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Further Mathematics

FM01 - Unit 1 Further Pure Mathematics

Mark scheme

9665

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct and answer only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Answer	Mark	Total	Comments
1(a)	$y_{3+h} = 6 + 8h + 2h^2$	B1	3	or $18 + 12h + h^2 - (12 + 4h)$
	Use of correct formula for gradient	M1		
	Gradient is $8 + 2h$	A1		
1(b)	As $h \to 0$ the gradient $\to 8$	E2, 1ft	2	E1 for "$h = 0$" E1 for "$gradient = 8$" with no limit seen
Total	5			

Q	Answer	Mark	Total	Comments
2	$\frac{3 - 2i}{x + iy} \times \frac{x - iy}{x - iy}$	M1	3	or $ax + by = 3$ **and** $ay + bx = -2$
	$a = \frac{3x - 2y}{x^2 + y^2}$	A1		
	$b = \frac{-2x - 3y}{x^2 + y^2}$	A1		
Total	3			

Q	Answer	Mark	Total	Comments
3(a)	$\frac{(r+3)-(r+2)}{(r+2)(r+3)} = \frac{1}{(r+2)(r+3)}$	B1	1	
3(b)(i)	$\sum_{r=11}^{30} \frac{1}{(r+2)(r+3)} = f(11) - f(12)$ $+f(12) - f(13)$ $+\ldots\ldots\ldots\ldots$ $+f(29) - f(30)$ $+f(30) - f(31)$	M1 A1	4	
	$= f(11) - f(31)$ or $= \frac{1}{13} - \frac{1}{33}$	A1		
	$= \frac{20}{429}$	A1		
3(b)(ii)	$\sum_{r=18}^{n} \frac{1}{(r+2)(r+3)} = f(18) - f(n+1)$	M1	4	PI
	$= \frac{1}{20} - \frac{1}{n+3}$	A1		
	$\lim_{n\to\infty}\left(\frac{1}{20} - \frac{1}{n+3}\right)$	E1		For correct explanation
	$= \frac{1}{20}$	B1		
Total	**9**			

Q	Answer	Mark	Total	Comments
4	$\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ Or $\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$	B1	5	
	Use of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $\left(3x+\frac{\pi}{6}\right)$ to x	dM1		including division of all terms by 3
	$x=\frac{2n\pi}{3}-\frac{5\pi}{36}$ or $x=\frac{2n\pi}{3}+\frac{\pi}{36}$	A1 A1		Alternative answers: 1. $x=\frac{2n\pi}{3}+\frac{19\pi}{36}$ or $x=\frac{2n\pi}{3}+\frac{\pi}{36}$ 2. $x=\frac{2n\pi}{3}-\frac{\pi}{18}\pm\frac{\pi}{12}$
Total	**5**			

Q	Answer	Mark	Total	Comments
5	$r = \frac{3}{4}h$	B1	8	
	$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2 h$ Or $\frac{3}{16}\pi h^3$	M1		
	$\frac{dV}{dh} = \frac{9}{16}\pi h^2$	M1 A1		M1 for differentiating "their" V A1 for correct derivative
	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$	B1		
	$\frac{dV}{dt} = 0.06$ seen	B1		Seen anywhere
	$0.06 = \frac{9}{16}\pi(2.5)^2 \times \frac{dh}{dt}$	M1		Correct use of 0.06 and 2.5
	$\frac{dh}{dt} = \frac{32}{1875\pi}$ oe	A1		Allow $\frac{dh}{dt} = \frac{0.0170\dot{6}}{\pi}$
Total	**8**			

Q	Answer	Mark	Total	Comments
6(a)	$4\sum_{r=1}^{45} r^2$	M1	2	PI
	125580	A1		
6(b)	$\sum_{r=1}^{90} r^2$ or 247065	B1	3	Seen anywhere
	247065 – 125580	M1		
	121485	A1		NMS 1/3
Total	**5**			

