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FM01 Further Pure Mathematics Unit FP1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark		Comments
1(a)	$16 + 32h + 24h^2 + 8h^3 + h^4$	B1	1	
(b)(i)	$y_{2+h} = 20 + 36h + 25h^2 + 8h^3 + h^4$	B1		
	Use of correct formula for gradient	M1		
	Gradient is $36 + 25h + 8h^2 + h^3$	A1	3	
(b)(ii)	As $h \to 0$ this $\to 36$	E2, 1ft	2	E1 for “$h = 0$"
	Total	**6**		

2	$3z - z^* = 4 + 20\mathrm{i}$	B1		
	Other root = $4 - 20\mathrm{i}$	M1		
	$b = -8$	A1		
	$c = 416$	A1	4	
	Total	**4**		

3(a)	$\mathrm{f}(r+1) - \mathrm{f}(r) = (r+1)^3 + (r+1)^2 - (r^3 + r^2)$	M1		
	$= 3r^2 + 5r + 2$	A1		or other reasonable simplification
	$= (r+1)(3r+2)$	A1	3	
(b)	$25 \times 74 = \mathrm{f}(25) - \mathrm{f}(24)$ $26 \times 77 = \mathrm{f}(26) - \mathrm{f}(25)$ ⋮	M1		
	$62 \times 185 = \mathrm{f}(62) - \mathrm{f}(61)$ $63 \times 188 = \mathrm{f}(63) - \mathrm{f}(62)$	M1		
	$\mathrm{f}(63) - \mathrm{f}(24)$	M1		Use of f(63) – f(23) gives 241320 (2 Marks)
	239616	A1	4	Correct answer using formulae for Σr^2 etc gains 2 marks (not done “hence”)
	Total	**7**		

Q	Answer	Mark		Comments
4(a)	$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ or $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$	B1		or $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$
	Use of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $\left(4x - \frac{\pi}{6}\right)$ to x	m1		including division of all terms by 4
	$x = \frac{n\pi}{2} - \frac{\pi}{24}$ or $x = \frac{n\pi}{2} - \frac{\pi}{8}$	A1 A1	5	oe
4(b)	Choice of $n = 15$	M1		oe; must be consistent with their answer to part (a)
	$\frac{59\pi}{8}$	A1	2	
	Total 7			

5	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2x^{-3}$	M1		
	$= -\frac{1}{4}$ when $x = 2$	A1		
	$\delta y \cong \frac{\mathrm{d}y}{\mathrm{d}x} \times \delta x$	M1		
	$\delta x = 0.08$	M1		
	$-\frac{1}{4} \times 0.08$ or -0.02	M1		PI
	0.23	A1		CSO
	Total 6			

Q	Answer	Mark		Comments
6(a)	$\alpha+\beta=-\frac{5}{3}$	B1		
	$\alpha\beta=3$	B1	2	
6(b)	$(\alpha+\beta)^2-2\alpha\beta$	M1		
	$-\frac{29}{9}$	A1	2	
6(c)	Sum of roots $=\frac{\alpha(\alpha+1)+\beta(\beta+1)}{(\alpha+1)(\beta+1)}$	M1		
	$=\frac{\alpha^2+\beta^2+\alpha+\beta}{\alpha\beta+\alpha+\beta+1}$	M1		
	$=-\frac{44}{21}$	A1		PI
	Product of roots $=\frac{\alpha\beta}{\alpha\beta+\alpha+\beta+1}$	M1		
	$=\frac{27}{21}$	A1		PI
	$21x^2+44x+27=0$	A1	6	oe (integer coefficients)
	Total	**10**		

7(a)	$y^2=\frac{6x}{k}$	B1		Must be stated explicitly
	$k(x+5)^2=6x$	M1		
	$k(x^2+10x+25)-6x=0$ $kx^2+(10k-6)x+25k=0$	A1	3	For both lines
7(b)	For equal roots $(10k-6)^2-4(25k)(k)=0$	M1		
	$100k^2-120k+36-100k^2=0$	M1		
	$-120k+36=0$ so $k=0.3$	A1		
	$y^2=20x$	A1	4	
	Total	**7**		

Q	Answer	Mark		Comments
8(a)	$\int_q^r x^{-\frac{1}{3}}\mathrm{d}x$	M1	3	
	$= \left[\frac{3}{2}x^{\frac{2}{3}}\right]_q^r$	M1		
	$\frac{3}{2}\left(r^{\frac{2}{3}} - q^{\frac{2}{3}}\right)$	A1		oe
8(b)	(A)$\lim_{q\to 0}\left(q^{\frac{2}{3}}\right) = 0$ and $8^{\frac{2}{3}} = 4$	M1	3	
	6	A1		
	(B) $\lim_{n\to\infty}(r^{\frac{2}{3}})$ is not defined, (or tends to infinity) so the integral has no finite value.	E1		Explanation needed, not just "Integral = ∞"
Total	**6**			

Q	Answer	Mark		Comments
9(a)	$y = 0$ $x = 1$ and $x = -4$	B1 B1	2	
9(b)	$k(x-1)(x+4) = x+2$ $kx^2 + (3k-1)x - (4k+2) = 0$ For real roots $(3k-1)^2 + 4k(4k+2) \geq 0$ $24k^2 + (k+1)^2 \geq 0$ (or > 0) Always true so there are real roots for all real k	M1 A1 m1 A1 E1	5	Shows as sum of squares
9(c)	$(x+2)(x^2+3x-5) = 0$ $x = -2, \dfrac{-3 \pm \sqrt{29}}{2}$ $(-2,0), \left(\dfrac{-3+\sqrt{29}}{2}, \dfrac{1+\sqrt{29}}{2}\right), \left(\dfrac{-3-\sqrt{29}}{2}, \dfrac{1-\sqrt{29}}{2}\right)$	M1 A1 A1	3	M1 A1 A0 for (1.19, 3.19) , (-4.19, -2.19) and (-2, 0) Max. 1 mark if (-2, 0) omitted
9(d)	Correct shape of curves Asymptotes Axis intercepts (0,-0.5) and (-2,0) All correct including line	B1 B1 B1 B1	4	Can be awarded without 3rd B1 or if asymptotes are drawn but not labelled
	Total 14			

Q	Answer	Mark		Comments
10(a)	$\lvert z-2-2\mathrm{i}\rvert = \lvert z-4-\mathrm{i}\rvert$	M1		
	$(x-2)^2+(y-2)^2=(x-4)^2+(y-1)^2$ $y=0 \Longrightarrow (x-2)^2+4=(x-4)^2+1$	M1		for either
	$x=\frac{9}{4}$	A1		
	$r^2=\left(\frac{1}{4}\right)^2+2^2$	M1		or $r^2=\left(\frac{7}{4}\right)^2+1^2$
	$r=\frac{\sqrt{65}}{4}$	A1		
	$\left\lvert z-\frac{9}{4}\right\rvert=\frac{\sqrt{65}}{4}$	A1	6	
10(b)	Circle to the right of imaginary axis	B1		
	Symmetrical about real axis	B1	2	
10(c)	$\sin\theta=\frac{\sqrt{65}}{9}$	M1		
	$\lvert z_1\rvert^2=\left(\frac{9}{4}\right)^2-\left(\frac{\sqrt{65}}{4}\right)^2$	M1		
	$\lvert z_1\rvert=1$	A1		
	$\cos\theta=\frac{4}{9}$	M1		
	$z_1=\frac{1}{9}\left(4+\mathrm{i}\sqrt{65}\right)$	A1	5	
		Total 13		