 5.19615$
$+ 15.58846) + 2(3 + 9)]$
$= 23.678$</td><td>B1

M1

m1

A1</td><td></td><td>All 7 correct x values (and no extra used) PI by 7 correct y values

At least 6 correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula) (PI by AWRT correct answer)

Correct substitution into formula with $h = 0.5$ oe and at least 6 correct y values either listed, with + signs, or totalled. (PI by AWRT correct answer)
CAO, must see this value exactly and no error seen</td></tr>
<tr><td></td><td></td><td></td><td>4</td><td></td></tr>
<tr><td>(b)(i)</td><td>$f(x) = 3^x - 12 + 4x$
$f(1.5) = 3^{1.5} - 12 + 4 \times 1.5 = -0.80...$
$f(1.6) = 3^{1.6} - 12 + 4 \times 1.6 = 0.199...$

Change of sign, $1.5 < \alpha < 1.6$</td><td>M1

A1</td><td></td><td>Or reverse
Both values rounded or truncated to at least 1sf

Must have both statement and interval in words or symbols
or comparing 2 sides:
at $1.5,\ 3^{1.5} < 6$;
at $1.6,\ 3^{1.6} > 5.6 = 0.8(\ldots)$ (M1)
Conclusion as before (A1)</td></tr>
<tr><td></td><td></td><td></td><td>2</td><td></td></tr>
<tr><td>(b)(ii)</td><td>$x_2 = 1.631$
$x_3 = 1.548$</td><td>B1
B1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>2</td><td></td></tr>
<tr><td></td><td>Total</td><td></td><td>8</td><td></td></tr>
</table>

Q2	Solution	Mark	Total	Comment
(a)(i)	48500	B1		
			1	
(a)(ii)	$50\,000\times(1-0.03)^{10}$ $=36900$	M1 A1		 Condone 36871
			2	
(a)(iii)	$50000 \div (1-0.03)^{10}$ $=67800$	M1 A1		 Condone 67803 or 67804
			2	
(b)	$25\,000\ \times 1.015^{t}$ $[50\,000\times 0.97^{t}=25\,000\ \times 1.015^{t}]$ $t=\dfrac{\ln(50\,000\ /\ 25\,000)}{\ln(1.015\ /\ 0.97)}$ $t=15.3...$ 2035	B1 M1 A1 A1F		 Attempt at solving an equation of the form $50\,000\times a^{t}=25\,000\ \times b^{t}$ using logarithms CAO FT (2019 + their t), but their t must be rounded up
			4	
	Total		9	

Q3	Solution	Mark	Total	Comment
(a)	$4(1.5)^3 + b(1.5)^2 + c(1.5) + 6 = -6$ $4(-0.5)^3 + b(-0.5)^2 + c(-0.5) + 6 = 10$	**M1**		One correct substitution OR for **M1** use of long division
	$\frac{9}{4}b + \frac{3}{2}c = -\frac{51}{2}$ $\frac{1}{4}b - \frac{1}{2}c = \frac{9}{2}$	**A1**		Both three-term equations correct PI
		m1		Attempt to solve
	$b = -4$ $c = -11$	**A1**		Both answers correct
			4	
(b)	$\frac{(2x-1)(2x+1)}{(2x-1)(2x+3)}$	**B1** **B1**		Factorising numerator Factorising denominator
	$= \frac{(2x+1)}{(2x+3)}$	**M1**		
	$= 1 - \frac{2}{2x+3}$	**A1**		
	OR			
	$\frac{4x^2 + 4x - 3 - 4x + 2}{4x^2 + 4x - 3}$	**(M1)**		
	$= 1 + \frac{-2(2x-1)}{(2x-1)(2x+3)}$	**(B1)** **(B1)**		
	$= 1 - \frac{2}{2x+3}$	**(A1)**		
			4	
	Total		**8**	

