OXFORD
INTERNATIONAL
AQA EXAMINATIONS

# INTERNATIONAL A-LEVEL
# **MATHEMATICS**
# **MA02**

Pure, Statistics and Mechanics Unit 1

Mark scheme

January 2019

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*I9IyMA02/MS*

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| √ **or ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **A2, 1** | 2 or 1 (or 0) accuracy marks |
| **–*x* EE** | Deduct *x* marks for each error |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **SCA** | Substantially correct approach |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 1(a)(i) | $180^{\circ} - b$ | B1 | Condone omission of units. |
| 1(a)(ii) | $360^{\circ} - b$ | B1 | Condone omission of units. |
| 1(b)(i) | $-k$ | B1 | |
| 1(b)(ii) | $360^{\circ}$ | B1 | Condone omission of units.<br>If answer to (a)(i) given as $\pi - b$ and answer to (a)(ii) given as $2\pi - b$, award B1 for $2\pi$ |
| | **Total** | 4 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 2(a)(i) | 4 | B1 | |
| 2(a)(ii) | 0 | B1 | |
| 2(b) | $\log_m x^2$ or $\log_m\left(\frac{5}{x+1}\right)$ | M1 | At least one correct application of the logarithm rules.<br>Condone omission of $m$. |
| | $\log_m x^2 = \log_m\left(\frac{5}{x+1}\right)$<br>and<br>$x^2 = \frac{5}{x+1}$ | M1 | For their $x^2$ and $\frac{5}{x+1}$<br>Dependent on previous M1 scored.<br>Must equate two single logarithmic terms then correctly eliminate the logarithms.<br>Condone omission of *m*.<br>May see alternatives such as $\log_m x^2(x+1) = \log_m 5$ and $x^2(x+1) = 5$<br>or $\log_m\left(\frac{x^2(x+1)}{5}\right) = 0$ and $\frac{x^2(x+1)}{5} = 1$ |
| | $x^3 + x^2 - 5 = 0$ | A1 | Must come from completely correct working with no errors seen |
| | **Total** | 5 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 3(a) | $\left(2\sqrt{7}\right)^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \cos\theta$<br>or<br>$\cos\theta = \dfrac{6^2 + 4^2 - \left(2\sqrt{7}\right)^2}{2 \times 6 \times 4}$<br>or<br>$36 - x^2 = 28 - (4 - x)^2 \Rightarrow x = \cdots$ | M1 | Correct substitution into Cosine Rule or use of Pythagoras to find distance of C to the foot of the perpendicular from D.<br>Condone powers evaluated. |
| | $\cos\theta = \dfrac{1}{2}$ | M1 | Correct order of evaluation to obtain value for $\cos\theta$.<br>Condone equivalent fractions provided numerator and denominator evaluated. |
| | $\theta = \dfrac{\pi}{3}$ (AG) | A1 | Conclusion stated.<br>Must follow from correct complete method. |
| 3(b) | (Area of triangle *CEF* =)<br>$\dfrac{1}{2} \times 9 \times 8 \times \sin\dfrac{\pi}{3}$ | M1 | oe.<br>Use of $\text{Area} = \frac{1}{2}ab\sin C$. |
| | (Area of triangle *CEF* =)<br>$18\sqrt{3}$ or 31.17691… | A1 | Allow decimal rounded or truncated to at least 1dp.<br>Condone omission of units.<br>PI |
| | (Area of sector *CDG* =)<br>$\dfrac{1}{2} \times 6^2 \times \dfrac{\pi}{3}$ | M1 | oe.<br>Use of $\text{Area} = \frac{1}{2}r^2\theta$. |
| | (Area of sector *CDG* =)<br>$6\pi$ or 18.84955… | A1 | Allow decimal rounded or truncated to at least 1dp.<br>Condone omission of units.<br>PI |
