# INTERNATIONAL AS
# Mathematics

MA02 –  Unit 2  Pure, Statistics and Mechanics
Mark scheme

9660

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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# Key to mark scheme abbreviations

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| **√ or ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **A2, 1** | 2 or 1 (or 0) accuracy marks |
| **–$x$ EE** | Deduct $x$ marks for each error |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **SCA** | Substantially correct approach |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **1 (a)** | $\log_a 1 = 0$ <br> or <br> $\log_a a^b = b$ | M1 | PI. Condone omission of base |
| | $-2b$ | A1 | CSO |
| **1 (b)** | $8 \log_a x$ <br> or <br> $6 \log_a x$ <br> or <br> $\log_a x^8$ <br> or <br> $\log_a x^6$ | M1 | Condone omission of base |
| | $14 \log_a x$ | A1 | CSO. Condone omission of base |
| | **Total** | **4** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| 2 (a) (i) | $(x - 11)^2 - 121 + (y - 8)^2 - 64$ $+ 135 = 0$ | M1 | $(x - 11)^2 - 121$ or $(y - 8)^2 - 64$ seen OE |
| | $(x - 11)^2 + (y - 8)^2 = 50$ | A1 | CSO |
| 2 (a) (ii) | $\sqrt{50},\ 5\sqrt{2},$ | B1ft | Simplified or unsimplified ft their $r^2$ provided it is positive AWRT 7.07 |
| | $(11,8)$ | B1ft | ft their centre $(a,b)$ provided M1 scored and their equation is of the form $(x - a)^2 + (y - b)^2 = r^2$ with $a$ and $b$ non-zero |
| 2 (b) | An indication that the distance required is from the midpoint of the chord to the centre of the circle | M1 | PI. May be evident in a right-angled triangle implying midpoint is 3 from one end of chord, or attempt at Pythagoras' theorem using 3. |
| | $(d^2) = (5\sqrt{2})^2 - 3^2$ | M1 | ft their $r^2$ provided that it is positive |
| | $d = \sqrt{41}$ | A1 | CSO. A0 for attempt to give $d$ as a decimal |
| 2 (c) | Translation | E1 | |
| | $\begin{pmatrix} -11 \\ -8 \end{pmatrix}$ | B1ft | ft minus their $a$ and minus their $b$ from their centre of $P$ |
| | **Total** | **9** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **3 (a)** | Area $= \frac{1}{2}r^2\theta$ | M1 | Area $= \frac{1}{2}r^2\theta$ seen or used. |
| | $0.648 = \frac{1}{2} \times 1.80^2 \times \theta \implies \theta$ <br> $= \frac{0.648}{1.62} = 0.4$ | A1 | $\theta = 0.4$ oe.  Be convinced |
| **3 (b)** | Length of arc *BC* $= r\theta = 1.80 \times 0.4$ | M1 | Arc $= r\theta$ seen or used in(b).  PI by correct length of arc. |
| | (Length of arc *BC* =)  0.72 | A1 | OE |
| | Angle $CAD = \frac{\pi}{2} - 0.4$ | B1 | AWRT 1.17 radians or 67.1 degrees |
| | $CD^2 = 2.42^2 + 1.80^2 - 2 \times 2.42$ <br> $\times 1.80 \times \cos 1.17$ | M1 | RHS of Cosine Rule used. Ft their angle *CAD* |
| | $5.8564 + 3.24 - 3.3926 \ldots$ | M1 | Correct order of evaluation |
| | $CD = \sqrt{5.7037\ldots} = 2.3882$ <br> $= 2.39$ m | A1 | AWRT 2.39 |
| | (Perimeter =) 7.33 | A1 | CAO |
| | **Total** | **9** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **4 (a)** | $\dfrac{\sin\theta}{10} = \dfrac{\sin18^o}{4}$ | M1 | A correct equation sine rule with all known values substituted |
| | $\sin\theta = \dfrac{10 \times \sin18^o}{4} = 2.5 \times \sin18^o$ | m1 | A correct rearrangement to make $\sin\theta$ the subject |
| | Angle ABC $= 50.5827\,...^o = 50.6^o$ | A1 | AWRT 51° |
