OXFORD

INTERNATIONAL AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

INTERNATIONAL AS

FURTHER MATHEMATICS

Further Pure, Statistics and Mechanics Unit 1

Monday 4 June 2018 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the booklet of formulae and statistical tables, which is included as an insert.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the box at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages. If you require extra space, use a supplementary answer book.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for Section A, 20 marks for Section B and 20 marks for Section C.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Section A

Answer **all** questions in the spaces provided.

1 The variables X and Y, where $X = \frac{1}{x}$ for $x > 0$ and $Y = y^2$ for $y \geq 0$, are related by the equation

$$Y = aX + b$$

where a and b are constants.

1 (a) The table shows some values of x and y.

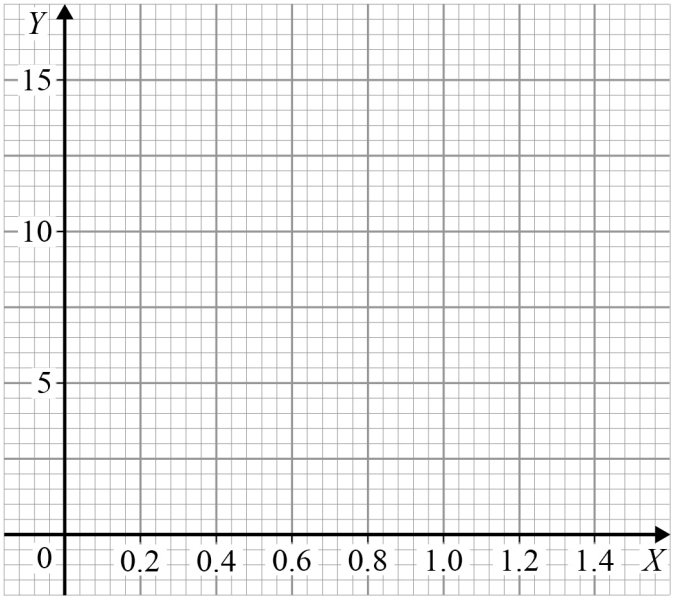
Complete the table to show the values of X and Y.

[2 marks]

x	1	2	5
y	3.7	3.1	2.6
X			
Y			

1 (b) On the grid below, draw a linear graph relating X and Y.

[3 marks]



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1 (c) Use your graph to estimate the values of a and b.

[3 marks]

$a =$ _____

$b =$ _____

1 (d) **Hence** write down an equation relating x and y.

[1 mark]

Answer _____

1 (e) Use your equation to estimate the value of y when $x = 0.5$

[1 mark]

Answer _____

10

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2 A curve passes through the point (2, 5) and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{x^3 - 3x}$$

Use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$

Give your answer to four decimal places.

[5 marks]

Answer _____

5

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3 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

3 (a) Name the type of transformation represented by $\mathbf{A}$.

[1 mark]

3 (b) The matrix $\mathbf{B}$ represents a clockwise rotation of 30° about the origin.

Write down the matrix $\mathbf{B}$.

[1 mark]

Answer

3 (c) Find the matrix $\mathbf{B}^{-1}\mathbf{A}^{-1}$, giving each element as an exact value.

[4 marks]

Answer

6

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4 The function $f(x)$ is defined as

$$f(x) = -0.1x^4 + x^2 - 2$$

4 (a) Show that the equation $f(x) = 0$ has a real root, α, in the interval $2 < x < 3$