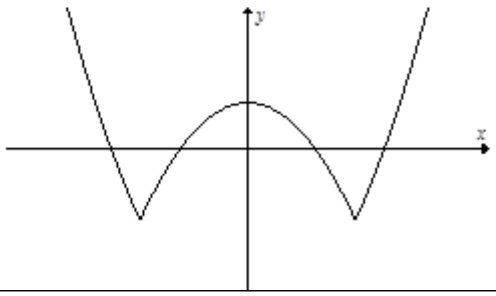
Q4	Solution	Mark	Total	Comment
(a)(i)	$[3\cos\theta - 4\sin\theta =]$ $R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	**M1**		PI by correct value for R or α
	$R = 5$	**A1**		
	$\alpha = 0.927$	**A1**		
			3	
(a)(ii)	$\cos(y - 0.1 + 0.927) = \frac{2.5}{5}$	**M1**		FT their **(a)**
	$y = -1.87$	**A1**		
	$y = 0.22$	**A1**		
			3	
(b)	$7\tan^2 x = 7\sec^2 x - 7 \quad [= 13 - 4\sec x]$ $7\sec^2 x - 7 = 13 - 4\sec x$ $7\sec^2 x + 4\sec x - 20 = 0$	**M1**		Correct use of trig identity PI
	$(7\sec x - 10)(\sec x + 2)[= 0]$	**m1**		Factorisation or correct use of formula
	$\sec x = \frac{10}{7}, -2$ $\cos x = 0.7,\ -0.5$	**A1**		Both correct and no errors seen
	$x = -46°,\ 46°,\ 120°,\ 240°$	**B1**		Sight of any one of these values correct or more accurate
		B1		All 4 correct and no extras in interval (ignore answers outside interval)
			5	
	Total		**11**	

Q5	Solution	Mark	Total	Comment
(a)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{3}{3x+2}$	M1 A1		
			2	
(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{x^2Ae^{3x}-Bxe^{3x}}{x^4}$	M1		Or: $\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]x^{-2}Ae^{3x}+Bx^{-3}e^{3x}$
	$A=3,\ B=2$	A1		$A=3,\ B=-2$
	$[=]\ \frac{3x^2-2x}{x^2}\times y$	A1		ACF, ISW, condone $\frac{e^{3x}}{x^2}$ in place of y
			3	
(c)	$2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y+2y\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}$	M1 A1		Either implicit differential correct All correct
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(-x^{-2}-2y)}{(2x+2y)}\right]$			
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\right]\quad 2y=-\frac{1}{x^2}\ \text{ or }\ y=-\frac{1}{2x^2}$	B1		
	$2x\times\frac{-1}{2x^2}+\frac{1}{4x^4}=\frac{1}{x}$	m1		Substituting their y into original equation
	$8x^3=1$	A1		
	$x=0.5,\quad y=-2$	A1		CAO
			6	
	Total		11	

Q6	Solution	Mark	Total	Comment
(a)	$\sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$	B1		
	$= 2\sin x\cos^2 x + (1-2\sin^2 x)\sin x$	M1		Correct use of double angle formulae
	$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$			Must see this line of working
	$= 3\sin x - 4\sin^3 x$	A1		AG, no errors seen
			3	
(b)	$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$	B1		PI
	$\left[\int\left(A\sin x - B\sin 3x\right)\mathrm{d}x =\right]$ $\left[\frac{1}{4}\right]\left(-A\cos x + \frac{B}{3}\cos 3x\right)\ [+c]$	M1		
	$-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x\ \ [+c]$	A1		
			3	
	Total		6	

Q7	Solution	Mark	Total	Comment
(a)	$\dfrac{dx}{d\theta}=\dfrac{[\cos\theta\times 0]-1(-\sin\theta)}{\cos^2\theta}$	M1		
	$\left[\dfrac{dx}{d\theta}=\right]\dfrac{\sin\theta}{\cos\theta}\times\dfrac{1}{\cos\theta}$			Must see this line
	$=\sec\theta\tan\theta$	A1		AG, no errors seen
			2	
(b)	$\left[\dfrac{dx}{d\theta}=\right]2\sec\theta\tan\theta$	B1		
	$\left[\int\dfrac{1}{x^2\sqrt{x^2-4}}\,dx=\right]\int\dfrac{2\sec\theta\tan\theta\,d\theta}{4\sec^2\theta\sqrt{4\sec^2\theta-4}}$	M1		All in terms of θ, condone omission of $d\theta$
		B1		Correct use of $\sqrt{4\sec^2\theta-4}=2\tan\theta$
	$=\int\dfrac{\tan\theta}{4\sec\theta\tan\theta}\,d\theta$	A1		Must have seen $d\theta$ here, or earlier
	$=\dfrac{1}{4}\int\cos\theta\,d\theta$			
	$=\dfrac{1}{4}\sin\theta$	A1		
	$\left[\dfrac{1}{4}\left[\left(1-\dfrac{4}{x^2}\right)^{1/2}\right]_{\frac{4}{3}\sqrt{3}}^{4}\right]$			
	$=\dfrac{1}{4}\left(1-\dfrac{1}{4}\right)^{1/2}-\dfrac{1}{4}\left(1-\dfrac{3}{4}\right)^{1/2}$	m1		Substituting the original limits into $\left[\dfrac{1}{4}\right]\left(1-\dfrac{4}{x^2}\right)^{1/2}$ or $\theta=\dfrac{\pi}{6}$ and $\theta=\dfrac{\pi}{3}$ into $\left[\dfrac{1}{4}\right]\sin\theta$
	$=\dfrac{1}{8}\left(\sqrt{3}-1\right)$	A1		
			7	
	Total		9	