| | Angle $ABC = 180^o - 50.5827\,...^o$<br>$= 129.4172\,...^o = 129.4^o$ | A1ft | ft their $50.6^o$<br>AWRT 129°<br>Extra solutions lose this mark |
| **4 (b)** | Area $= \dfrac{1}{2} \times 10 \times 4$<br>$\qquad \times \sin(180^o - 18^o$<br>$\qquad - 50.6^o)$<br>$= 20 \times \sin111.4^o$ | M1 | Correct use of formula for area of a triangle using their acute angle from part (a) |
| | Area $= 18.6189\,... = 18.6 \; [\text{cm}^2]$ | A1 | AWRT 19 [cm$^2$] |
| | **Total** | **6** | |

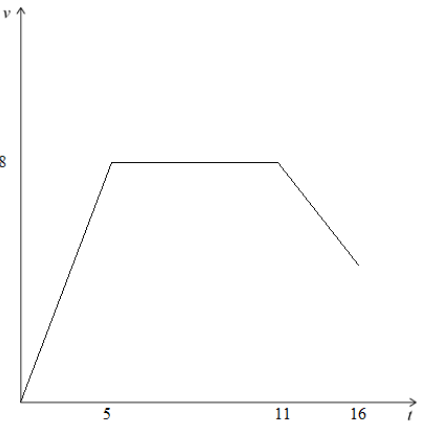| Q | Answer | Mark | Comments |
|---|---|---|---|
| **5 (a)** | $2^{8x-4\sqrt{x}}$ or $2^{6x-4\sqrt{x}+14}$ | B1 | For giving either numerator or denominator of LHS as correct power of $2$. Accept equivalent as along as it is a power of $2$. |
| | $2^{4(2x-\sqrt{x})-2(3x-2\sqrt{x}+7)}$ or $2^{8x-4\sqrt{x}-6x+4\sqrt{x}-14}$ oe | M1 | Correctly applies index rules to LHS. Ft their numerator and denominator dep on B1 scored. Power must not be fully simplified at this stage |
| | $\dfrac{16^{2x-\sqrt{x}}}{4^{3x-2\sqrt{x}+7}} \equiv 2^{2x-14}$ | A1 | CSO. Must show method otherwise award zero. |
| **5(a) ALT** | $\dfrac{4^{4x-2\sqrt{x}}}{4^{3x-2\sqrt{x}+7}} = 4^{(4x-2\sqrt{x})-(3x-2\sqrt{x}+7)} = 2^{\cdots}$ | M1 | Numerator given as power of 4 and correctly applies index rules to LHS then attempts to write as a power of 2 |
| | $4^{x-7} = 2^{2(x-7)}$ | B1 | Uses $4 = 2^2$ |
| | $\dfrac{16^{2x-\sqrt{x}}}{4^{3x-2\sqrt{x}+7}} \equiv 2^{2x-14}$ | A1 | CSO. Must show method otherwise award zero. |
| **5 (b)** | $2x - 14 = \log_2 6$ (do not condone omission of base) | M1 | Equates RHS of part (a) to $6$ and takes $\log_2$ of both sides. Do not condone omission of base. Must appear as an equation with correct linear expression |
| | $(x =) 7+\dfrac{1}{2}\log_2 6$ | A1 | Accept *b* as $0.5$ Do not condone omission of base. |
| | **Total** | **5** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 6 (a) | $(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$ | M1 | LHS factorized as the difference of two squares |
| | $\sin^2\theta - \cos^2\theta$ <br> $= \sin^2\theta - (1 - \sin^2\theta)$ | M1 | $\sin^2\theta + \cos^2\theta = 1$ used at least once |
| | $(=)\ 2\sin^2\theta - 1$ | A1 | CSO. AG |
| 6(a) ALT 1 | $\sin^2\theta(1 - \cos^2\theta)$ <br> $-\cos^2\theta(1 - \sin^2\theta)$ <br> $= \sin^2\theta - \sin^2\theta\cos^2\theta - \cos^2\theta$ <br> $+\cos^2\theta\sin^2\theta$ | M1 | $\sin^2\theta + \cos^2\theta = 1$ used twice in LHS and result multiplied out |
| | $\sin^2\theta - \cos^2\theta$ <br> $= \sin^2\theta - (1 - \sin^2\theta)$ | M1 | $\sin^2\theta + \cos^2\theta = 1$ used at least once |
| | $(=)\ 2\sin^2\theta - 1$ | A1 | CSO. AG |
| 6(a) ALT 2 | $\sin^4\theta - (1 - \sin^2\theta)^2$ | M1 | $\sin^2\theta + \cos^2\theta = 1$ used once in LHS |
| | $\sin^4\theta - (1 - 2\sin^2\theta + \sin^4\theta)$ | M1 | Bracket multiplied out |
| | $(=)\ 2\sin^2\theta - 1$ | A1 | CSO. AG |
| 6 (b) | $2\sin^2\theta - 1 = -0.28$ <br> $2\sin^2\theta = 0.72$ <br> $\sin^2\theta = 0.36$ | M1 | Uses result from part (a) to obtain the value of $\sin^2\theta$. Condone one arithmetic slip in manipulation. |
| | $\sin\theta = 0.6$ and $\sin\theta = -0.6$ | A1 | Need both |