Q	Answer	Mark	Total	Comments
7(a)	$\alpha + \beta = 4$	B1	**2**	
	$\alpha\beta = 7$	B1		
7(b)	$(\alpha + \beta)^2 - 2\alpha\beta$	M1	**2**	
	$= 2$	A1		
7(c)	$\alpha^4 + \beta^4 = \left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2$	M1	**2**	
	$= 2^2 - 2 \times 49 = -94$	A1		
7(d)	Sum of roots $= \alpha^2 + \beta^2 + \frac{\beta^2+\alpha^2}{\alpha\beta}$	M1	**6**	
	$= \frac{16}{7}$	A1		
	Product of roots $= \alpha^2\beta^2 + 1 + \frac{\alpha^3}{\beta} + \frac{\beta^3}{\alpha}$	M1		
	$= \alpha^2\beta^2 + 1 + \frac{\alpha^4 + \beta^4}{\alpha\beta}$	M1		
	$= \frac{256}{7}$	A1		
	$7x^2 - 16x + 256 = 0$ oe	A1		
Total	**12**			

Q	Answer	Mark	Total	Comments
8(a)	$a = 6$	B1	**2**	$a = \pm 6$ and $b = \pm 4$ SC1
	$b = 4$	B1		$a = 3$ and $b = 2$ SC1
8(b)(i)	$\dfrac{(x-4)^2}{36} - \dfrac{y^2}{16} = 1$	B1F	**1**	oe FT their a and b from **8(a)**
8(b)(ii)	$\dfrac{(x-4)^2}{36} - \dfrac{m^2x^2}{16} = 1$	M1	**3**	
	$4(x-4)^2 - 9m^2x^2 = 144$	M1		
	$4(x^2 - 8x + 16) - 9m^2x^2 = 144$ $4x^2 - 32x + 64 - 9m^2x^2 = 144$			
	$(4 - 9m^2)x^2 - 32x - 80 = 0$	A1		for last line and either of the two preceding lines (oe)
8(b)(iii)	For equal roots $32^2 + 4(4 - 9m^2)(80) = 0$	M1	**5**	oe
	$1024 + 1280 - 2880m^2 = 0$	M1		
	$m^2 = \dfrac{4}{5}$	A1		
	$y = \dfrac{2\sqrt{5}}{5}x,\ y = -\dfrac{2\sqrt{5}}{5}x$	A1, A1		
Total	**11**			

Q	Answer	Mark	Total	Comments
9(a)	Line in 1st quadrant with negative gradient	B1	**4**	Or clearly shows the perpendicular bisector of the line connecting 0 and 4 + 4i
	Crosses axes at 4 and 4i **and** extends into quadrants 2 and 4	B1		
	Circle centre 4 + 4i	B1		
	Touches their L	B1		
9(b)	$r = 2\sqrt{2}$ or furthest point = 6 + 6i	B1	**3**	PI
	$\|z\|_{max} = OP + r$ or $\|6 + 6\text{i}\|$	M1		
	$6\sqrt{2}$	A1		
9(c)	Radius perpendicular to tangent	M1	**3**	
	Angle $TOP = 30°$ or π/6	A1		
	15° or π/12	A1		
Total	**10**			

Q	Answer	Mark	Total	Comments
10(a)	$y = 1$	B1	3	
	$x = 0$	B1		
	$x = 4$	B1		
10(b)(i)	$k(x^2 - 4x) = x^2 + 6x + 5$	M1	4	
	$(k-1)x^2 - (4k+6)x - 5 = 0$	A1		
	$(4k+6)^2 + 20(k-1) \geq 0$	M1		
	$16k^2 + 68k + 16 \geq 0$ $4k^2 + 17k + 4 \geq 0$	A1		for both lines
10(b)(ii)	Equal roots: $4k^2 + 17k + 4 = 0$	M1	5	
	$k = -4$ Or $-\frac{1}{4}$	A1		
	Substituting at least one value of k into $(k-1)x^2 - (4k+6)x - 5 = 0$	M1		
	$(1, -4)$ or $\left(-2, -\frac{1}{4}\right)$	A1		
	$(1, -4)$ and $\left(-2, -\frac{1}{4}\right)$ and no extras	A1		
Total	**12**			