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MATHEMATICS

9665

FM02 Further Pure Mathematics Unit 2

Mark scheme

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Q	Answer	Mark	Comments
1	$hf(x,y) = \dfrac{0.1}{1.2^2 + 0.8^2}$ $= 0.048077$	M1 A1	PI
	$y_2 = 0.8 + 0.048077 = 0.848077$	m1	0.8+ their value of $hf(x,y)$
	$y_3 = 0.848077 + \dfrac{0.1}{1.3^2 + 0.848077^2}$ $(= 0.88958)$	m1	
	0.890	A1	CAO – candidate's **final** answer
	Total	**5**	

Q	Answer	Mark	Comments
2	[f(3) =] –9 and [f(4) =] 20	B1	PI
	$x_1 = 3 + \frac{9}{20+9}$	M1	PI
	$x_1 = \frac{96}{29}$	A1	
	$\mathrm{f}(x_1)[= -2.2067] < 0$	B1	
	$\mathrm{f}(x_1) < 0$ so $\alpha > \frac{96}{29}$ and $\frac{96}{29} < \alpha < 4$	E1	
	Total	**5**	

	Answer	Mark	Comments
3(a)	$y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$ $\log_{10} y = \log_{10} a + \log_{10} x^n$	M1	Take logs and apply one log law correctly PI
	$\log_{10} y = \log_{10} a + n\log_{10} x$	m1	Apply a further log law correctly.
	$Y = \log_{10} a + nX$ (which is a linear relationship between *Y* and *X*.)	A1	Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here)
3(b)	0.602, 0.778, 0.903	B1	At least 2 sig fig
	1.96, 2.37, 2.66	B1	At least 2 sig fig
3(c)	Their four points plotted correctly	B1ft	
	Straight line drawn through their points	B1ft	
3(d)	$a = 10^{0.58}$	M1	10 to the power of their intercept
	$a = 3.8$	A1	[3.3, 4.2]
	Gradient found	M1	
	$n = 2.3$	A1	[2.1, 2.5]
	Total	**11**	

Q	Answer	Mark	Comments
4(a)	$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	B1	
4(b)	W in correct place	B1	Two vertices correct B1B0
	(–3, –3), (–1, –3) and (–3, –6)	B1	
4(c)	**Method 1**		
	det(**P**) = 3	B1ft	
	det (**Q**) = –1	B1	PI
	det (**QP**) = –3	B1	
	Alternative method		
	$\mathbf{Q} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	B1	
	$\mathbf{QP} = \begin{bmatrix} 0 & -1 \\ -3 & 0 \end{bmatrix}$	M1	
	det (**QP**) = –3	A1	
	Total	**6**	

Q	Answer	Mark	Comments
5(a)	**Method 1**		
	det(**B**) = 2	M1	
	$\mathbf{B}^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$	A1	
	A = $\mathbf{CB}^{-1}$	M1	
	$\mathbf{A} = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$	A1	
	Alternative method		
	Let $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 6 \end{bmatrix}$	M1	
	$-a-b=2$ $a-b=4$ $-c-d=2$ $c-d=6$	M1	For four equations
	$a=1$ and $b=-3$ or $c=2$ and $d=-4$	M1	For solving one pair of equations
	$\mathbf{A} = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$	A1	

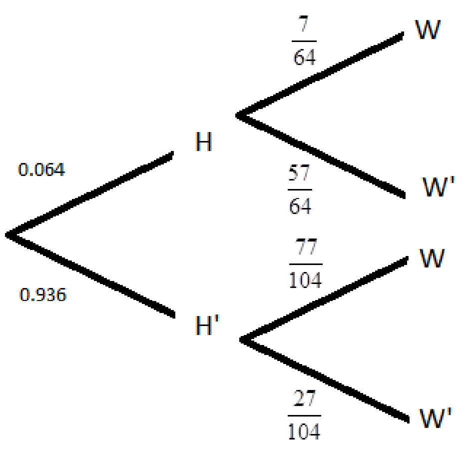
5(b)	$\mathbf{B^2} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$	M1	
	$\mathbf{B^4} = (\mathbf{B}^2)^2 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$	M1 A1	
	$\mathbf{B^4} = k\mathbf{I}$ where $k = -4$	E1	

5(c)(i)	Scale factor = √2	B1	

5(c)(ii)	– 135°	B1	Or 135°clockwise oe. Do not accept just 135°

5(d)	$\mathbf{B^{21}} = (\mathbf{B}^4)^5 \times \mathbf{B}$	M1	
	$= (-4)^5 \times \mathbf{B}$ or $-1024\mathbf{B}$	M1	
	$\mathbf{B^{21}} = \begin{bmatrix} 1024 & -1024 \\ 1024 & 1024 \end{bmatrix}$	A1	
	Total	**13**	

Q	Answer	Mark	Comments
6(a)	$\frac{2}{3}$	B1	
6(b)	$G_X(t) = \sum_{n=1}^{3} t^n \left(\frac{1}{3}\right)$	M1	Applies formula for $G_X(t)$
	$= \frac{1}{3}(t + t^2 + t^3)$ oe	A1	Ignore subsequent incorrect working
6(c)	**Alternative Method 1**		
	$G_{X+Y}(t) = \frac{1}{3}(t + t^2 + t^3)(0.6 + 0.4t)$	M1	Multiplies their $G_X(t)$ and $G_Y(t)$
	$\frac{1}{3}t \times 0.6$	M1	Multiplies out the required terms to find the coefficient of t Implied by correct answer
	$= 0.2$	A1	Accept 1/5 oe
	Alternative method 2		
	Y Bernoulli, $p = 0.4$ or $Y \sim B(1, 0.4)$	M1	Identifies distribution of Y
	$P(X + Y = 1)$ means $X = 1$ and $Y = 0$ or $P(X + Y = 1) = P(X = 1)P(Y = 0)$	M1	Identifies possible combinations of X and Y
	$= \frac{1}{3}(1 – 0.4) = 0.2$	A1	Accept 1/5 oe
	Total	**6**	