| | (Area of shaded region=)<br>$18\sqrt{3} - 6\pi$<br>or<br>31.17691 ... − 18.84955… | m1 | Subtracts their area of sector from their area of triangle<br>Dependent on previous two method marks |
| | $12.3\ \text{cm}^2$ | A1 | CAO |
| | **Total** | 9 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4(a) | $\frac{\cos\theta}{\tan\theta} = \sin\theta$<br>or<br>$\cos\theta = \sin\theta\tan\theta$ | M1 | For use of $\sin^2\theta = 1 - \cos^2\theta$<br>Possible seen in later manipulation. |
| | $\frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}} = \sin\theta$<br>or<br>$\cos\theta = \sin\theta\frac{\sin\theta}{\cos\theta}$<br>or<br>$\frac{1}{\tan\theta} = \tan\theta$ | M1 | For use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ or $\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ in an equation.<br>Must be clearly seen before the final answer |
| | $\cos^2\theta = \sin^2\theta$<br>or<br>$\frac{\sin^2\theta}{\cos^2\theta} = 1$ | M1 | Allow correct working leading to $\frac{1}{\tan\theta} = \tan\theta$<br>Allow working leading to $\tan^4\theta = 1$ after squaring the original equation |
| | $\tan^2\theta = 1$ | A1 | Must come from fully correct working.<br>Be convinced.<br>Condone square rooting both sides of $\tan^4\theta = 1$ without consideration of possible negative roots |
| 4(b) | $(\tan\theta = 1 \Rightarrow \theta =)\quad 45^{\circ}$<br>or<br>$(\tan\theta = -1 \Rightarrow \theta =)\quad -45^{\circ}$ | B1 | |
| | $(\tan\theta = 1 \Rightarrow \theta =)\quad 45^{\circ}$<br>and<br>$(\tan\theta = -1 \Rightarrow \theta =)\quad -45^{\circ}$ | B1 | B0 if extra solutions within range<br>Ignore extra solutions out of range |
| | **Total** | 6 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **5(a)** | Exponential curve and increasing function in the first and second quadrants with the correct form, asymptotic to the negative *x*-axis from above. | B1 | |
| | $\frac{1}{9}$ marked on *y*-intercept on positive *y*-axis. | B1 | Condone correct coordinates |
| **5(b)** | $\log_3 3^{p-2} = \log_3 7$ | M1 | Substitution of coordinates into the equation of *C* and taking $\log_3$ of both sides. |
| | $p - 2$ | M1 | Application of logarithm rules to reduce LHS of equation to this stage. |
| | $(p =)\ 2 + \log_3 7$ | A1 | Condone log instead of $\log_3$. |
| **5(b) ALT** | $\frac{3^p}{3^2} = 7$ | M1 | Substitution of coordinates into the equation of *C* and apply law of indices to LHS |
| | $3^p = 63$ | M1 | Rearrange to form $3^p = k$ |
| | $(p =)\ \log_3 63$ | A1 | Condone log instead of $\log_3$. |
| | **Total** | 5 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 6(a) | $\sqrt{(5\sqrt{2})^2 + (\sqrt{30})^2} = \sqrt{80} = 4\sqrt{5}$<br>or<br>$(5\sqrt{2})^2 + (\sqrt{30})^2 = 80 = (4\sqrt{5})^2$ | B1 | Must have $4\sqrt{5}$ or $(4\sqrt{5})^2$ and must come from clear application of Pythagoras' Theorem.<br>If Cosine Rule used then must be clear attempt to find angle *Q* |
| | Triangle *PQR* has a right-angle at *Q*. | E1 | Dependent on previous B1 scored.<br>oe. Statement that angle at *Q* is a right-angle or $90^{\circ}$ |
| | PR is a diameter<br>and<br>Angle in a semicircle is $90^{\circ}$ | E1 | Dependent on previous E1 scored.<br>oe. |