Q8	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$	M1		Either derivative correct
	$\frac{\mathrm{d}y}{\mathrm{d}t} = 3 - 2t$	A1		Both derivatives correct
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3-2t}{\frac{1}{(t+1)^2}}$			
	$y = 0 \quad \Rightarrow \quad t = 0, 3$	B1		
	$t = 0 \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -3$	m1		Either value correct
	$t = 3 \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = 48$	A1		Both values correct and no others
			5	
(b)	$t = \frac{1}{x} - 1$	M1		oe, isolating t
	$y = 3\left(\frac{1}{x} - 1\right) - \left(\frac{1}{x} - 1\right)^2$	A1		
	$yx^2 = 3x(1-x) - (1-x)^2$	m1		Eliminates fractions by multiplying throughout by x^2
	$yx^2 = (1-x)(4x-1)$	A1		oe
			4	
	Total		9	

Q9	Solution	Mark	Total	Comment
(a)(i)	$-3 \le \mathrm{f}(x) \le 17$	B1		Condone use of y or f
			1	
(a)(ii)		B1 M1 A1		y-intercept: 2 or $(0, 2)$ Graph symmetrical about y-axis with three distinct sections Correct graph with cusps and correct curvature in all three sections
			3	
(a)(iii)	$\left\|x^2-5\right\|=4$ $x^2-5=4$ or $5-x^2=4$ $x=1,\ 3$ $x=-1,\ -3$	 M1 A1 A1		 PI by at least 2 correct values
			3	
(b)(i)	$[\mathrm{fg}(x)=]\left\|\dfrac{1}{x^2}-5\right\|-3$	B1		
			1	
(b)(ii)	$\left\|\dfrac{1}{x^2}-5\right\|[=]3$ $\dfrac{1}{x^2}-5[=]3$ or $\dfrac{1}{x^2}-5[=]-3$ $x^2[=]\dfrac{1}{8},\ \dfrac{1}{2}$ $\mathrm{fg}(x)<0$ $-\dfrac{1}{\sqrt{2}}<x<-\dfrac{1}{\sqrt{8}}$ $\dfrac{1}{\sqrt{8}}<x<\dfrac{1}{\sqrt{2}}$	 B1 M1 A1		 oe, one correct interval Both correct intervals and no others
			3	
	Total		11	

Q10	Solution	Mark	Total	Comment
(a)	$\int \frac{\mathrm{d}y}{y} = \int \frac{\mathrm{d}x}{\sqrt{2x-1}}$	**M1**		Separate variables
	$\ln y = (2x-1)^{0.5} \times 2 \times \frac{1}{2}[+c]$	**A1**		Integrating correctly
	$y = 1,\ x = 5$ $\ln 1 = 3 + c$ $c = -3$	**m1**		Attempt to find c
	$\ln y = \sqrt{(2x-1)} - 3$	**A1**		ACF, ISW
			4	
(b)	$y = \mathrm{e}^4$ $\ln \mathrm{e}^4 = \sqrt{(2x-1)} - 3$ $7 = \sqrt{(2x-1)}$	**M1**		Substitution into their answer to part **(a)** and attempt to isolate
	$x = 25$	**A1**		CAO
			2	
	Total		**6**	