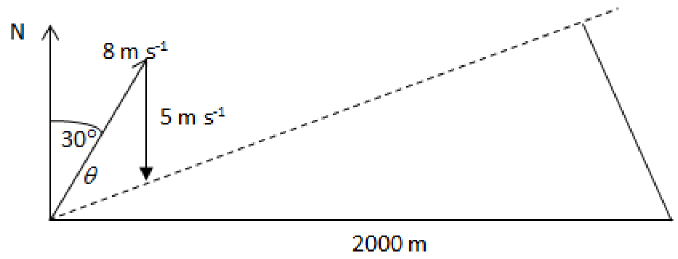
Q	Answer	Mark	Comments
7(a)	$(1 - 0.6)^3 = 0.064$	B1	Accept 8/125 oe
7(b)	$P(W \mid H) = \dfrac{P(H \mid W)P(W)}{P(H)}$ $= \dfrac{0.01 \times 0.7}{0.064}$	M1	Applies Bayes Theorem to find P(W|H)
	$= \dfrac{7}{64}$ or AWRT 0.109	A1ft	ft their P(H) from (a) provided 0 < P(H) < 1
	$P(H' \cap W) = P(W) - P(H \cap W)$ $P(H')P(W \mid H') = P(W) - P(H)P(W \mid H)$ $(1 - 0.064)P(W \mid H') = 0.7 - 0.064 \times \dfrac{7}{64}$	M1	Method to find P(W|H')
	$P(W \mid H') = \dfrac{77}{104}$ or 0.740 AWRT	A1ft	ft their P(H) from (a) provided 0 < P(H) < 1
		A1ft	ft their probabilities Need to score at least one M1
	Total	**6**	

Q	Answer	Mark	Comments
8(a)	$E(2P^2 - 5) = 2E(P^2) - 5$ or $E(PQ) = 2E(P^3) - 5E(P)$	M1	One correct formula seen
	$E(2P^2 - 5) = 2 \times 5 - 5 = 5$ or $E(PQ) = 2 \times 14.6 - 5 \times 2 = 19.2$	A1	One correct value Accept 96/5 oe for 19.2 Can be implied by correct final answer
	$E(2P^2 - 5) = 2 \times 5 - 5 = 5$ and $E(PQ) = 2 \times 14.6 - 5 \times 2 = 19.2$	A1	Both correct values Accept 96/5 oe for 19.2 Can be implied by correct final answer
	Cov (P, Q) = E(PQ) – E(P)E(Q) $= 19.2 - 2 \times 5$	M1	Applies formula for Cov (P, Q)
	$= 9.2$	A1	Accept 46/5 oe
8(b)	$\text{Var}(P) = E(P^2) - (E(P))^2$ $= 5 - 2^2$ $= 1$	B1	Finds Var (P) Can be implied by correct final answer
	Var (P + Q) = Var (P) + Var (Q) + 2 Cov (P, Q) $= 1 + 8 + 2 \times 9.2$	M1	Applies formula for Var (P + Q)
	$= 27.4$	A1	Accept 137/5 oe
	Total	**8**	

Q	Answer	Mark	Comments
9	$MLT^{-2} = [k]L^{\frac{3}{2}}T^{-\frac{3}{2}}$ $[k] = ML^{-\frac{1}{2}}T^{-\frac{1}{2}}$	M1 A1	
	Total	**2**	

Q	Answer	Mark	Comments
10(a)	$I = \frac{1}{2} \times 2500 \times 10$ $= 12500$ Ns	B1	
10(b)	$-12500 = 2000U - 2000 \times 20$ $U = \frac{2000 \times 20 - 12500}{2000}$ $U = 13.75$	M1 A1 A1	M1: Equation with correct terms but any signs. A1: Correct equation. A1: Correct *U*.
	Total	**4**	

Q	Answer	Mark	Comments
11(a)	$5mU + 4mU = mv_A + 4mv_B$	M1	
	$9U = v_A + 4v_B$	A1	
	$v_A - v_B = -e(5U - U)$	M1	
	$v_A - v_B = -4eU$	A1	
	$9U = v_B - 4eU + 4v_B$		
	$v_B = \dfrac{U(9+4e)}{5}$	A1	
11(b)	$v_A = \dfrac{U(9+4e)}{5} - 4eU$		
	$v_A = \dfrac{U(9-16e)}{5}$	B1	
	$9 - 16e < 0$	M1	
	$e > \dfrac{9}{16}$	A1	
	Total	**8**	

Q	Answer	Mark	Comments
12	$v_{MY}^2 = 8^2 + 5^2 - 2 \times 8 \times 5\cos 30°$	M1	Accept 869
	$v_{MY}^2 = 89 - 40\sqrt{3}$	A1	
	$\frac{\sin\theta}{5} = \frac{\sin 30°}{\sqrt{89 - 40\sqrt{3}}}$	M1	
	$\theta = \sin^{-1}\frac{5}{2\sqrt{89 - 40\sqrt{3}}}$	A1	
	$\theta = 34.26357\,...°$		
	Min Sep $= 2000 \times \sin(60-\theta) = 868$ m	M1	
		A1	
	Additional Guidance		
			
	Total	**6**	