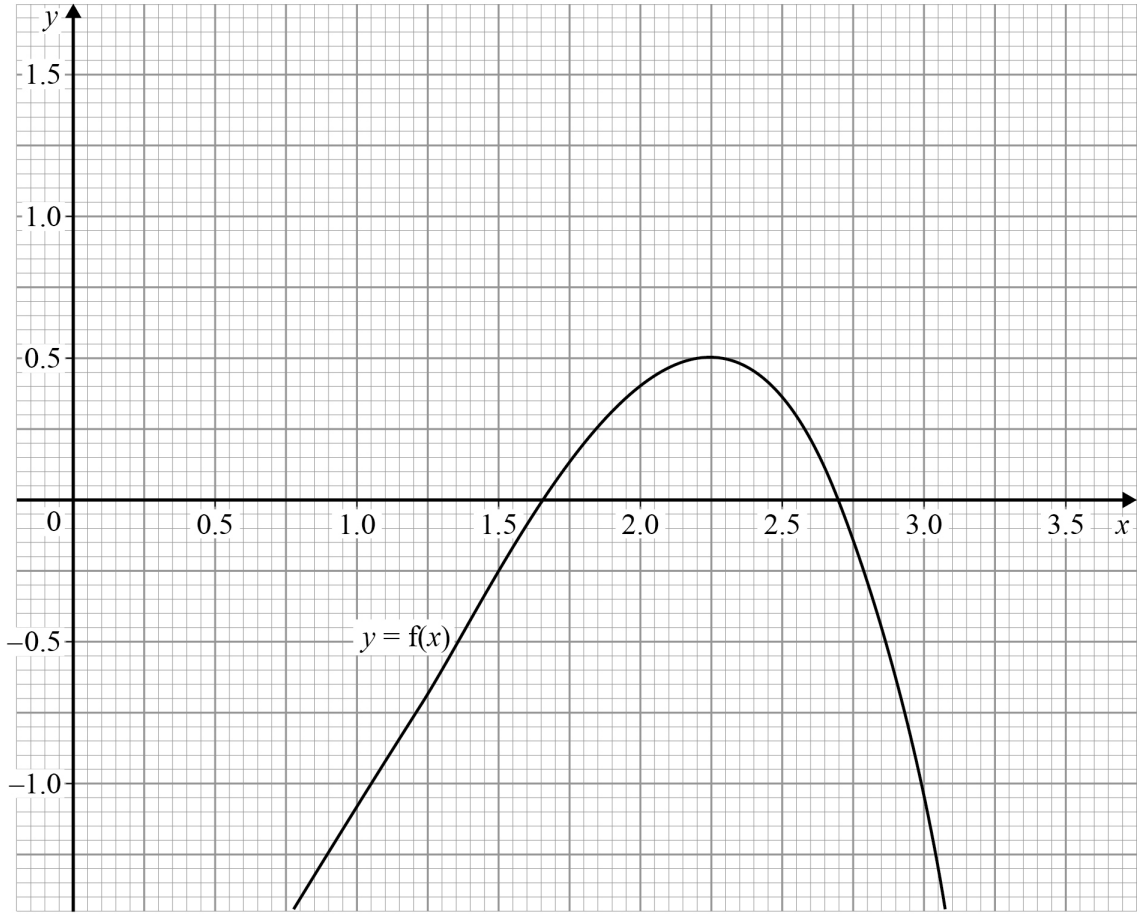
[2 marks]

Do not write outside the box

4 (b) The Newton-Raphson method can be used to estimate α, but it will only work for one of the two initial values $x_1 = 2$ and $x_1 = 3$

Explain, with the help of lines which you draw on the diagram below, why this is true.

[4 marks]



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4 (c) Using $x_1 = 2.5$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α.

[4 marks]

Answer _____

10

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

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5 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

5 (a) Describe the transformation represented by $\mathbf{A}$.

[1 mark]

5 (b) It is given that $y = mx + c$ is an invariant line of the transformation represented by $\mathbf{A}$.

Show that m satisfies the equation

$$m^2 \sin 2\theta + 2m \cos 2\theta - \sin 2\theta = 0$$

[4 marks]

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5 (c) In the case where $\theta = \frac{\pi}{6}$

5 (c) (i) show that $m = -\sqrt{3}$ is one solution of the equation in part **(b)** and find the other solution.