| 6(b) | $-2$ | B1 | For gradient of *PR*.<br>PI by later working.<br>Condone use in straight line formula |
| | $\frac{1}{2}$ or $0.5$ | B1ft | For gradient of tangent at *P*.<br>ft $-\frac{1}{\text{Their gradient of } PR}$ |
| | $y - 5 = \frac{1}{2}(x - 1)$ | A1ft | oe. e.g. $y = \frac{1}{2}x + \frac{9}{2}$<br>ft their gradient of tangent. |
| 6(c) | (Centre of $C_1$ =) $(3,1)$<br>or<br>(Centre of $C_2$ =) $(9,9)$ | B1 | For one correct centre.<br>May be seen in parts (a) or (b)<br>PI |
| | $\sqrt{(9-3)^2 + (9-1)^2}$ | M1 | oe.<br>ft their centres.<br>Correct method to find distance between centres |
| | 10 | A1ft | ft their centres. |
| | $10 - 2\sqrt{5} - \sqrt{5}$ | M1 | oe.<br>ft their 10 provided greater than $3\sqrt{5}$ .<br>Radii of circles must be correct. |
| | $10 - 3\sqrt{5}$ | A1 | |
| | **Total** | 11 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 7(a) | $E(X^2) = 2^2 \times 0.6 + 4^2 \times 0.3 + 6^2 \times 0.1$ | M1 | Use of E(g(x)) formula |
| | = 10.8 | A1 | Accept 54/5 oe |
| 7(b) | P(X > 3) = 0.4 | B1 | Accept 2/5 oe |
| 7(c) | E(X) = 3 or 2 × 0.6 + 4 × 0.3 + 6 × 0.1 | B1 | Value or calculation must be seen<br>Can be embedded in calculation for variance |
| | $Var(X) = E(X^2) - (E(X))^2$<br>$= 10.8 - 3^2$ | M1 | Substitutes their values of $E(X^2)$ and $E(X)$ into the formula for $Var(X)$<br>Can be implied by correct value for their values of $E(X^2)$ and $E(X)$.<br>Actual value 1.8 or 9/5 oe |
| | Var(S) = Var(X) + Var (Y)<br>= 1.8 + 3 | M1 | Substitutes their value of $Var(X)$ and 3 into $Var(X) + Var(Y)$ |
| | = 4.8 | A1 | Accept 24/5 oe |
| | **Total** | **7** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 8(a) | $M$ and $S$ are mutually exclusive | B1 | Correct statement<br>Both independent and mutually exclusive scores B0<br>Condone poor spelling but both words must be seen |
| 8(b)(i) | = 1 – P(S)<br>= 1 - $\frac{100}{1000}$ = 0.9 | M1 | Uses mutually exclusive property to find P($M \cup D$)<br>PI |
| | P(M ∩ D) = P(M) + P(D) – P(M ∪ D)<br>$= \frac{650}{1000} + \frac{300}{1000} - 0.9$<br>or<br>P(M ∩ D) = P(M) + P(D) + P(S) – 1<br>$= \frac{650}{1000} + \frac{300}{1000} + \frac{100}{1000} - 1$ | M1 | Uses Addition Formulae to find P($M \cap D$) |
| | = 0.05 | A1 | Accept 1/20 oe |
| 8(b)(i) ALT | 1000 – 100 = 900<br>or<br>600 + 300 + 100 = 1050 | M1 | Finds number of employees working in either the manufacturing or delivery departments or total of employees in manufacturing, employees in delivery and employees in services |
| | 650 + 300 – 900 = 50<br>or<br>1050 – 1000 = 50 | M1 | Finds number of employees working in both the manufacturing and the delivery departments |
| | $\frac{50}{1000}$ = 0.05 | A1 | Finds probability<br>Accept 1/20 oe |
| 8(b)(ii) | $\mathrm{P}(M\|D) = \frac{\mathrm{P}(M \cap D)}{\mathrm{P}(D)}$<br>$= \frac{0.05}{\frac{300}{1000}}$ | M1 | Use of the Multiplication Law of probability to calculate the required conditional probability<br>May be implied by a correct answer |
| | $= \frac{1}{6}$ or AWRT 0.17 (0.166666…) | A1 | CSO |
| | **Total** | **6** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 9(a) | p(1 – p) = 0.16 | M1 | Forms equation using variance of Bernoulli formula<br>If by first principles, $p$ must be assigned correctly |