| | $\theta = 37^\text{o}, 143^\text{o}, \theta = 217^\text{o}, 323^\text{o}$ | B2 | Condone more accurate answers. (36.8698…, 143.1301…; 216.8698…, 323.1301…) <br> Ignore answers outside interval. <br> If more than 2 answers for each inside interval, -1 for each extra from Bs to a min of 0 |
| | **Total** | **7** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 7 (a) (i) | $P(G \cup S) = P(G) + P(S)$ <br> $\qquad\qquad - P(G \cap S)$ <br><br> $= 0.3 + 0.8 - 0.25$ | M1 | Use of the Addition Rule of probability <br> May be implied by a correct answer |
| | $= 0.85$ | A1 | (Accept 17/20 OE) |
| 7 (a) (ii) | $P(G\|S) = \dfrac{P(G \cap S)}{P(S)}$ <br><br> $= \dfrac{0.25}{0.8}$ | M1 | Use of the Multiplication Law of probability to calculate the required conditional probability <br> May be implied by a correct answer |
| | $= 0.3125$ | A1 | (Accept 5/16 OE) |
| 7(b) | $P(G) \times P(S) = 0.3 \times 0.8 = 0.24$ | B1 | Correct multiplication <br> Value must be seen <br> (Accept 6/25 OE) |
| | $P(G) \times P(S) \neq P(G \cap S)$ <br> G and S are not independent | E1 | Correct statement and conclusion <br> Statement may be in words |
| 7(b) ALT | $P(G) = 0.3$ <br> $P(G\|S) = 0.3125$ | (B1) | Gives values of $P(G)$ and their $P(G\|S)$ <br> May find $P(G\|S') = 0.25$ instead of $P(G)$ |
| | $P(G\|S) \neq P(G)$ or $P(G\|S')$ <br> G and S are not independent | (E1) | Correct statement and conclusion <br> Statement may be in words |
| | **Total** | **6** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **8 (a)** | $\binom{25}{18} 0.612^{18}(1 - 0.612)^{25\text{-}18}$ | M1 | Correct expression<br>Can be implied by **correct answer**<br>Ignore extra terms |
| | $= 0.0923$ | A1 | AWRT<br>Accept percentage equivalent |
| **8 (b) (i)** | $\text{E}(X) = 7.5$ | B1 | (Accept 15/2 OE) |
| | $\text{Var}(X) = 6.375$ | B1 | (Accept 51/8 OE) |
| **8 (b) (ii)** | $\text{P}(6 \leq X < 9) = \text{P}(X \leq 8)$<br>$\qquad\qquad\qquad - \text{P}(X \leq 5)$ | M1 | Correct expression<br>Can be implied by further correct working |
| | $= 0.6681 - 0.2194$ | M1 | Correct values for their two Poisson probabilities given to at least 4 s.f.<br>(From calculator 0.66810072 – 0.21935333) |
| | $= 0.449$ | A1 | AWRT<br>Accept percentage equivalent |
| **8 (b) (ii) ALT** | $\text{P}(6 \leq X < 9) = \text{P}(X = 6) + \text{P}(X = 7) + \text{P}(X = 8)$ | (M1) | Correct expression<br>Can be implied by further correct working |
| | $= 0.1419 + 0.1575 + 0.1493$ | (M1) | Correct values for $\text{P}(X = 6)$, $\text{P}(X = 7)$ and $\text{P}(X = 8)$ given to at least 4 s.f.<br>(From calculator 0.1419460509 + 0.1574527623 + 0.149348576) |
| | $= 0.449$ | (A1) | AWRT<br>Accept percentage equivalent |
| | **Total** | **7** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **9(a)** | $0.3 + k + 0.25 + 2k = 1$ | M1 | Uses sum of probabilities equals one to form an equation<br>Accept equivalent expressions |
| | $k = 0.15$  AG | A1 | CSO  AG |
| **9(b)** | $\mathrm{E}(X)$<br>$= 1 \times 0.3 + 2 \times k + 3 \times 0.25 + 4 \times 2k$<br>$= 2.55$ | B1 | Award on value only<br>(Accept 51/20 OE) |
| **9(c)** | $\mathrm{Var}(X)$<br>$= 1^2 \times 0.3 + 2^2 \times k + 3^2 \times 0.25 + 4^2 \times 2k$ – their $2.55^2$ | M1 | Correct expression with their $\mathrm{E}(X)$<br>Can be implied by a correct answer |