[3 marks]

Answer _____

5 (c) (ii) find the determinant of $\mathbf{A}$.

[1 mark]

Answer _____

9

Turn over for the next section

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Section B

Answer **all** questions in the spaces provided.

6 Jagdeep often catches the bus to school. Sometimes he is late for school.

The probability that Jagdeep catches the bus is 0.875

The probability that Jagdeep is late given that he catches the bus is 0.12

The probability that Jagdeep is late given that he does not catch the bus is 0.34

6 (a) Find the probability that Jagdeep catches the bus and is not late.

[2 marks]

Answer _____

6 (b) Find the probability that Jagdeep catches the bus given that he is late.

[4 marks]

Answer _____

6

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7 Let T be the random variable with probability function

$$\mathrm{P}(T = t) = \begin{cases} \frac{1}{4} & t = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

7 (a) Find $\mathrm{E}(T)$ and $\mathrm{Var}(T)$.

[2 marks]

$\mathrm{E}(T) =$ _____

$\mathrm{Var}(T) =$ _____

7 (b) Let S be the random variable such that $S = T^2 + 1$

Find $\mathrm{E}(T + S)$.

[4 marks]

Answer _____

6

Turn over ►

Do not write outside the box

8 On a production line, components are checked for damage.

The random variable X represents the number of components that are checked before the first damaged component is found and can be modelled by a geometric distribution with $p = 0.045$

8 (a) (i) Find the probability that exactly eight products are checked before the first damaged product is found.

[2 marks]

Answer _____

8 (a) (ii) Find the probability that more than five products are checked before the first damaged product is found.

[2 marks]

Answer _____

Do not write outside the box

8 (b) Derive the probability generating function, $G_X(t)$, of X in the form $\dfrac{at}{1-bt}$

where a and b are constants.

Fully justify your answer.

[4 marks]

Answer _____

8

Turn over for the next section

Do not write outside the box

Section C

Answer **all** questions in the spaces provided.

9 An aeroplane is flying due north at a constant speed of 150 m s^{-1} relative to the air.

The air is moving south-east at 40 m s^{-1}

Find the resultant speed of the aeroplane.

[2 marks]

Answer _____

2

Do not write outside the box

10 The magnitude of the gravitational attraction, F newtons, between two planets is given by

$$F = \frac{GMm}{d^2}$$

G is a constant.

M is the mass of the first planet in kilograms.

m is the mass of the second planet in kilograms.

d is the distance between the centres of the planets in metres.

Find the units of G in the form $m^a\ s^b\ kg^c$

[3 marks]

Answer _____

3

Turn over for the next question

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11 Two boats, A and B, are moving on a large lake.

Boat A moves with constant velocity of $(\mathbf{i} - 2\mathbf{j})$ m s^{-1} and is initially at the point with position vector $(30\mathbf{i} + 90\mathbf{j})$ m.

Boat B moves with constant velocity of $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and is initially at the point with position vector $(5\mathbf{i} + 70\mathbf{j})$ m.

Find the shortest distance between the two boats as they move on the lake.

[6 marks]

Answer _____

6

Do not write outside the box

12 Two smooth spheres, P and Q, are moving in the same direction on a straight line on a smooth, horizontal surface.

Sphere P has mass 1 kg and is moving at 4 m s^{-1}

Sphere Q has mass 5 kg and is moving at 1 m s^{-1}

Sphere P collides directly with sphere Q.

The coefficient of restitution between P and Q is 0.8

12 (a) Find the speeds of P and Q after the collision.

[6 marks]

Answer _____

Turn over ►

Do not write outside the box

12 (b) The sphere Q collides with a vertical wall that is perpendicular to its path.

The coefficient of restitution between Q and this wall is e.

After colliding with the wall, Q collides with P for a second time.

Deduce an inequality for e.

[3 marks]

Answer _____

9

END OF QUESTIONS