| | $p^2 - p + 0.16 = 0$<br>(p – 0.8)(p – 0.2) = 0 | M1 | Attempts to solve their quadratic<br>Implied by correct values for $p$ |
| | p = 0.2 or 0.8 | A1 | Both required<br>Accept 1/5 oe or 4/5 oe<br>CSO |
| 9(b)(i) | np(1 – p) = 10 × 0.16<br>or $\sqrt{np(1-p)} = \sqrt{10 \times 0.16}$ | M1 | Applies formula for variance or standard deviation of binomial using their value of $p$ from part (a)<br>Implied by correct answer |
| | = 1.26 | A1 | AWRT<br>Accept 1.265 |
| 9(b)(ii) | P(Y > 1) = 1 – P(Y ≤ 1)<br>= 1 – 0.3758 | M1 | Correct expression using their value of $p < 0.5$<br>Can be implied by further correct working<br>Accept<br>1 – 0.1074 – 0.2684 |
| | = 0.624 | A1 | AWRT |
| | **Total** | **7** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 10(a) | $6^2 = u^2 + 2 \times 9.8 \times 1.2$ | M1 | Use of $v^2 = u^2 + 2as$ ignoring signs with $v = 6$, $a = 9.8$ or $g$ and $s = 1.2$. |
| | | A1 | Correct equation. |
| | 3.5 | A1 | (3.53270…)<br>AWRT 3.5 |
| 10(b) | $(I =)\ 0.6 \times 3.53270$ | M1 | ft their 3.5 if positive.<br>Allow within incorrect impulse equation e.g. 0.6(6 – 3.53270) |
| | 2.1 | A1 | (2.11962…)<br>AWRT 2.1 |
| | **Total** | 5 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 11(a) | $\int_3^6 (0.5t^2 - 4t + 11)\, dt$ | M1 | Intention to integrate expression for $v$.<br>At least one term increased in power by 1<br>Condone missing limits. |
| | $\left[\frac{1}{6}t^3 - 2t^2 + 11t\right]_3^6$ | A1 | oe.<br>Condone missing limits.<br>Ignore $+c$. |
| | $(36 - 72 + 66) - \left(\frac{9}{2} - 18 + 33\right)$ | m1 | oe.<br>Substitute in limits and subtract the correct way round<br>Dependent on first M1 mark<br>PI |
| | 10.5 | A1 | oe. |
| 11(b) | Distance travelled by the particle is the same as its displacement ….. | B1 | States the value is the same with an attempt at an explanation |
| | Since the velocity is always positive the particle is always travelling in the same direction. | E1 | Statement saying that no change in sign indicates no change in direction.<br>Be convinced. |
| | **Total** | 6 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 12(a) | $T - 6g = 2.4$ | M1 | Three term equation of motion ignoring signs with 6g or 58.8 or 58.86 and 2.4 or $6 \times 0.4$. |
| | | A1 | For correct equation. |
| | $T = 61.2\text{N}$ | A1 | For correct $T$.<br>If g = 9.81 used, accept 61.26 or 61.3 |
| 12(b) | $80 - 61.2 - F = 5 \times 0.4$ | M1 | Four term equation of motion ignoring signs with friction $F$ and 2 or $5 \times 0.4$.<br>ft their $T$. |
| | | A1ft | For correct equation.<br>May be simplified to three term equation.<br>ft their $T$. |
| | $F = 16.8\text{N}$ | A1 | Correct value for frictional force.<br>Ignore omission of units.<br>PI<br>If g = 9.81 used accept 16.74 or 16.7 |
| | $R = 5g\text{N}$ or $49\text{N}$ | B1 | Correct normal reaction $R$ at block $A$.<br>Ignore omission of units.<br>PI<br>If g = 9.81 used accept 49.05 or 49.1 |
| | $16.8 = 49\mu$<br>or<br>$16.8 = 5g\mu$ | M1 | Use of $F = \mu R$. |
| | $\mu = 0.343$ | A1 | AWRT 0.343<br>If g = 9.81 used accept 0.341 or 0.340 |
| | **Total** | 9 | |