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FM05

(9665/FM05) – Further Mechanics Unit 2

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1 (a)	$0.1 \times 9.8 \times 0.8(1-\cos 40°) = \frac{1}{2} \times 0.1v^2$ $v = \sqrt{15.68(1-\cos 40°)} = 1.9 \text{ m s}^{-1}$	M1 A1 A1 **3**	M1: GPE found using either cos40° or sin 40° A1:Correct energy equation A1: Correct speed. AWRT 1.9
1 (b)	$T - 0.1 \times 9.8 = 0.1 \times \frac{15.68(1-\cos 40°)}{0.8}$ $T = 1.4 \text{ N}$	M1 A1 A1 **3**	M1: Equation of motion using radial acceleration formula. Allow their speed from part (a). A1: Correct three term equation of motion. A1: Correct tension. AWRT 1.4
	Total	**6**	

Q	Answer	Mark	Comments
2 (a)	$\frac{\lambda \times 30^2}{2 \times 20} = 75 \times 9.8 \times 50$ $\lambda = \frac{1470000}{900} = \frac{4900}{3} = 1600 \text{ N}$	M1 B1 A1 **3**	M1: Correct GPE used in two term energy equation. B1: Correct initial EPE. A1: Correct modulus. AWRT 1600
2 (b)	Max speed when: $\frac{4900 \times e}{3 \times 20} = 75 \times 9.8$ $e = 9 \text{ m}$ Speed given by: $29 \times 75 \times 9.8 = \frac{1}{2} \times 75v^2 + \frac{4900 \times 9^2}{2 \times 3 \times 20}$ $v = \sqrt{480.2} = 22 \text{ m s}^{-1}$	M1 B1 A1 M1A1 A1 A1 **7**	M1: Tension and weight equated. B1: Correct tension A1: Correct extension for equilibrium. M1: Energy equation with at least two terms correct with any signs A1: Correct equation. A1: Correct speed. AWRT 22
2 (c)	No air resistance. Bungee Jumper is a particle	B1 **1**	B1: Two appropriate assumptions.
	Total	**11**	

Q	Answer	Mark	Comments
3 (a)	$5\cos 30° = v\cos\alpha$ $0.4 \times 5\sin 30° = v\sin\alpha$ $\frac{\sin\alpha}{\cos\alpha} = \frac{2\sin 30°}{5\cos 30°}$ $\tan\alpha = \frac{2\sqrt{3}}{15}$ $\alpha = 13.0039°$ $\alpha = 13°$	M1A1 M1A1 M1A1 A1 **7**	M1: Equation for motion parallel to wall. A1: Correct equation. M1: Equation for motion perpendicular to wall. Must include 0.4 A1: Correct equation. M1: Expression for tan *α* A1: Correct expression. A1: Correct angle. AWRT 13°
3 (b)	$v = \frac{5\cos 30°}{\cos\alpha}$ $v = 4.4$ Or $v = \frac{2\sin 30°}{\sin\alpha}$ $v = 4.4$	M1A1 A1 (M1A1) (A1) **3**	M1: Equation with v as the only unknown. A1: Correct equation. A1: Correct value for v. AWRT 4.4
3 (c)	$I = 0.5 \times 4.44\sin 13° - 0.5(-5\sin 30°)$ $I = 1.7$ N s	M1A1 A1 **3**	M1: Impulse equation with correct values and any signs. A1: Correct equation. A1: Correct impulse AWFW [1.7, 1.8]
	Total	**13**	

Q	Answer	Mark	Comments
4 (a)	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = -kv$ $m\frac{\mathrm{d}v}{\mathrm{d}x} = -k$	M1 A1 **2**	M1: Differential equation with correct terms and any signs. A1: Simplified correct differential equation.
4 (b)	$mv = -kx + c$ Using $x = 0, v = U$ $mU = c$ Using $v = 0$ $0 = -kx + mU$ $x = \frac{mU}{k}$	M1 A1 M1 A1 **4**	M1: Integrating their equation. Condone omission of *c*. A1: Correct integration. M1: Initial values used to find *c*. A1: Correct value of *c* and correct final answer from correct working.
4 (c)	$m\frac{dv}{dt} = -kv$ $\frac{m}{v}\frac{dv}{dt} = -k$ $m\ln(v) = -kt + c$ Using t= $0, v = U$ $m\ln(U) = c$ Using $v = \frac{U}{2}$ $m\ln\left(\frac{U}{2}\right) = -kt + m\ln(U)$ $t = \frac{m}{k}\ln(2)$	M1 M1 A1 M1 A1 M1 A1 **7**	M1: Differential equation with correct terms and any signs. M1: Integrating their equation. Condone omission of *c*. A1: Correct integrals. M1: Initial values used to find *c*. A1: Correct value of *c.* M1: Substitutes $\frac{U}{2}$. A1: Correct time.
	Total	**13**	