Q11	Solution	Mark	Total	Comment
(a)	$\mathrm{f}(x) = \dfrac{A}{1-x} + \dfrac{B}{(1-x)^2} + \dfrac{C}{1+2x}$			
	$6 = A(1-x)(1+2x) + B(1+2x) + C(1-x)^2$	B1		Correctly eliminating fractions
	$x = 1,\ 6 = 3B,$	M1		Attempt at finding one constant
	$B = 2$	A1		
	$x = -0.5,\ 6 = \dfrac{9}{4}C,$			
	$C = \dfrac{8}{3}$	A1		
	$\text{Coef of } x^2 : 0 = -2A + C,\quad A = \dfrac{4}{3}$	A1		
	$\mathrm{f}(x) = \dfrac{4}{3(1-x)} + \dfrac{2}{(1-x)^2} + \dfrac{8}{3(1+2x)}$			
			5	
(b)	$(1-x)^{-1} = 1 + x + x^2 + x^3$	B1		
			1	
(c)	$\mathrm{f}(x):$			
	$\dfrac{4}{3}(1-x)^{-1} = \dfrac{4}{3}(1 + x + x^2 + x^3)$			
	$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3$	M1 A1		At least three terms correct All correct
	$(1+2x)^{-1} = 1 - 2x + 4x^2 - 8x^3$	M1 A1		At least three terms correct All correct
	$\mathrm{f}(x):$			
	$\dfrac{4}{3}(1 + x + x^2 + x^3) + 2(1 + 2x + 3x^2 + 4x^3)$ $+\dfrac{8}{3}(1 - 2x + 4x^2 - 8x^3)$	m1		Correct substitution of their expansions into their part **(a)** & **(b)**, but must have scored at least **M1 A0 M1 A0**
	$\mathrm{f}(x) = 6 + 18x^2 - 12x^3$	A1		
			6	
	Total		**12**	

Q12	Solution	Mark	Total	Comment
	[Vol =] $\pi\int_0^1 (xe^{-1.5x})^2\ dx$	**B1**		Correct including π, limits, dx
	$(xe^{-1.5x})^2 = x^2e^{-3x}$			
	$u = x^2 \quad dv = e^{-3x}$ $du = 2x \quad v = -\frac{1}{3}e^{-3x}$	**M1**		Correct identification of u and dv, du correct and attempt at v PI
	$\left[\int \ =\right] -\frac{1}{3}x^2e^{-3x} + k\int xe^{-3x}dx$	**A1**		$k \neq 0$
	$\int xe^{-3x}dx = -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}$	**B1**		Correct integration
	$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x}$	**A1**		Correct simplified integral
	$= \left(-\frac{1}{3}e^{-3} - \frac{2}{9}e^{-3} - \frac{2}{27}e^{-3}\right) - \left(-\frac{2}{27}\right)$	**m1**		Correct substitution of correct limits into their integral of the form $= ax^2e^{-3x} + bxe^{-3x} + ce^{-3x}$
	$\text{Vol} = \pi\left(\frac{2}{27} - \frac{17}{27}e^{-3}\right)$	**A1**		
	Total		**7**	

Q13	Solution	Mark	Total	Comment
(a)	$-4-3\lambda=6+4\mu$ $1-4\lambda=10+\mu$	**M1**		Pair of simultaneous equations
	$\lambda=-2,$ $\mu=-1$	**A1**		
	$-5-5\lambda=c+\mu$ $c=6$	**A1**		
	$(2,\ 9,\ 5)$	**A1**		
			4	
(b)	$\begin{pmatrix}-3\\-4\\-5\end{pmatrix}.\begin{pmatrix}4\\1\\1\end{pmatrix}=-21$	**M1** **A1**		For forming scalar product Correct value of scalar product
	$\cos\theta=\pm\dfrac{-21}{\sqrt{3^2+4^2+5^2}\times\sqrt{4^2+1^2+1^2}}$	**M1**		oe, with their –21, PI by AWRT $\theta=134°$
	$\cos\theta=\dfrac{7}{10}$	**A1**		ISW, PI by AWRT $\theta=46°$
			4	
(c)	Direction of perpendicular: $\begin{pmatrix}10+3p\\15+4p\\12+5p\end{pmatrix}$	**B1**		oe
	$\begin{pmatrix}10+3p\\15+4p\\12+5p\end{pmatrix}.\begin{pmatrix}-3\\-4\\-5\end{pmatrix}[=0]$	**M1**		
	$50p=-150$ $p=-3$	**A1**		oe
	$\begin{pmatrix}-4\\1\\-5\end{pmatrix}+(-3)\times\begin{pmatrix}-3\\-4\\-5\end{pmatrix}$	**m1**		Substituting their p into l_1
	$(5,\ 13,\ 10)$	**A1**		CAO
			5	
	Total		**13**	
	TOTAL		**120**	