| | $= 1.4475$ | A1 | (Accept 579/400 OE) |
| **9(d)** | $\mathrm{Var}(4X - 5) = 4^2 \, \mathrm{Var}(X)$ | M1 | Correct expression<br>$\mathrm{Var}(X)$ can be replaced with their $\mathrm{Var}(X) > 0$<br>Can be implied by a correct answer |
| | $= 23.16$ | A1ft | ft their $\mathrm{Var}(X) > 0$<br>(Accept 579/25 OE) |
| | **Total** | **7** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **10 (a)** |  | B1 | Correct shape of graph and $5$, $11$ and $16$ marked on t axis |
| | | B1 | Horizontal part of graph at $8$ ms$^{-1}$ (could be indicated on vertical axis) |
| **10 (b)** | $(11 - 5) \times$ their $8 = 48$ | M1 | Finds area under rectangle |
| | $\frac{1}{2}(8 + v) \times 5 = 100 - 20 -$ their $48$ | m1 | Finds area of trapezium and sets equal to $100 - 20 -$ their $48$ OE Accept use of $s = ut + \frac{1}{2}at^2$ to find $a$ ($-0.64$ ms$^{-2}$) $100 - 20 -$ their $48 = 8 \times 5 + \frac{1}{2}a \times 5^2$ |
| | $v = 4.8$ ms$^{-1}$ (could be indicated on vertical axis) | A1 | |
| | **Total** | **5** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **11** | Solution<br>Let mass of $A = m$<br>Mass of $B = m + 2$ | | |
| | Momentum before $= 6m + 2(m + 2) = 8m + 4$ | M1 | Can be unsimplified |
| | Momentum after $= 3.6\,(m + m + 2) = 7.2m + 7.2$ | M1 | Can be unsimplified |
| | Conservation of momentum<br>$8m + 4 = 7.2m + 7.2$ | M1 | for equation for conservation of momentum |
| | $m = 4$ kg | A1 | |
| | **Total** | **4** | |

| Q | Answer | Mark | Comments |
|---|--------|------|----------|
| **12** | $a = \dfrac{dv}{dt}$ | M1 | Accept at least one term reduced in power by one |
| | $= 6t^2 - 30t + 24$ | A1 | May be unsimplified |
| | $6t^2 - 30t + 24 = 0$ | M1 | Set their $a$ equal to zero |
| | $t^2 - 5t + 4 = 0$ $(t-1)(t-4) = 0$ $t = 1$ [or $t = 4$] | A1 | Solves to find $t = 1$ |
| | when $t = 1$, $v = 20$ m s$^{-1}$ | A1 | Mark not awarded if $v$ = -7 m s$^{-1}$ (or other) given but not rejected |
| | **Total** | **5** | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **13(a)** | Whole system<br>$\pm 2200 \pm 600 \pm S = \pm 5000 \times 0.15$<br><br>**or**<br><br>Van<br>$\pm 2200 \pm S \pm T = \pm 3000 \times 0.15$<br>**and**<br>Ride<br>$\pm T \pm 600 = \pm 2000 \times 0.15$ | M1 | Attempt at equation of motion for whole system or equations of motion for both the van and the ride<br><br>Allow slips in signs for this mark |
| | Whole System<br>$2200 - 600 - S = 5000 \times 0.15$<br><br>**or**<br><br>Van<br>$2200 - S - 900 = 3000 \times 0.15$ | A1 | For a correct equation in terms of $S$ only OE |
| | $S = 850$ [N] | A1 | |
| **13(b)** | Van<br>$\pm 2200 \pm S \pm T = \pm 3000 \times 0.15$<br><br>**or**<br><br>Ride<br>$\pm T \pm 600 = \pm 2000 \times 0.15$ | M1 | Attempt at equation of motion for the van or the ride<br><br>Allow slips in signs for this mark<br><br>May be seen in part (a) |
| | Van<br>$2200 - $ their $S - T = 3000 \times 0.15$<br><br>**or**<br><br>Ride<br>$T - 600 = 2000 \times 0.15$ | A1 | Correct equation of motion for the van or the ride<br>ft for their $S$ for the correct equation for the van<br><br>May be seen in part (a) |
| | $T = 900$ N | A1ft | for correct $T$<br>ft their $S$ from part (a) if equation of motion for the van is used<br><br>Must be seen in part (b) |
| | **Total** | **6** | |