Q	Answer	Mark	Comments
5 (a)	$1.5 = 0.05\omega$ $\omega = 30$ $\text{Period} = \frac{2\pi}{30} = 0.209\text{ s}$	M1 A1 A1 **3**	M1: Max speed used to form an equation to find ω. A1: Correct ω. A1: Correct period. AWRT 0.21
5 (b)	As SHM $0.5\ddot{x} = -kx$ $2k = 30^2$ $k = \frac{900}{2} = 450\text{ N m}^{-1}$	 M1 A1 A1 **3**	M1: Differential equation in terms of k. A1: Correct equation for k. A1: Correct k.
5 (c)	In equilibrium $0.5 \times 9.8 = 450e$ $e = \frac{4.9}{450} = 0.0109\text{ m}$ $\text{Max extension} = 0.0109 + 0.05$ $= 0.0609\text{ m}$ $\text{Max length} = 0.0609 + 0.5 = 0.561\text{ m}$	M1 A1 A1 A1 **4**	M1: Equation to find extension in equilibrium. A1: Correct extension. A1: Correct maximum extension. A1: Correct maximum length. AWRT 0.56
5 (d)	$v^2 = 30^2(0.05^2 - 0.0109^2)$ $v = 1.46\text{ m s}^{-1}$	M1A1 A1 **3**	M1: Use of SHM equation with correct values. The terms x and a may be interchanged. A1: Correct equation. A1: Correct speed. AWRT 1.5
	Total	**13**	

Q	Answer	Mark	Comments
6 (a)	$y = U\sin\theta t - \frac{1}{2}g\cos 30° t^2$ $0 = U\sin\theta t - \frac{\sqrt{3}}{4}gt^2$ $t = \frac{4U\sin\theta}{g\sqrt{3}}$ $\dot{x} = U\cos\theta - g\sin 30° t$ $0 = U\cos\theta - \frac{g}{2} \times \frac{4U\sin\theta}{g\sqrt{3}}$ $\sqrt{3}\cos\theta = 2\sin\theta$ $\tan\theta = \frac{\sqrt{3}}{2}$	M1A1 M1 A1 M1 A1 A1 **7**	M1: Equation for distance from the plane. Allow sign / angle errors. A1: Correct equation. M1: Solving *their* quadratic for t A1: Correct time. M1: Equation for velocity parallel to the plane. Allow sign / angle errors. A1: Correct equation. A1: AG, CSO.
6 (b)	$\dot{y} = U\sin\theta - g\cos 30° t$ $\dot{y} = U\sqrt{\frac{3}{7}} - \frac{\sqrt{3}}{2}g \times \frac{4U}{g\sqrt{3}} \times \sqrt{\frac{3}{7}}$ $\dot{y} = -U\sqrt{\frac{3}{7}}$ Speed = $U\sqrt{\frac{3}{7}} = \frac{U\sqrt{21}}{7} = 0.65U$	M1 A1 A1 A1 **4**	M1: Equation for velocity perpendicular to the plane. Allow sign / angle errors. A1: Correct equation. A1: Correct velocity A1: Correct speed.
	Total	**11**	

Q	Answer	Mark	Comments
7 (a)	*B* will move along the line of centres.	B1 **1**	B1: Correct statement about the line of centres.
7 (b)	Conservation of momentum along line of centres: $3 \times 4\cos 60° = 2v_B + 3v_A$ $6 = 2v_B + 3v_A$ Use of law of restitution: $v_A - v_B = -0.6(4\cos 60° - 0)$ $v_A - v_B = -1.2$ $v_A = v_B - 1.2$ $6 = 2v_B + 3(v_B - 1.2)$ $v_B = \frac{9.6}{5} = 1.9 \text{ m s}^{-1}$	M1 A1 M1 A1 M1 A1 **6**	M1: Three term equation for conservation of momentum. Allow trig errors. A1: Correct equation. M1: Restitution equation. Allow sign / trig errors. A1: Correct equation. M1: Solving their equations. A1: Correct speed to 2 sf. Accept 1.92
7 (c)	Velocity along line of centres: $= 1.92 - 1.2 = 0.72$ Velocity perpendicular to line of centres: $= 4\sin 60 = 2\sqrt{3}$ Magnitude of velocity: $\sqrt{0.72^2 + \left(2\sqrt{3}\right)^2} = 3.5 \text{ m s}^{-1}$ Direction θ to line of centres: $\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{0.72}\right) = 78°$	M1 M1 A1 A1 **4**	M1: Finding velocity along line of centres. M1: Finding velocity perpendicular to the line of centres. A1: Correct magnitude of velocity. AWRT 3.5 A1: Correct direction. AWRT 78
7 (d)	$I = 2 \times 1.92 = 3.8 \text{ N s}$	M1A1F **2**	M1: Impulse equation with correct values and any signs. A1F: Correct impulse. AWRT 3.8 FT their velocity.
	Total	**13**	